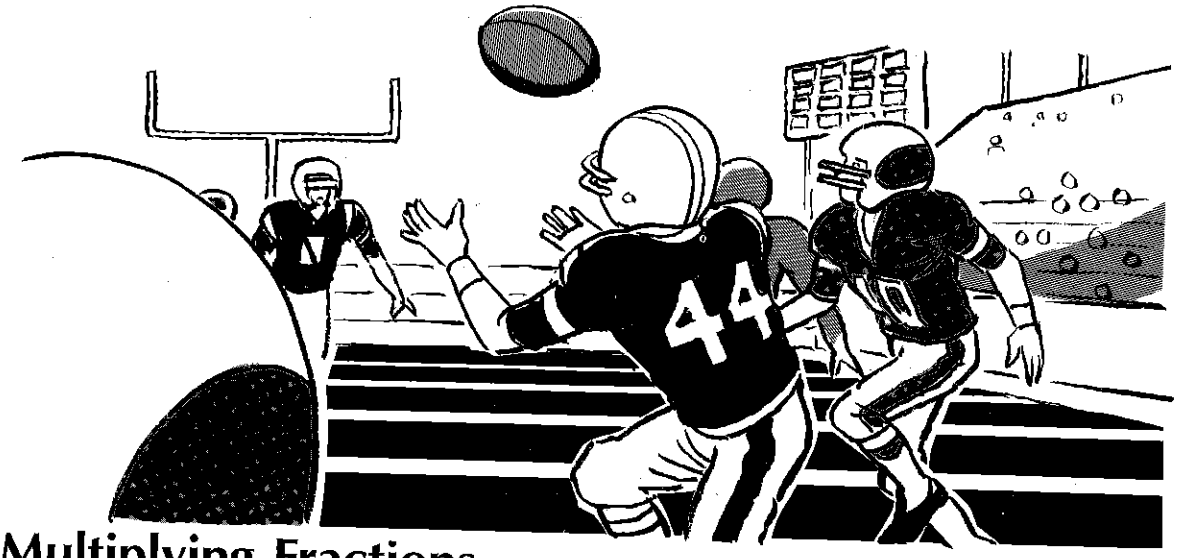
UNIT 7
Operations with Rational Numbers



Multiplying Fractions

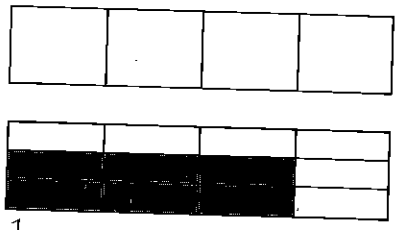
Twenty-four glasses of juice were poured for the football team. They drank $\frac{7}{8}$ of the glasses. How many glasses of juice did the team drink?

$\frac{7}{8}$ of $24 = \frac{7}{8} \times 24 = \frac{7 \times 24}{8} = \frac{168}{8} = 21$

The team drank 21 glasses of juice.

Three fourths of the football players are playing for their third year.

Two thirds of these players will graduate in June. What fraction of the team will graduate in June?



$\frac{2}{3}$ of $\frac{3}{4} = \frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12} = \frac{1}{2}$ Simplest terms

$\frac{1}{2}$ of the team will graduate in June.

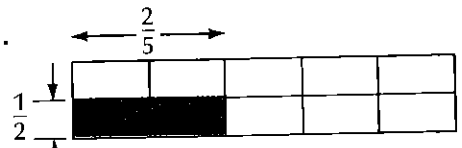
Whenever possible, simplify *before* multiplying.

Divide by the common factor 3. Divide by the common factor 2. Multiply.

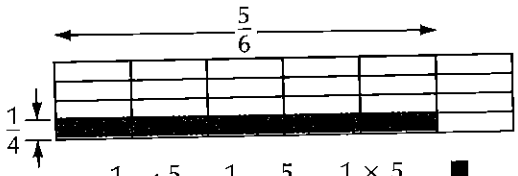
$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times \cancel{3}^{1}}{\cancel{3} \times 4_{1}} = \frac{\cancel{2}^{1} \times 1}{1 \times \cancel{4}_{2}} = \frac{1 \times 1}{1 \times 2} = \frac{1}{2}$$

EXERCISES

Find the product in simplest terms.

1. 

$\frac{1}{2}$ of $\frac{2}{5} = \frac{1}{2} \times \frac{2}{5} = \frac{1 \times 2}{2 \times 5} = \frac{\blacksquare}{\blacksquare}$

2. 

$\frac{1}{4}$ of $\frac{5}{6} = \frac{1}{4} \times \frac{5}{6} = \frac{1 \times 5}{4 \times 6} = \frac{\blacksquare}{\blacksquare}$

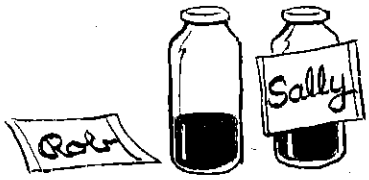
3. $\frac{1}{4}$ of 32
4. $\frac{2}{4}$ of 32
5. $\frac{3}{4}$ of 32
6. $\frac{4}{4}$ of 32
7. $\frac{2}{3} \times \frac{1}{6}$
8. $\frac{2}{3} \times \frac{2}{6}$
9. $\frac{2}{3} \times \frac{3}{6}$
10. $\frac{2}{3} \times \frac{4}{6}$
11. $\frac{3}{4}\left(\frac{1}{3}\right)$
12. $10\left(\frac{5}{6}\right)$
13. $\frac{3}{8} \times \frac{2}{3} \times \frac{4}{5}$
14. $7 \times \frac{2}{3} \times \frac{5}{7}$

PRACTICE

Find the product in simplest terms.

1. $\frac{5}{8} \times \frac{3}{4}$
2. $\frac{2}{3} \times \frac{1}{6}$
3. $4 \times \frac{5}{6}$
4. $\frac{1}{2} \times \frac{3}{5}$
5. $0 \times \frac{6}{8}$
6. $\frac{1}{7} \times \frac{1}{4}$
7. $\frac{5}{8} \times \frac{4}{7}$
8. $\frac{4}{9} \times 9$
9. $\frac{1}{2} \times \frac{3}{4}$
10. $\frac{1}{3} \times \frac{2}{5}$
11. $\frac{3}{4} \times 5$
12. $\frac{3}{5} \times \frac{1}{2}$
13. $\frac{5}{9}\left(\frac{3}{5}\right)$
14. $\frac{7}{8}\left(\frac{1}{7}\right)$
15. $\frac{3}{4}\left(\frac{2}{3}\right)$
16. $\frac{3}{10}\left(\frac{5}{9}\right)$
17. $\frac{7}{8} \times 8 \times 2$
18. $\frac{2}{3} \times \frac{2}{5} \times \frac{5}{6}$
19. $5 \times \frac{3}{4} \times \frac{4}{5}$
20. $\frac{4}{5} \times \frac{2}{3} \times \frac{3}{4}$

21. Rob drank $\frac{2}{3}$ of a bottle of orange juice. Sally drank $\frac{3}{4}$ as much as Rob. How much juice did Sally drink?



22. Wallace scored 27 points for his entries in an art competition. Two-thirds of the points were for his water-colour landscapes. How many points did he score on his other entries?

23. Janet can finish a jigsaw puzzle in 40 min. It takes Renate $\frac{3}{4}$ as long to complete the same puzzle. What fraction of an hour does it take Renate to finish a puzzle?

Multiplying Mixed Numerals

A lemonade recipe for 9 people calls for $3\frac{1}{2}$ lemons. If lemonade is made for 30 people, $\frac{30}{9}$ or $3\frac{1}{3}$ times more lemons are required.

How many lemons are needed to make lemonade for 30 people?

Lemonade for 9 people	
Boil for 10 min:	150 mL water 150 g sugar
Cool. Add juice of:	$3\frac{1}{2}$ lemons
Stir in:	one 355 mL can orange juice
Add:	2 sliced oranges 1.5 L water
Chill. Serve over ice.	

Multiply $3\frac{1}{3} \times 3\frac{1}{2}$.

Change to fractions. Divide by the common factor 2. Multiply.

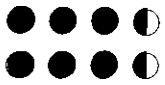
$$3\frac{1}{3} \times 3\frac{1}{2} = \frac{10}{3} \times \frac{7}{2} = \frac{\overset{5}{\cancel{10}}}{3} \times \frac{7}{\cancel{2}_{1}} = \frac{5 \times 7}{3 \times 1} = \frac{35}{3} = 11\frac{2}{3}$$

For 30 people, $11\frac{2}{3}$ lemons are needed.

EXERCISES

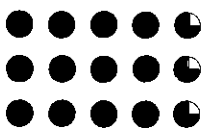
Find the product in simplest terms.

1.



$2 \times 3\frac{1}{2} = 2 \times \frac{7}{2}$

2.



$3 \times 4\frac{3}{4} = 3 \times \frac{19}{4}$

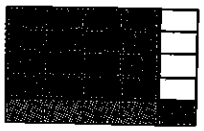
3. $1\frac{1}{3}$

$\frac{1}{6}$

$\frac{1}{6} \times 1\frac{1}{3} = \frac{1}{6} \times \frac{4}{3}$

4. $1\frac{1}{4}$

$1\frac{1}{4}$



$1\frac{1}{4} \times 1\frac{1}{4} = \frac{5}{4} \times \frac{5}{4}$

5. $5 \times 1\frac{1}{3}$

6. $2 \times 5\frac{2}{3}$

7. $2\frac{3}{5} \times 4$

8. $6\frac{1}{3} \times 5$

9. $\frac{3}{4} \times 2\frac{1}{2}$

10. $\frac{5}{8} \times 1\frac{3}{5}$

11. $2\frac{4}{5} \times \frac{3}{4}$

12. $3\frac{4}{7} \times \frac{3}{5}$

13. $5\frac{1}{2} \times 3\frac{1}{3}$

14. $7\frac{1}{3} \times 3\frac{2}{3}$

15. $2\frac{5}{6} \times 7\frac{1}{2}$

16. $6\frac{2}{3} \times 3\frac{1}{2}$

17. $\frac{3}{7}\left(5\frac{3}{4}\right)$

18. $9\left(3\frac{1}{3}\right)$

19. $2\frac{1}{7}(5)$

20. $8\frac{1}{3}\left(\frac{3}{5}\right)$

21. $3\frac{1}{3} \times 1\frac{1}{5} \times 2\frac{1}{2}$

22. $4 \times 3\frac{1}{4} \times 1\frac{1}{3}$

23. $5 \times 2\frac{2}{3} \times \frac{5}{8}$

PRACTICE

Multiply. Write the product in simplest terms.

1. $6 \times 1\frac{3}{8}$
2. $7\frac{1}{3} \times \frac{1}{4}$
3. $6\frac{5}{6} \times 2$
4. $3\frac{3}{8} \times \frac{5}{6}$
5. $5\frac{1}{3} \times 2\frac{1}{4}$
6. $\frac{7}{9} \times 6\frac{3}{5}$
7. $2\frac{1}{6} \times 7\frac{1}{2}$
8. $8 \times 1\frac{3}{8}$
9. $\frac{4}{9}\left(3\frac{3}{4}\right)$
10. $2\frac{5}{8}(7)$
11. $6\frac{2}{3}\left(3\frac{1}{2}\right)$
12. $\left(7\frac{1}{2}\right)\frac{5}{6}$
13. $1\frac{1}{2} \times 3\frac{1}{3} \times 5$
14. $4\frac{1}{2} \times \frac{5}{9} \times 2\frac{1}{2}$
15. $4\frac{2}{3} \times 1\frac{3}{5} \times \frac{5}{7}$
16. $2\frac{3}{8} \times 4 \times 1\frac{3}{7}$
17. $3\frac{1}{8} \times 3\frac{4}{5} \times 3\frac{1}{3}$
18. $2\frac{4}{5} \times 2\frac{1}{7} \times 2\frac{3}{4}$
19. $3 \times \frac{4}{9} \times 5\frac{3}{8}$
20. $\frac{7}{10} \times 3\frac{1}{5} \times \frac{1}{8}$
21. $5\frac{1}{2} \times 3\frac{1}{2} \times 2\frac{2}{5}$

Solve.

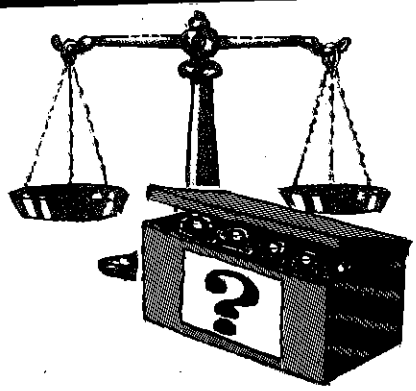
22. Using the recipe on the previous page, how many lemons are required to make lemonade for 24 people?

23. The cooking time for hot cereal must be increased by one and one half minutes for every 300 m of elevation above 900 m. The elevation of Lake Louise, Canada's highest town, is about 1500 m. If the cooking time is normally 8 min, how long will it be in Lake Louise?

24. It takes Joan $2\frac{1}{2}$ hours to assemble a graphic art design at her father's printing business. On Saturday, she had time to assemble 5 designs. How long did she work?

25. Tony works $\frac{3}{4}$ of an hour to paint lines on 1 km of roadway. How long will it take him to paint 5.3 km?

A Delicate Balance



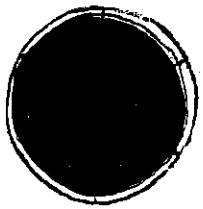
Lawrence has a set of 5 masses and a two-pan balance. With them, he can find the mass of any object from 1 g to 121 g.

He can use the masses in any combination on one pan or both pans.

What are Lawrence's 5 masses?

Adding and Subtracting Fractions

Tom and Doris ordered a pizza.
Tom took half of it and Doris took a third.
What fraction of the pizza did they take?



The pizza can be cut into 6 equal pieces.

Decide.

Add: $\frac{1}{2} + \frac{1}{3}$.

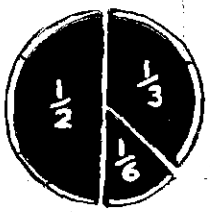
Find the **Least Common Denominator (LCD)** of $\frac{1}{2}$ and $\frac{1}{3}$

$2 = \{2, 4, 6, 8, \ldots\}$

$3 = \{3, 6, 9, 12, \ldots\}$

LCD = 6

$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{3+2}{6} = \frac{5}{6}$$

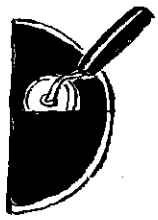


Tom and Doris took $\frac{5}{6}$ of the pizza.

Tom gave a part of his half pizza to Joe.

Joe's piece was $\frac{1}{4}$ of the whole pizza.

How much pizza was left for Tom?



Decide.

Find the difference $\frac{1}{2} - \frac{1}{4}$ using the **LCD**, 4.

$$\frac{1}{2} - \frac{1}{4} = \frac{2}{4} - \frac{1}{4} = \frac{2-1}{4} = \frac{1}{4}$$

$\frac{1}{4}$ of the pizza is left for Tom.

EXERCISES

Write the LCD for each pair of fractions.

1. $\frac{1}{7}, \frac{3}{4}$ **2.** $\frac{5}{6}, \frac{7}{9}$ **3.** $\frac{11}{12}, \frac{1}{18}$ **4.** $\frac{3}{10}, \frac{14}{15}$ **5.** $\frac{3}{20}, \frac{7}{8}$

Add. Write the sum in simplest terms.

6. $\frac{7}{11} + \frac{6}{11}$ **7.** $\frac{5}{8} + \frac{7}{8}$ **8.** $\frac{9}{20} + \frac{16}{20}$ **9.** $\frac{41}{50} + \frac{34}{50}$

10. $\frac{2}{3} + \frac{1}{4}$ **11.** $\frac{5}{6} + \frac{3}{4}$ **12.** $\frac{1}{8} + \frac{7}{12}$ **13.** $\frac{7}{9} + \frac{11}{12}$

Subtract. Write the sum in simplest terms.

14. $\frac{5}{6} - \frac{1}{6}$ **15.** $\frac{9}{10} - \frac{4}{10}$ **16.** $\frac{5}{7} - \frac{2}{7}$ **17.** $\frac{18}{25} - \frac{13}{25}$

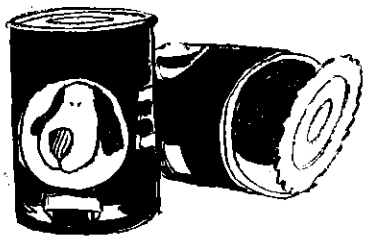
18. $\frac{7}{12} - \frac{1}{3}$ **19.** $\frac{3}{4} - \frac{2}{5}$ **20.** $\frac{3}{8} - \frac{1}{4}$ **21.** $\frac{5}{6} - \frac{3}{4}$

PRACTICE

Compute. Write the result in simplest terms.

1. $\frac{3}{5} + \frac{1}{5}$
2. $\frac{7}{10} + \frac{1}{10}$
3. $\frac{2}{9} + \frac{5}{9}$
4. $\frac{4}{9} + \frac{2}{9}$
5. $\frac{2}{3} + \frac{2}{3}$
6. $\frac{7}{8} + \frac{5}{8}$
7. $\frac{3}{4} + \frac{3}{4}$
8. $\frac{5}{9} + \frac{4}{9}$
9. $\frac{1}{3} + \frac{1}{12}$
10. $\frac{3}{5} + \frac{3}{10}$
11. $\frac{3}{4} + \frac{5}{12}$
12. $\frac{5}{6} + \frac{1}{3}$
13. $\frac{1}{3} + \frac{1}{4}$
14. $\frac{1}{2} + \frac{3}{5}$
15. $\frac{9}{10} + \frac{1}{8}$
16. $\frac{3}{4} + \frac{7}{10}$
17. $\frac{7}{8} + \frac{1}{3}$
18. $\frac{5}{6} + \frac{3}{4}$
19. $\frac{5}{6} + \frac{1}{10}$
20. $\frac{2}{3} + \frac{3}{10}$
21. $\frac{7}{10} - \frac{3}{10}$
22. $\frac{3}{4} - \frac{1}{4}$
23. $\frac{5}{6} - \frac{1}{6}$
24. $\frac{7}{12} - \frac{5}{12}$
25. $\frac{4}{5} - \frac{3}{10}$
26. $\frac{5}{6} - \frac{2}{3}$
27. $\frac{3}{8} - \frac{1}{4}$
28. $\frac{3}{5} - \frac{1}{10}$
29. $\frac{3}{4} - \frac{2}{3}$
30. $\frac{1}{2} - \frac{1}{3}$
31. $\frac{2}{3} - \frac{1}{4}$
32. $\frac{5}{8} - \frac{1}{3}$
33. $\frac{3}{10} - \frac{1}{4}$
34. $\frac{9}{10} - \frac{1}{3}$
35. $\frac{7}{10} - \frac{1}{6}$
36. $\frac{2}{3} - \frac{3}{10}$
37. $\frac{2}{3}\left(\frac{1}{2} - \frac{1}{4}\right)$
38. $\frac{5}{8} - \left(\frac{1}{6} + \frac{1}{3}\right)$
39. $\frac{1}{8} + \left(\frac{3}{4} - \frac{1}{3}\right)$
40. $\frac{5}{9} - \left(\frac{1}{3} - \frac{1}{6}\right)$
41. $\frac{2}{3} - \frac{4}{7} + \frac{1}{2}$
42. $\frac{7}{8} - \frac{2}{3} + \frac{5}{12}$
43. $\frac{1}{3}\left(\frac{5}{6} - \frac{1}{2}\right)$
44. $\frac{7}{9} \times \frac{3}{4} + \frac{1}{5}$
45. $\frac{1}{2}\left(\frac{5}{12} - \frac{5}{18}\right)$

Solve.



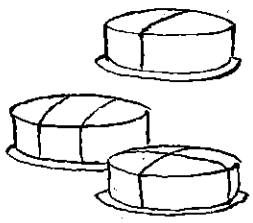
46. Ron's terrier eats $\frac{1}{2}$ can of dog food every day.
His poodle eats $\frac{2}{3}$ of a can.
How much dog food does Ron need in a week?

47. John invested in a mining stock.
Over three days, the stock gained $\frac{5}{8}$, lost $\frac{1}{4}$, and gained $\frac{1}{2}$.
What was the net increase in the value of the stock?

Pattern Predictions

When 1 cut is made in a cake,
2 pieces are formed.
Two cuts can form 3 or 4 pieces.

Predict the greatest number of pieces that can be formed by 10 straight cuts.



Cuts	Greatest Number of Pieces

Adding Mixed Numerals

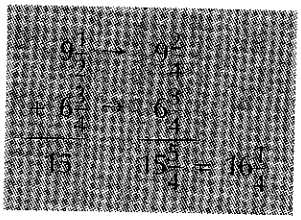
Michelle swims for $9\frac{1}{2}$ hours during the school week and $6\frac{3}{4}$ hours on weekends. How much time does Michelle swim each week in all?

Change to fractions. Find the **LCD**. Add the numerators.

$$9\frac{1}{2} + 6\frac{3}{4} = \frac{19}{2} + \frac{27}{4} = \frac{38}{4} + \frac{27}{4} = \frac{38 + 27}{4} = \frac{65}{4} = 16\frac{1}{4}$$

Alternatively, you can add the whole numbers and fractions separately.

$$9\frac{1}{2} + 6\frac{3}{4} = (9 + 6) + \left(\frac{1}{2} + \frac{3}{4}\right)$$
$$= 15 + \frac{2}{4} + \frac{3}{4}$$
$$= 15 + \frac{5}{4} = 15 + 1\frac{1}{4} = 16\frac{1}{4}$$



Michelle swims $16\frac{1}{4}$ hours each week.

EXERCISES

Find the LCD for each pair of mixed numerals.

1. $2\frac{7}{9}, 4\frac{3}{4}$ 2. $5\frac{1}{8}, 1\frac{19}{20}$ 3. $6\frac{1}{5}, 3\frac{3}{4}$ 4. $9\frac{1}{3}, 2\frac{7}{20}$ 5. $1\frac{5}{6}, 3\frac{11}{15}$

Change the fraction to a mixed numeral in simplest terms.

6. $\frac{50}{6}$ 7. $\frac{105}{25}$ 8. $\frac{42}{4}$ 9. $\frac{78}{8}$ 10. $\frac{94}{12}$

Add. Write the sum in simplest terms.

11. $1\frac{2}{3} + 1\frac{2}{3} = \frac{\blacksquare}{3} + \frac{\blacksquare}{3} = \frac{\blacksquare + \blacksquare}{\blacksquare} = \blacksquare$ 12. $1\frac{1}{5} + 1\frac{2}{5} = \frac{\blacksquare}{5} + \frac{\blacksquare}{5} = \frac{\blacksquare + \blacksquare}{5} = \blacksquare$

13. $3\frac{2}{3} + 1\frac{2}{3}$ 14. $6\frac{7}{10} + 8\frac{9}{10}$ 15. $2\frac{3}{4} + 5\frac{3}{4}$ 16. $3\frac{5}{7} + 4\frac{3}{7}$

17. $5\frac{4}{5} + 3\frac{7}{10} = \frac{\blacksquare}{5} + \frac{\blacksquare}{10} = \frac{\blacksquare}{10} + \frac{\blacksquare}{10} = \frac{\blacksquare + \blacksquare}{10} = \blacksquare$

18. $2\frac{7}{8} + 1\frac{1}{4} = \frac{\blacksquare}{8} + \frac{\blacksquare}{4} = \frac{\blacksquare}{8} + \frac{\blacksquare}{8} = \frac{\blacksquare + \blacksquare}{8} = \blacksquare$

19. $3\frac{1}{6} + 8\frac{1}{4}$ 20. $3\frac{3}{10} + 4\frac{5}{8}$ 21. $5\frac{3}{16} + 7\frac{3}{4}$ 22. $6\frac{2}{3} + 6\frac{1}{2}$

PRACTICE

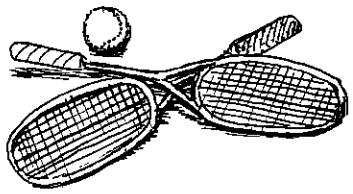
Compute. Write the result in simplest terms.

1. $7\frac{5}{8} + 5\frac{7}{8}$
2. $3\frac{5}{6} + 2\frac{2}{3}$
3. $4\frac{5}{12} + 3\frac{7}{12}$
4. $4\frac{1}{2} + 1\frac{3}{8}$
5. $1\frac{11}{20} + \frac{3}{4}$
6. $4\frac{7}{9} + 4\frac{5}{9}$
7. $3\frac{1}{6} + 8$
8. $10\frac{5}{6} + 1\frac{4}{9}$
9. $3\frac{7}{15} + 4\frac{7}{12}$
10. $2\frac{5}{7} + \frac{3}{4}$
11. $7\frac{4}{5} + 3\frac{5}{6}$
12. $14 + 12\frac{4}{9}$
13. $2\frac{2}{3} + 7\frac{1}{6} + 9\frac{1}{3}$
14. $7\frac{1}{2} + \frac{3}{4} + 5\frac{3}{4}$
15. $4\frac{3}{5} + 8\frac{7}{10} + 3\frac{1}{2}$
16. $9\frac{3}{8} + 1\frac{1}{4} + 6\frac{5}{16}$
17. $3\frac{7}{8} + \frac{5}{6} + 7\frac{3}{4}$
18. $12\frac{3}{4} + 23\frac{2}{3} + 14\frac{1}{2}$
19. $3\frac{3}{4} + 8\frac{1}{2} \times 6$
20. $\frac{7}{8}\left(5\frac{1}{2} + 5\frac{5}{6}\right)$
21. $10\frac{1}{2} + 3\frac{4}{5} + \frac{1}{3}$

Solve.

22. After the bake sale, there were 3 whole pies and 9 pieces left over. Each piece was one sixth of a pie. What was the total amount of pie left over.

23. Mindy played in singles tennis tournaments for $2\frac{1}{4}$ years. For the last 18 months, she has only entered doubles tournaments. For how many years altogether has Mindy been playing tournament tennis?



24. Anita worked $3\frac{1}{2}$ hours on Thursday night, $2\frac{1}{4}$ hours Friday night, and $5\frac{3}{4}$ hours on Saturday. Altogether, how many hours did Anita work?

25. Paul usually practises the piano for $1\frac{3}{4}$ hours each day during the week and for $3\frac{1}{2}$ hours on each day of the weekend. How many hours in all does Paul usually practise in a week?

Computer Program Mixup

The statements at the right form a BASIC computer program that changes mixed numerals to fractions. Put the statements in order and assign them line numbers so the program will work.

- `INPUT "ENTER THE WHOLE NUMBER:";A`
- `END`
- `LET M = A * C`
- `INPUT "ENTER THE DENOMINATOR:";C`
- `PRINT "THE FRACTION IS ";N;"/";C`
- `LET N = M + B`
- `INPUT "ENTER THE NUMERATOR:";B`

Subtracting Mixed Numerals

At age 50, the average male is expected to live about $24\frac{3}{4}$ years more and the average female is expected to live about $30\frac{1}{2}$ years more.

About how much longer is the average 50-year-old female expected to live than the average 50-year-old male?

Change to fractions. Find the **LCD**. Subtract the numerators.

$$30\tfrac{1}{2} - 24\tfrac{3}{4} = \tfrac{61}{2} - \tfrac{99}{4} = \tfrac{122}{4} - \tfrac{99}{4} = \tfrac{122 - 99}{4} = \tfrac{23}{4} = 5\tfrac{3}{4}$$

Alternatively, you can subtract whole numbers and fractions separately. You may have to **regroup.**

Regroup.

$$30\tfrac{1}{2} - 24\tfrac{3}{4} = (30 - 24) + \left(\tfrac{1}{2} - \tfrac{3}{4}\right) = (29 - 24) + \left(1\tfrac{1}{2} - \tfrac{3}{4}\right)$$
$$= 5 + \left(\tfrac{3}{2} - \tfrac{3}{4}\right) = 5 + \tfrac{3}{4} = 5\tfrac{3}{4}$$

$$\begin{array}{rrr} 30\frac{1}{2} \rightarrow & 30\frac{2}{4} \rightarrow & 29\frac{6}{4} \\ -24\frac{3}{4} \rightarrow & -24\frac{3}{4} \rightarrow & -24\frac{3}{4} \\ \hline & & 5\frac{3}{4} \end{array}$$

The average female is expected to live $5\frac{3}{4}$ years longer.

EXERCISES

Express each whole number as a fraction with the denominator given.

1. $1 = \frac{\blacksquare}{4}$
2. $3 = \frac{\blacksquare}{5}$
3. $2 = \frac{\blacksquare}{6}$
4. $9 = \frac{\blacksquare}{3}$
5. $20 = \frac{\blacksquare}{2}$
6. $32 = \frac{\blacksquare}{4}$
7. $24 = \frac{\blacksquare}{8}$
8. $30 = \frac{\blacksquare}{3}$

Subtract. Write the difference in simplest terms.

9. $4 - 2\frac{1}{3} = \frac{\blacksquare}{3} - \frac{\blacksquare}{3} = \frac{\blacksquare - \blacksquare}{3} = \blacksquare$
10. $3 - 1\frac{3}{4} = \frac{\blacksquare}{4} - \frac{\blacksquare}{4} = \frac{\blacksquare - \blacksquare}{4} = \blacksquare$
11. $1\frac{5}{6} - \frac{1}{3} = \frac{\blacksquare}{6} - \frac{\blacksquare}{6} = \frac{\blacksquare - \blacksquare}{6} = \blacksquare$
12. $1\frac{3}{10} - 1\frac{1}{5} = \frac{13}{10} - \frac{12}{10} = \frac{13 - 12}{10} = \blacksquare$
13. $4 - 1\frac{2}{9}$
14. $11 - 7\frac{5}{6}$
15. $3 - 1\frac{3}{4}$
16. $18 - 9\frac{5}{8}$
17. $5\frac{7}{10} - 3\frac{9}{10}$
18. $15\frac{1}{3} - 7\frac{2}{3}$
19. $18\frac{5}{11} - 7\frac{10}{11}$
20. $14\frac{3}{5} - 8\frac{4}{5}$
21. $5\frac{2}{3} - 2\frac{5}{6}$
22. $7\frac{1}{4} - \frac{5}{8}$
23. $6\frac{3}{10} - 4\frac{4}{5}$
24. $3\frac{3}{8} - \frac{2}{3}$

PRACTICE

Compute. Write the result in simplest terms.

1. $7 - 2\frac{7}{12}$
2. $9 - 3\frac{1}{4}$
3. $5\frac{3}{10} - 2\frac{7}{10}$
4. $6\frac{3}{7} - 2\frac{5}{7}$
5. $11\frac{5}{12} - 2\frac{7}{12}$
6. $10 - 4\frac{5}{11}$
7. $8\frac{3}{5} - 3\frac{3}{10}$
8. $7\frac{3}{7} - 4\frac{10}{14}$
9. $8\frac{5}{8} - 1\frac{3}{4}$
10. $3\frac{9}{10} - 1\frac{1}{5}$
11. $6 - 2\frac{7}{15}$
12. $3\frac{1}{3} - 2\frac{5}{6}$
13. $7\frac{3}{4} - 5\frac{5}{12}$
14. $10\frac{8}{15} - 7\frac{4}{15}$
15. $7\frac{2}{3} - 5\frac{4}{9}$
16. $9\frac{1}{6} - 5\frac{3}{4}$
17. $4\frac{1}{8} - 2\frac{5}{8} + 1\frac{3}{8}$
18. $12\frac{1}{2} - 3\frac{5}{8} - 2\frac{1}{4}$
19. $20\frac{1}{4} - 3\frac{1}{2} + \frac{7}{8}$
20. $15\frac{1}{2} + 2\frac{7}{8} + 3\frac{1}{4}$
21. $10 - 1\frac{7}{8} + 2\frac{5}{16}$
22. $4\frac{1}{2} - 2\frac{3}{4} + 5$
23. $12\frac{1}{2} - 6\frac{1}{4} \times \frac{1}{2}$
24. $\left(8 - 5\frac{2}{3}\right)4\frac{1}{4}$
25. $5\frac{1}{5} + \left(9\frac{1}{2} - 6\frac{3}{4}\right)$

26. Golden Meridian stocks opened one day at $\$15\frac{1}{8}$. During the day, it dropped $\$1\frac{1}{4}$, gained $\$2\frac{3}{8}$, and then went down another $\$1\frac{1}{2}$. What was its closing price?

27. Gina lived in Switzerland for $1\frac{1}{3}$ years and in Spain for $3\frac{1}{2}$ years. For the rest of her life, Gina has lived in Canada. If she is now 14 years old, for how many years has Gina lived in Canada?

REVIEW

Find the product in simplest terms.

1. $\frac{5}{8} \times 10$
2. $\frac{1}{3} \times \frac{1}{7}$
3. $\frac{12}{25} \times \frac{5}{9}$
4. $\frac{3}{5} \times \frac{7}{2} \times \frac{10}{21}$
5. $7 \times 3\frac{1}{4}$
6. $\frac{7}{8} \times 1\frac{1}{3}$
7. $3\frac{4}{5} \times 1\frac{3}{10}$
8. $3 \times 7\frac{1}{2} \times 1\frac{2}{5}$

Find the sum or difference in simplest terms.

9. $\frac{5}{8} + \frac{1}{2}$
10. $\frac{5}{9} + \frac{1}{2}$
11. $\frac{2}{3} - \frac{5}{12}$
12. $\frac{3}{4} - \frac{2}{5}$
13. $2\frac{3}{8} + 4\frac{1}{2}$
14. $3\frac{5}{6} + 2\frac{7}{4}$
15. $10\frac{11}{12} + 2\frac{1}{15}$
16. $3\frac{5}{8} + \frac{1}{2} + 2\frac{1}{2}$

Find the difference in simplest terms.

17. $8 - 3\frac{2}{3}$
18. $3\frac{4}{5} - \frac{9}{10}$
19. $9\frac{1}{9} - 3\frac{1}{3}$
20. $4\frac{7}{15} - 1\frac{5}{6}$

Dividing Fractions

Two numbers are **reciprocals** if their product is 1.

5 and $\frac{1}{5}$ are reciprocals. $5 \times \frac{1}{5} = \frac{\cancel{5}^{1} \times 1}{\cancel{5}_{1}} = \frac{1}{1} = 1$

$2\frac{3}{4}$ and $\frac{4}{11}$ are reciprocals. $2\frac{3}{4} \times \frac{4}{11} = \frac{11}{4} \times \frac{4}{11} = \frac{\cancel{11}^{1} \times \cancel{4}^{1}}{\cancel{4}_{1} \times \cancel{11}_{1}} = \frac{1}{1} = 1$

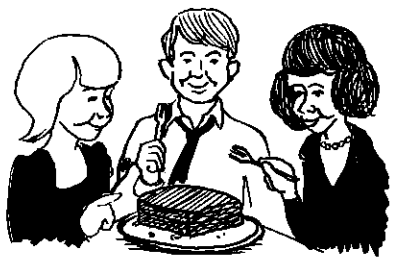
To divide fractions, multiply by the *reciprocal* of the divisor.

If 20 dollars are equally shared by 4 people, how much money does each person get?

$20 \div 4 = 5$ or $20 \times \frac{1}{4} = 5$ 4 and $\frac{1}{4}$ are reciprocals.

Each person gets 5 dollars.

After a school party, three eighths of a cake is left. If the remaining portion is equally divided among 3 people, what part of the whole cake does each person get?



Decide. Divide $\frac{3}{8}$ by 3.

Multiply by the reciprocal. Simplify.

$$\frac{3}{8} \div 3 = \frac{3}{8} \times \frac{1}{3} = \frac{\cancel{3}^{1} \times 1}{8 \times \cancel{3}_{1}} = \frac{1 \times 1}{8 \times 1} = \frac{1}{8}$$

Each person gets $\frac{1}{8}$ of the cake.

Study these other examples of division by fractions and mixed numerals.

How many sixths are there in $\frac{2}{3}$? $\frac{2}{3} \div \frac{1}{6} = \frac{2}{3} \times \frac{6}{1} = \frac{2 \times \cancel{6}^{2}}{\cancel{3}_{1} \times 1} = \frac{4}{1} = 4$

Share $2\frac{1}{3}$ pies among 7 people. $2\frac{1}{3} \div 7 = \frac{7}{3} \div 7 = \frac{7}{3} \times \frac{1}{7} = \frac{\cancel{7}^{1} \times 1}{3 \times \cancel{7}_{1}} = \frac{1}{3}$

Split $3\frac{3}{4}$ hours into $2\frac{1}{2}$ time periods. $3\frac{3}{4} \div 2\frac{1}{2} = \frac{15}{4} \div \frac{5}{2} = \frac{\cancel{15}^{3}}{4} \times \frac{2}{\cancel{5}_{1}} = \frac{\cancel{15}^{3} \times \cancel{2}^{1}}{\cancel{4}_{2} \times \cancel{5}_{1}} = \frac{3}{2} = 1\frac{1}{2}$

EXERCISES

Are the pairs of numbers reciprocals of each other? Write *yes* or *no*.

1. $\frac{1}{3}, 1$ **2.** $\frac{5}{6}, 1\frac{1}{5}$ **3.** $2\frac{7}{8}, \frac{8}{23}$ **4.** $9, \frac{1}{9}$ **5.** $\frac{5}{9}, 1\frac{3}{5}$

Write the reciprocal of each in simplest terms.

6. $\frac{3}{8}$ **7.** $1\frac{5}{8}$ **8.** $\frac{5}{6}$ — **9.** $2\frac{1}{2}$ **10.** 7

Evaluate n in each equation.

11. $\frac{7}{9} \times n = 1$ **12.** $19n = 1$ **13.** $3\frac{2}{3} \times \frac{3}{11} = n$

14. $\frac{3}{4}n = 1$ **15.** $\frac{2}{7} \times 3\frac{1}{2} = n$ **16.** $n \times 1\frac{4}{5} = 1$

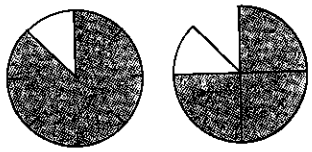
Find the quotient in simplest terms.

17. Divide $\frac{2}{3}$ into 4 parts. **18.** How many $\frac{1}{3}$s in 3?

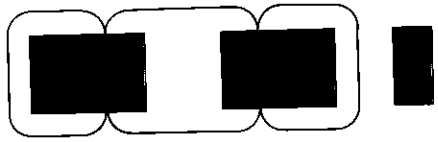
$\frac{2}{3} \div 4 = \frac{2}{3} \times \frac{1}{4} = \frac{2 \times 1}{3 \times 4} =$ ■

$3 \div \frac{1}{3} = 3 \times \frac{3}{1} = \frac{3 \times 3}{1} =$ ■

19. How many $\frac{1}{4}$s in $\frac{7}{8}$? **20.** How many $\frac{2}{3}$s in $2\frac{1}{3}$?



$\frac{7}{8} \div \frac{1}{4} =$ ■ $\times$ ■ $=$ $\frac{\blacksquare \times \blacksquare}{\blacksquare} =$ ■



$2\frac{1}{3} \div \frac{2}{3} =$ ■ $\div$ ■ $=$ ■ $\times$ ■ $=$ $\frac{\blacksquare \times \blacksquare}{\blacksquare \times \blacksquare} =$ ■

21. $\frac{3}{8} \div 4$ **22.** $\frac{2}{3} \div 2$ **23.** $\frac{5}{7} \div 10$ **24.** $\frac{9}{10} \div 4$

25. $8 \div \frac{2}{3}$ **26.** $6 \div \frac{3}{4}$ **27.** $9 \div \frac{5}{6}$ **28.** $12 \div \frac{7}{8}$

29. $\frac{3}{5} \div \frac{2}{3}$ **30.** $\frac{5}{7} \div \frac{3}{4}$ **31.** $\frac{3}{10} \div \frac{2}{5}$ **32.** $\frac{6}{11} \div \frac{1}{3}$

33. $4\frac{1}{2} \div \frac{1}{7}$ **34.** $1\frac{3}{5} \div 2\frac{3}{4}$ **35.** $12 \div 1\frac{1}{5}$ **36.** $10\frac{2}{3} \div 2\frac{5}{8}$

37. How many $\frac{2}{3}$s in $\frac{3}{4}$? **38.** How many $\frac{2}{5}$s in $\frac{9}{10}$?

39. How many $\frac{1}{2}$s in $2\frac{7}{8}$? **40.** How many $\frac{1}{6}$s in $4\frac{2}{3}$?

PRACTICE

Write the reciprocal for each.

1. $\frac{3}{5}$
2. $3\frac{8}{9}$
3. 5
4. $\frac{2}{3}$
5. 4
6. $3\frac{1}{8}$
7. $\frac{6}{7}$
8. $1\frac{3}{5}$
9. $\frac{3}{16}$
10. $4\frac{7}{10}$

Evaluate n in each equation.

11. $1 = 3\frac{7}{8} \times n$
12. $50n = 1$
13. $\frac{5}{4} \times \frac{4}{5} = 10n$
14. $\frac{3}{4} \times \frac{9}{8} \times \frac{8}{9} = n$
15. $n \times \frac{6}{1} \times \frac{1}{6} = \frac{2}{3}$
16. $\frac{11}{4} \times \frac{4}{11} = n \times 2\frac{1}{2}$

Find the quotient in simplest terms.

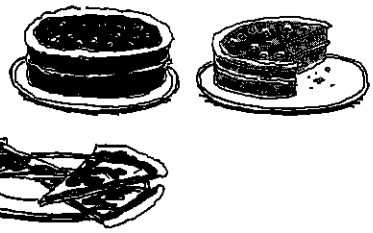
17. $\frac{10}{11} \div 5$
18. $\frac{5}{8} \div \frac{3}{4}$
19. $\frac{3}{8} \div 12$
20. $4 \div \frac{5}{6}$
21. $\frac{5}{9} \div \frac{2}{3}$
22. $\frac{7}{9} \div 6$
23. $\frac{4}{7} \div \frac{3}{8}$
24. $15 \div \frac{5}{16}$
25. $\frac{10}{21} \div 8$
26. $\frac{5}{6} \div \frac{2}{5}$
27. $2 \div \frac{11}{25}$
28. $\frac{3}{8} \div \frac{1}{2}$
29. $\frac{7}{12} \div \frac{4}{9}$
30. $\frac{5}{12} \div \frac{1}{6}$
31. $8\frac{3}{4} \div \frac{1}{3}$
32. $\frac{11}{21} \div \frac{1}{7}$
33. $7\frac{1}{3} \div 3\frac{2}{3}$
34. $\frac{11}{12} \div \frac{7}{25}$
35. $6\frac{2}{3} \div 2$
36. $8\frac{4}{5} \div 2\frac{1}{2}$
37. $6\frac{2}{7} \div 2\frac{4}{5}$
38. $1\frac{7}{10} \div \frac{1}{2}$
39. $4\frac{2}{3} \div 4\frac{1}{5}$
40. $4\frac{5}{9} \div 4\frac{5}{9}$

Compute. Write the result in simplest terms.

41. $16 \times \frac{3}{5} \div \frac{8}{9}$
42. $3\frac{2}{5} \div \left(2\frac{1}{2} + 2\frac{1}{4}\right)$
43. $\frac{3}{8} \div 2\frac{1}{2} + \frac{4}{5}$
44. $8\frac{1}{3} \div \left(6\frac{1}{2} - 4\frac{3}{4}\right)$
45. $\left(7\frac{1}{5} - 4\frac{2}{3}\right) \div \left(6 \times \frac{3}{5}\right)$
46. $\dfrac{\frac{2}{3} \times 7\frac{1}{2} - \frac{5}{6}}{\frac{6}{25}}$

Solve.

47. Ten people share equally $1\frac{2}{3}$ cakes. What is the size of each piece?



48. Pat plans to serve each of his guests $\frac{3}{8}$ of a pizza. If he bakes 4 pizzas, how many guests can he serve? How much pizza will be left?

49. What number has no reciprocal?

Calculator Fractions

Complicated problems can be completed on calculators with memory keys such as [M+] , [MR] , and [MC] .

[M+] adds the display to the memory.

[MR] displays the contents of the memory.

To calculate: *Press:*

$1\frac{1}{2} + 3\frac{1}{3} = 4\frac{5}{6}$ [1] [÷] [2] [+] [1] [M+] [1] [−] [3] [+] [3] [+] [MR] [=] → 4.8333333

$6\frac{1}{5} - 1\frac{3}{4} = 4\frac{9}{20}$ [3] [÷] [4] [+] [1] [M+] [1] [÷] [5] [+] [6] [−] [MR] [=] → 4.45

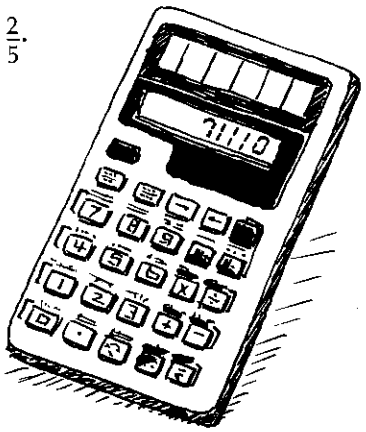
$7\frac{1}{8} \times 2\frac{2}{3} = 19$ [1] [÷] [8] [+] [7] [M+] [2] [÷] [3] [+] [2] [×] [MR] [=] → 18.999999

$3\frac{1}{5} \div 1\frac{3}{5} = 2$ [3] [÷] [5] [+] [1] [M+] [1] [÷] [5] [+] [3] [÷] [MR] [=] → 2

Compute on a calculator using [M+] and [MR] .
Write the result as a fraction in simplest terms.

1. $2\frac{1}{4} + 3\frac{2}{5}$
2. $16\frac{1}{2} - 11\frac{1}{3}$
3. $4\frac{3}{4} \times 9\frac{2}{5}$
4. $3\frac{6}{20} \div 1\frac{1}{2}$
5. $12\frac{1}{2} - 6\frac{7}{8}$
6. $\frac{11}{16} \div \frac{1}{8}$
7. $4\frac{1}{2} + 2\frac{2}{3}$
8. $4\frac{1}{4} \times 3\frac{1}{3}$

Barb, Roberto, and Todd used a calculator to multiply $\frac{7}{8} \times \frac{2}{5}$.
Each used a different way to find the product.



9. Which keys did Barb press?
   [7] [?] [8] [?] [2] [?] [5] [=]
10. Which keys did Roberto press?
   [8] [?] [5] [?] [7] [?] [2] [?] [?] [=]
11. Which keys did Todd press?
   [.] [?] [?] [?] [×] [.] [?] [=]
12. Which method do you prefer? Why?

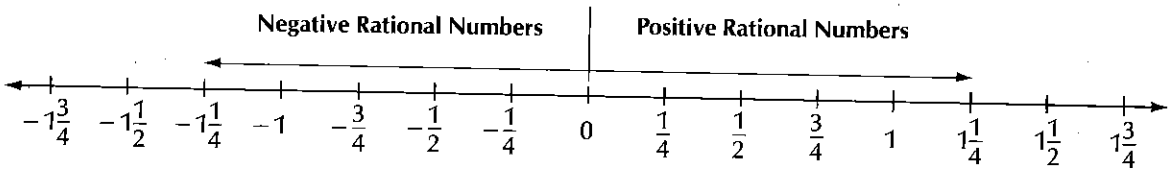
Use either Barb's, Roberto's, or Todd's method to multiply these fractions. Write the result as a fraction in simplest terms.

13. $\frac{3}{4} \times \frac{3}{8}$
14. $\frac{11}{25} \times \frac{4}{2}$
15. $\frac{4}{5} \times \frac{11}{8}$
16. $\frac{3}{50} \times \frac{7}{5}$
17. $\frac{5}{8} \times \frac{1}{2}$
18. $\frac{7}{8} \times \frac{45}{50}$
19. $\frac{14}{8} \times \frac{5}{4}$
20. $\frac{30}{25} \times \frac{74}{50}$

Rational Numbers

The number line shows that for every positive fraction there is an **opposite** negative fraction.

$-\frac{3}{4}$ is the opposite of $\frac{3}{4}$, and vice versa.



A **rational number** is a number that can be expressed as the *quotient* of two integers, the divisor not being zero. All positive and negative fractions belong to the set of rational numbers.

Examples of rational numbers

$\frac{8}{2} = 4$ $\quad \frac{4}{5} = 0.8$ $\quad \frac{20}{-4} = -5$ $\quad \frac{-18}{9} = -2$

$\frac{0}{2} = 0$ $\quad \frac{-1}{3} = -0.\overline{3}$ $\quad \frac{5}{7} = 0.\overline{714\,285}$

Integers, like 8 and -6, are rational numbers since they can be written as a *quotient*.

$8 = \frac{16}{2}$ $\quad -6 = \frac{-36}{6}$

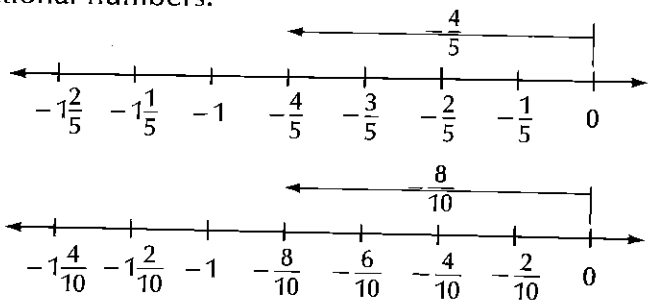
Negative rational numbers can be written *in fraction form* in three ways.

$\frac{-2}{3} = \frac{2}{-3} = -0.\overline{6} = -\frac{2}{3}$ This is the usual way.

The number lines show *equivalent* rational numbers.

$-\frac{4}{5} = \frac{-4}{5} = \frac{-4 \times 2}{5 \times 2} = \frac{-8}{10} = -\frac{8}{10}$

$-\frac{4}{5}$ is equivalent to $-\frac{8}{10}$.



Just as with positive fractions, we can compare rational numbers by writing equivalent fractions with a common denominator.

$-\frac{1}{2} < -\frac{1}{3}$ since $-\frac{3}{6} < -\frac{2}{6}$

$-\frac{3}{6}$ is to the left of $-\frac{2}{6}$ on the number line.

EXERCISES

1. Write each rational number in *decimal form.*

   a. $\frac{1}{2}$ **b.** $\frac{1}{3}$ **c.** $\frac{-5}{4}$ **d.** $-\frac{7}{8}$ **e.** $\frac{-1}{9}$

2. Write each rational number in *fraction form.*

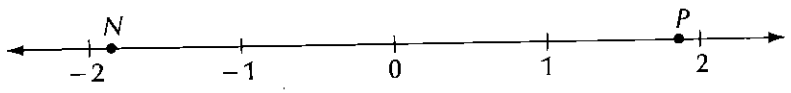
   a. 0.25 **b.** -0.6 **c.** 0.6 **d.** -4.5 **e.** -2.3

3. Is the rational number *positive* or *negative*?

   a. $\frac{6}{5}$ **b.** $\frac{-2}{3}$ **c.** $\frac{-3}{-4}$ **d.** $\frac{1}{-8}$ **e.** $\frac{7}{9}$

   f. $-\frac{12}{25}$ **g.** $\frac{-7}{-100}$ **h.** $\frac{-4}{15}$ **i.** $\frac{30}{-7}$ **j.** $\frac{-9}{-5}$

4. Would the rational number be graphed at point *P*, or point *N*?



   a. $\frac{15}{8}$ **b.** $-\frac{15}{8}$ **c.** $\frac{-15}{8}$ **d.** $\frac{-15}{-8}$ **e.** $\frac{15}{-8}$

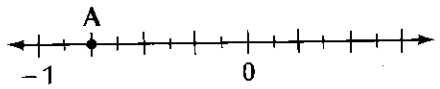
5. Write each rational number in fraction form with a *positive denominator.*

   a. $7 \div (-3)$ **b.** $(-1) \div (-8)$ **c.** $4 \div (-5)$ **d.** $(-12) \div (-7)$

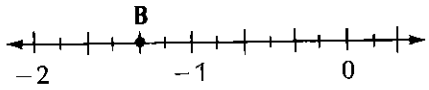
   e. -0.8 **f.** -5.75 **g.** -3.3 **h.** -4.8

6. Name two equivalent rational numbers for each point on the number line.

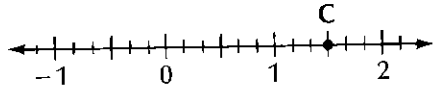
a.



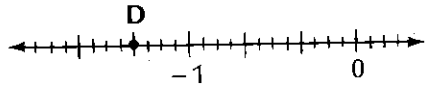
b.



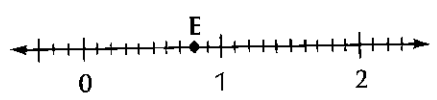
c.



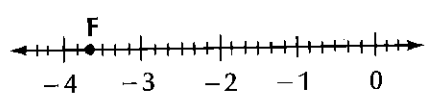
d.



e.



f.



7. Identify the pairs of equivalent rational numbers.

   a. $\frac{-3}{4}, \frac{6}{-8}$ **b.** $\frac{7}{9}, \frac{-28}{-36}$ **c.** $-1\frac{1}{2}, -1\frac{25}{50}$ **d.** $\frac{2}{3}, \frac{-6}{9}$

   e. $\frac{-3}{8}, \frac{12}{-32}$ **f.** $\frac{-5}{9}, \frac{25}{45}$ **g.** $\frac{16}{-5}, -1.2$ **h.** $\frac{32}{9}, 3.5$

PRACTICE

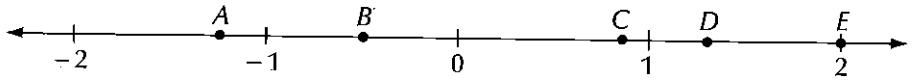
1. Write each rational number in decimal form.

   a. $\frac{-3}{5}$ b. $\frac{-15}{-2}$ c. $4\frac{1}{3}$ d. $\frac{4}{-9}$ e. $-\frac{7}{8}$

2. Write each rational number in fraction form.

   a. 0.6 b. −2.375 c. −0.4 d. 0.0625 e. −0.09

3. At which point on the number line is each rational number graphed?



   a. $\frac{-5}{4}$ b. $\frac{4}{5}$ c. $-\frac{1}{2}$ d. 2 e. $-1\frac{1}{4}$

   e. −0.5 f. $\frac{-4}{-5}$ g. −1.25 h. $\frac{1}{-2}$ i. $\frac{-6}{-3}$

   j. 0.8 k. 1.25 l. $\frac{-5}{-4}$ m. $\frac{-1}{2}$ n. $-\frac{5}{4}$

4. *True* or *false*?

   a. $\frac{-7}{9} = -\frac{7}{9}$ b. $-\frac{3}{2} = \frac{-3}{-2}$ c. $\frac{18}{5} = \frac{-18}{-5}$ d. $-6.4 = \frac{-32}{5}$

5. Compare using $<$, $=$, or $>$.

   a. $\frac{-4}{5} \bullet \frac{-8}{-10}$ b. $\frac{6}{-7} \bullet -\frac{24}{28}$ c. $\frac{9}{8} \bullet \frac{18}{-16}$ d. $\frac{-3}{-4} \bullet \frac{9}{12}$

   e. $-0.7 \bullet \frac{-3}{4}$ f. $-\frac{2}{5} \bullet -0.375$ g. $-2\frac{9}{10} \bullet \frac{-14}{5}$ h. $-0.625 \bullet \frac{2}{-3}$

6. Evaluate n for each pair of equivalent rational numbers.

   a. $\frac{-11}{25} = \frac{n}{100}$ b. $\frac{1}{-5} = \frac{n}{15}$ c. $\frac{3}{8} = \frac{n}{-64}$ d. $\frac{7}{-11} = \frac{n}{77}$

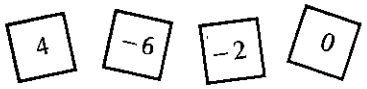
7. List the set of rational numbers between −5 and −2 with 4 as denominator.

8. List the set of rational numbers between −1 and 2 with 2 as denominator.

9. All rational numbers can be written in the form $\frac{a}{b}$, $b \neq 0$.
   Which are not rational numbers?

   a. $\frac{0}{7}$ b. $\frac{9}{0}$ c. $\frac{9-9}{3}$ d. $\frac{3(2)}{3-3}$

10. Make all possible rational numbers using the integers at the right.

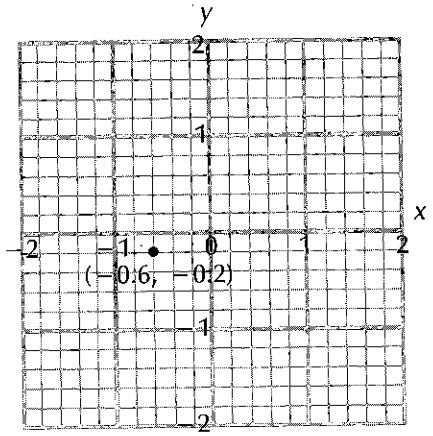


Graphing Rational Numbers

The location of point A on the grid is described by the **coordinates**, or **ordered pair**, $(-0.6, -0.2)$.

The x-coordinate, -0.6, describes the horizontal distance of point A from the origin $(0, 0)$ or the y-axis.

The y-coordinate, -0.2, describes the vertical distance of point A from the origin $(0, 0)$ or the x-axis.

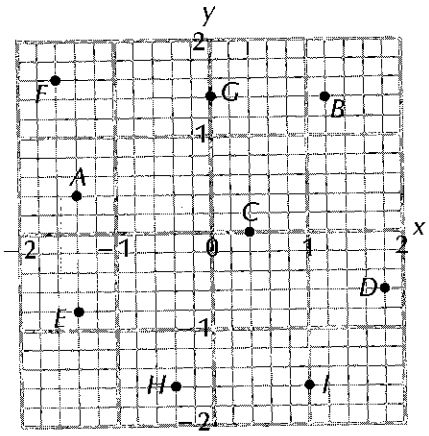


1. Write the ordered pair for each point on the grid below.

a. D **b.** F **c.** B

d. A **e.** G **f.** I

g. E **h.** C **i.** H

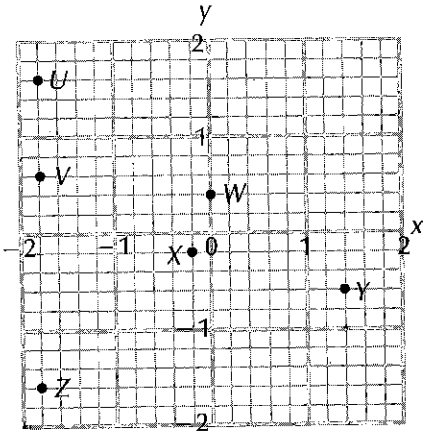


2. Name the point in the grid described by each ordered pair.

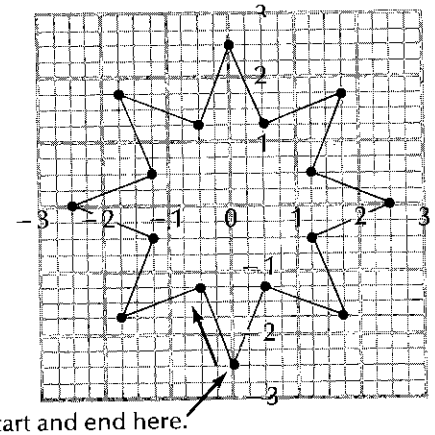
a. $(-1.8, 0.6)$ **b.** $(0, 0.4)$

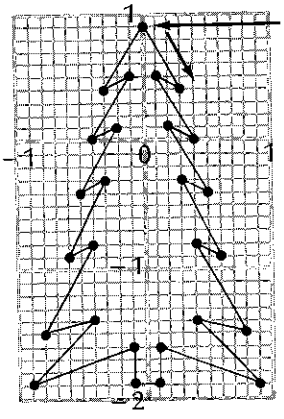
c. $(-0.2, -0.2)$ **d.** $(-1.8, -1.6)$

e. $(1.4, -0.6)$ **f.** $(-1.8, 1.6)$



3. For each figure, write the ordered pairs which were joined in order.





Adding and Subtracting Rational Numbers

Rational numbers are added or subtracted with a common denominator. It is easier to work with a *positive* denominator.

$$\frac{3}{7} + \left(-\frac{4}{7}\right) = \frac{3}{7} + \frac{-4}{7} = \frac{3 + (-4)}{7} = \frac{-1}{7} = -\frac{1}{7}$$

$$\frac{2}{3} + \left(-\frac{1}{5}\right) = \frac{2}{3} + \frac{-1}{5} = \frac{10}{15} + \frac{-3}{15} = \frac{10 + (-3)}{15} = \frac{7}{15}$$

$$\left(-2\frac{1}{4}\right) + 1\frac{1}{2} = \left(-\frac{9}{4}\right) + \frac{3}{2} = \frac{-9}{4} + \frac{6}{4} = \frac{(-9) + 6}{4} = \frac{-3}{4} = -\frac{3}{4}$$

These are the related subtractions for each addition above.

$$\left(-\frac{1}{7}\right) - \left(-\frac{4}{7}\right) = \frac{-1}{7} - \frac{-4}{7} = \frac{(-1) - (-4)}{7} = \frac{(-1) + 4}{7} = \frac{3}{7}$$

$$\frac{7}{15} - \left(-\frac{1}{5}\right) = \frac{7}{15} - \frac{-1}{5} = \frac{7}{15} - \frac{-3}{15} = \frac{7 - (-3)}{15} = \frac{7 + 3}{15} = \frac{10}{15} = \frac{2}{3}$$

$$\left(-\frac{3}{4}\right) - 1\frac{1}{2} = \frac{-3}{4} - \frac{3}{2} = \frac{-3}{4} - \frac{6}{4} = \frac{(-3) + (-6)}{4} = \frac{-9}{4} = -2\frac{1}{4}$$

EXERCISES

Find the sum in simplest terms.

1. $\frac{3}{5} + \left(-\frac{1}{5}\right) = \frac{\blacksquare + \blacksquare}{5} = \blacksquare$

2. $\left(-\frac{5}{8}\right) + \frac{7}{8}$

3. $\frac{7}{8} + \left(-\frac{3}{4}\right) = \frac{7}{8} + \frac{\blacksquare}{4} = \frac{7}{8} + \frac{\blacksquare}{8} = \blacksquare$

4. $\frac{3}{10} + \left(-\frac{1}{2}\right)$

5. $2\frac{3}{10} + \left(-1\frac{4}{5}\right) = \frac{\blacksquare}{10} + \frac{\blacksquare}{5} = \frac{\blacksquare}{10} + \frac{\blacksquare}{10} = \blacksquare$

6. $\left(-5\frac{1}{15}\right) + \left(-2\frac{1}{3}\right)$

Find the difference in simplest terms.

7. $\frac{11}{12} - \left(-\frac{1}{12}\right) = \blacksquare + \blacksquare = \frac{\blacksquare + \blacksquare}{12} = \blacksquare$

8. $\left(-\frac{7}{10}\right) - \frac{1}{10}$

9. $\frac{7}{9} - \left(-\frac{1}{6}\right) = \blacksquare + \blacksquare = \frac{\blacksquare}{18} + \frac{\blacksquare}{18} = \frac{\blacksquare}{18}$

10. $\frac{7}{10} - \left(-\frac{4}{5}\right)$

11. $5\frac{1}{3} - \left(-\frac{3}{8}\right) = \blacksquare + \blacksquare = \frac{\blacksquare}{24} + \frac{\blacksquare}{24} = \frac{\blacksquare}{24} = \blacksquare$

12. $4\frac{2}{3} - \left(-2\frac{1}{6}\right)$

PRACTICE

Find the sums in simplest terms.

1. $\frac{3}{4} + \left(-\frac{3}{4}\right)$
2. $\frac{2}{5} + \frac{2}{5}$
3. $\frac{7}{9} + \left(-\frac{4}{9}\right)$
4. $\frac{3}{5} + \left(-\frac{2}{3}\right)$
5. $\left(-\frac{7}{8}\right) + \frac{5}{8}$
6. $\frac{11}{12} + \left(-\frac{7}{16}\right)$
7. $\left(-2\frac{1}{2}\right) + 2\frac{1}{2}$
8. $\left(-\frac{7}{9}\right) + 6\frac{1}{3}$
9. $\left(-12\frac{3}{5}\right) + \left(-6\frac{3}{4}\right)$

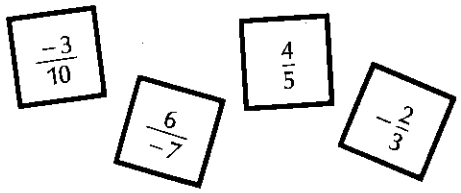
Find the differences in simplest terms.

10. $\frac{8}{9} - \left(-\frac{8}{9}\right)$
11. $\frac{5}{6} - \left(-\frac{1}{6}\right)$
12. $\left(-\frac{14}{15}\right) - \left(-\frac{7}{15}\right)$
13. $\frac{3}{7} - \left(-\frac{1}{5}\right)$
14. $\left(-\frac{4}{9}\right) - \left(-\frac{1}{3}\right)$
15. $\left(-\frac{16}{3}\right) - \left(-\frac{10}{8}\right)$
16. $3\frac{1}{8} - \left(-1\frac{5}{6}\right)$
17. $\left(-5\frac{1}{2}\right) - \left(-5\frac{1}{2}\right)$
18. $\left(-2\frac{3}{4}\right) - 1\frac{1}{8}$

Compute. Write the result in simplest terms.

19. $\frac{1}{4} + \left(-\frac{1}{2}\right) - \frac{3}{4}$
20. $\left(-2\frac{3}{5}\right) - 1\frac{1}{10} + 4\frac{7}{10}$
21. $\frac{5}{6} - \left(-\frac{2}{3}\right) + \frac{1}{2}$
22. $\left(-\frac{7}{8}\right) + \frac{7}{8} - 4\frac{1}{2}$
23. $\frac{3}{7} - \frac{3}{7} + \left(-\frac{3}{5}\right)$
24. $\left(-4\frac{5}{8}\right) + 4\frac{5}{8} - \left(-2\frac{3}{5}\right)$

25. What is the opposite of each rational number at the right?
26. What is the sum of each rational number and its opposite?

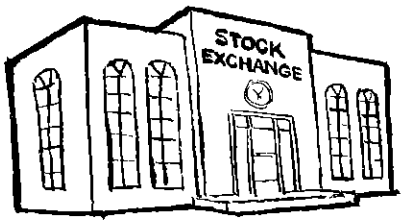


Fluctuating Stock Prices

Refer to the stock chart.

a. In dollars and cents, how much did each stock change from yesterday's closing to today's closing?

b. What was the price of 100 shares of each stock yesterday?



Stock Exchange Quotations		
Stock	Day's Close	Change From Yesterday's Close
Acme Hardware	$\$15\frac{1}{8}$	$+2\frac{5}{8}$
Megaco	$\$38\frac{1}{2}$	$-\frac{1}{4}$
Seaway Yachts	$\$6\frac{3}{4}$	$+1\frac{1}{4}$
Pacific Tech	$\$28\frac{1}{2}$	$-3\frac{7}{8}$
Gemco	$\$52\frac{1}{2}$	$-\frac{3}{4}$

Multiplying and Dividing Rational Numbers

To find the product of rational numbers, multiply the numerators and then multiply the denominators.

$$\left(-\frac{5}{7}\right) \times (-7) = \frac{(-5) \times (-7)}{7} = \frac{35}{7} = 5$$

$$\left(-\frac{3}{4}\right) \times \left(-\frac{1}{6}\right) = \frac{(-3) \times (-1)}{4 \times 6} = \frac{3}{24} = \frac{1}{8}$$

It is possible to *simplify* before multiplying if the numerators and denominators have a common factor.

$$1\frac{1}{5} \times \left(-2\frac{2}{3}\right) = \frac{6}{5} \times \frac{-8}{3} = \frac{\overset{2}{\cancel{6}} \times (-8)}{5 \times \underset{1}{\cancel{3}}} = \frac{-16}{5} = -3\frac{1}{5}$$

To divide rational numbers, multiply by the reciprocal of the divisor.

$$\frac{1}{3} \div \left(-1\frac{1}{3}\right) = \frac{1}{3} \div \left(-\frac{4}{3}\right) = \frac{1}{3} \times \left(-\frac{3}{4}\right) = \frac{1 \times (\overset{-1}{\cancel{-3}})}{\underset{1}{\cancel{3}} \times 4} = \frac{-1}{4} = -\frac{1}{4}$$

EXERCISES

Find the product in simplest terms.

1. $\left(-\frac{2}{3}\right) \times 4 = \frac{\blacksquare \times 4}{3} = \blacksquare$
2. $\left(-\frac{3}{7}\right) \times 21$
3. $\frac{5}{8} \times (-2)$
4. $\left(-\frac{3}{7}\right) \times \frac{1}{6} = \frac{\blacksquare \times \blacksquare}{7 \times 6} = \blacksquare$
5. $\left(-\frac{2}{5}\right) \times \left(-\frac{3}{8}\right)$
6. $\left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)$
7. $3\frac{1}{8} \times \left(-5\frac{1}{10}\right) = \frac{\blacksquare \times \blacksquare}{8 \times 10} = \blacksquare$
8. $\left(-2\frac{3}{5}\right) \times \left(-4\frac{1}{10}\right)$
9. $\left(-4\frac{1}{3}\right) \times 2\frac{1}{2}$

What is the *reciprocal* of each?

10. $-\frac{3}{10}$
11. $4\frac{1}{8}$
12. $-\frac{7}{11}$
13. $-\frac{6}{17}$
14. $-2\frac{2}{3}$

Find the quotient in simplest terms.

15. $\left(-\frac{3}{5}\right) \div \frac{1}{10} = \frac{\blacksquare \times \blacksquare}{5 \times 1} = \blacksquare$
16. $\frac{7}{8} \div \left(-\frac{1}{2}\right)$
17. $\left(-\frac{4}{5}\right) \div \left(-\frac{3}{10}\right)$
18. $\left(-2\frac{1}{7}\right) \div 2\frac{1}{2} = \frac{\blacksquare}{7} \times \frac{\blacksquare}{5} = \frac{\blacksquare \times \blacksquare}{\blacksquare \times \blacksquare} = \blacksquare$
19. $1\frac{5}{6} \div \left(-3\frac{1}{3}\right)$

PRACTICE

Find the product in simplest terms.

1. $\left(-\frac{5}{9}\right) \times 9$
2. $\frac{3}{7} \times (-35)$
3. $\left(-\frac{6}{3}\right) \times (-9)$
4. $\left(-\frac{5}{6}\right) \times \frac{2}{7}$
5. $\left(-\frac{3}{4}\right) \times \left(-\frac{2}{5}\right)$
6. $\frac{5}{9} \times \frac{3}{10}$
7. $\left(-3\frac{1}{5}\right) \times 4\frac{1}{2}$
8. $\left(-4\frac{2}{7}\right) \times 2\frac{2}{5}$
9. $\left(-3\frac{1}{2}\right) \times \left(-4\frac{2}{3}\right)$

Find the quotient in simplest terms.

10. $\left(-\frac{5}{7}\right) \div 5$
11. $\left(-\frac{3}{8}\right) \div (-6)$
12. $7 \div \left(-\frac{3}{8}\right)$
13. $\left(-\frac{3}{8}\right) \div \frac{1}{4}$
14. $\left(-\frac{7}{9}\right) \div \left(-\frac{1}{3}\right)$
15. $\left(-\frac{3}{10}\right) \div \left(-\frac{4}{5}\right)$
16. $2\frac{4}{5} \div \left(-4\frac{2}{3}\right)$
17. $\frac{9}{10} \div \left(-5\frac{3}{5}\right)$
18. $\left(-3\frac{3}{4}\right) \div \left(-4\frac{1}{2}\right)$

Find the value of x in each equation.

19. $\left(-\frac{6}{15}\right) \times x = 1$
20. $\left(-\frac{7}{11}\right) \div x = 1$
21. $\left(-\frac{1}{6}\right) + x = (-9) \times \frac{0}{2}$
22. $\left(-4\frac{1}{2}\right) \times \left(\frac{2}{3} + \left(-\frac{2}{3}\right)\right) = x$

Solve.

23. The product of $-\frac{7}{12}$ and a mystery number is $-\frac{1}{2}$. What is the number?
24. The quotient of $2\frac{3}{5}$ and a mystery number is $-\frac{5}{6}$. What is the number?

Closure

The set of whole numbers, W, is **closed under addition** because the sum of any two whole numbers is unique and is still a whole number.

$W = \{0, 1, 2, 3, \ldots\}$ $\quad 0 + 6 = 6 \in W, 4 + 21 = 25 \in W$, and so on.

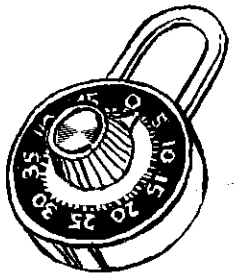
Decide which of the sets at the right are *closed* under the following operations.

a. addition
b. subtraction
c. multiplication
d. division

$N = \{1, 2, 3, 4, 5, \ldots\}$
$I = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
$W = \{0, 1, 2, 3, \ldots\}$
$A = \{0, 1\}$
$B = \{-1, 0, 1\}$
$Q =$ the set of rational numbers

Combinations

Sometimes calculations involve rational numbers in both fraction and decimal form.



The numbers often need to be converted all to the same form.

Examples:

$\frac{1}{3} + 0.5 = \frac{1}{3} + \frac{1}{2}$
$= \frac{2}{6} + \frac{3}{6}$
$= \frac{5}{6}$

$1\frac{1}{4} - 1.329 = 1.25 - 1.329$
$= -0.079$

$\frac{1}{2} \times (-7.6) = -\frac{7.6}{2}$
$= -3.8$

Compute the answers.

1. $\frac{5}{6} + 0.3$
2. $4\frac{3}{8} - 1.75$
3. $\frac{3}{4} \div 0.6$
4. $0.78 \times \frac{2}{5}$
5. $(-5.6) - \left(-3\frac{1}{4}\right)$
6. $3.75 + \left(-4\frac{9}{10}\right)$
7. $0.75 - \frac{2}{3}$
8. $1\frac{1}{2} \times (-0.058)$
9. $(-1.7) \div \left(-5\frac{2}{3}\right)$
10. $\left(-2\frac{2}{5}\right) \div 3$
11. $1.3 + \frac{1}{3}$
12. $0.6 \times \left(-1\frac{1}{2}\right)$
13. $\frac{5}{8} \times 0.2 - 0.125$
14. $\left(-\frac{1}{2}\right) \div 2 + 0.25$
15. $\left(\frac{2}{3} - 0.6\right) \times 0.3$

Solve.

16. The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

    Use $\pi = 3.14$ to calculate the volume of spheres with the following radii.

    a. 3 cm **b.** 3.3 cm **c.** 9.5 cm **d.** 0.85 cm

17. If an object is dropped from 1000 m above sea level, its height in metres after t seconds is given by the formula $1000 - 4.9\,t^2$.

    Calculate the object's height after:

    a. 10 s **b.** 12 s **c.** 1.5 s **d.** 15 s

    What is the meaning of a negative height?

REVIEW

Find the quotient in simplest terms.

1. $\frac{1}{2} \div \frac{3}{8}$
2. $\frac{3}{4} \div 6$
3. $\frac{3}{10} \div \frac{2}{5}$
4. $\frac{1}{10} \div \frac{2}{3}$
5. $1\frac{1}{2} \div \frac{2}{3}$
6. $3\frac{2}{3} \div \frac{5}{6}$
7. $4\frac{1}{2} \div 9$
8. $1\frac{3}{4} \div 2\frac{1}{8}$

Copy and complete.

9. $-\frac{3}{8} = -\frac{\blacksquare}{24}$
10. $-\frac{6}{7} = -\frac{\blacksquare}{49}$
11. $-\frac{2}{3} = -\frac{\blacksquare}{9}$
12. $-\frac{3}{4} = -\frac{\blacksquare}{16}$
13. $-\frac{15}{9} = -1\frac{\blacksquare}{3}$
14. $-2\frac{1}{4} = -\frac{\blacksquare}{4}$
15. $-\frac{4}{5} = \frac{-4}{\blacksquare}$
16. $-\frac{2}{3} = \frac{\blacksquare}{-3}$

Compute. Express the answer in simplest terms.

17. $\frac{2}{9} + \left(-\frac{5}{9}\right)$
18. $\left(-\frac{3}{8}\right) + \left(-\frac{1}{8}\right)$
19. $\left(-\frac{3}{4}\right) + \frac{1}{4}$
20. $\left(-\frac{3}{8}\right) + \frac{1}{4}$
21. $\left(-\frac{4}{5}\right) + \frac{1}{8}$
22. $\frac{2}{3} + \left(-\frac{3}{4}\right)$
23. $3\frac{4}{5} + \left(-\frac{2}{3}\right)$
24. $2\frac{1}{4} + \left(-\frac{1}{2}\right)$
25. $\left(-6\frac{1}{5}\right) + 2\frac{1}{2}$
26. $\left(-\frac{1}{8}\right) - \left(-\frac{1}{8}\right)$
27. $\left(-\frac{2}{3}\right) - \frac{2}{3}$
28. $\frac{3}{4} - \left(-\frac{1}{4}\right)$
29. $\left(-\frac{3}{5}\right) - \frac{1}{15}$
30. $\frac{1}{10} - \frac{3}{4}$
31. $\left(-\frac{4}{5}\right) - \left(-\frac{5}{6}\right)$
32. $2\frac{1}{2} - \left(-2\frac{1}{4}\right)$
33. $3\frac{2}{3} - 4\frac{1}{3}$
34. $\left(-2\frac{1}{4}\right) - 5\frac{1}{6}$
35. $10 \times \left(-\frac{3}{5}\right)$
36. $\left(-\frac{3}{4}\right) \times \left(-\frac{3}{8}\right)$
37. $\left(-\frac{4}{5}\right) \times \frac{5}{6}$
38. $(-2) \times \left(-5\frac{1}{2}\right)$
39. $\left(-1\frac{2}{3}\right) \times 1\frac{4}{5}$
40. $6\frac{1}{4} \times \left(-2\frac{2}{3}\right)$
41. $6 \div \left(-\frac{5}{6}\right)$
42. $\left(-\frac{3}{8}\right) \div \left(-\frac{1}{2}\right)$
43. $\left(-\frac{2}{3}\right) \div (-2)$
44. $3\frac{1}{7} \div \left(-\frac{2}{3}\right)$
45. $\left(-5\frac{3}{4}\right) \div \left(-2\frac{1}{2}\right)$
46. $\left(-1\frac{9}{10}\right) \div \frac{3}{10}$
47. $4 \times \left(-\frac{3}{4}\right) + 2\frac{1}{2}$
48. $\left(-\frac{2}{3}\right) + \frac{3}{4} \div 1\frac{1}{2}$
49. $\left(-\frac{5}{6}\right) \times \frac{3}{4} \div \frac{1}{2}$
50. $\left(-2\frac{1}{2}\right) \div \frac{5}{9} + \left(-\frac{4}{9}\right)$

TEST UNIT 7

Find the product in simplest terms.

1. $7 \times \frac{5}{6}$
2. $\frac{5}{12} \times \frac{4}{7}$
3. $\frac{3}{5} \times \frac{2}{3} \times \frac{10}{12}$
4. $3\frac{5}{6} \times 9$
5. $2\frac{1}{8} \times 6\frac{3}{4}$
6. $\frac{4}{5} \times \frac{5}{6} \times \frac{7}{8}$

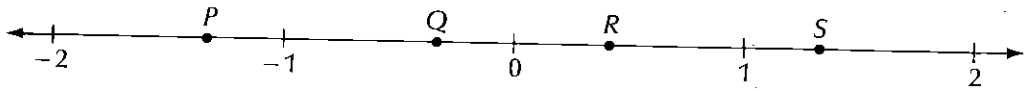
Find the sum or difference in simplest terms.

7. $\frac{3}{4} + \frac{1}{6}$
8. $\frac{7}{9} - \frac{5}{12}$
9. $\frac{7}{8} + \frac{2}{3} - \frac{1}{2}$
10. $3\frac{2}{3} + 6\frac{4}{5}$
11. $12\frac{5}{9} + \frac{5}{6}$
12. $\frac{1}{3} + \frac{5}{9} + \frac{3}{4}$
13. $10 - 6\frac{7}{9}$
14. $6\frac{3}{8} - 1\frac{3}{4}$
15. $9\frac{1}{8} - 6\frac{1}{2}$

Find the quotient in simplest terms.

16. $\frac{4}{7} \div \frac{8}{9}$
17. $6 \div \frac{2}{3}$
18. $3\frac{1}{2} \div 2\frac{2}{5}$

In Questions 19–28, identify each rational number as a point on the number line.



19. $\frac{2}{5}$
20. $\frac{-4}{-3}$
21. $-1\frac{1}{3}$
22. 0.4
23. $-1.\overline{3}$
24. $\frac{-4}{3}$
25. $1\frac{1}{3}$
26. -0.4
27. $\frac{4}{-3}$
28. $-\frac{2}{5}$

Compute. Write the answer in simplest terms.

29. $\left(-\frac{3}{5}\right) + \frac{7}{10}$
30. $\left(-\frac{5}{8}\right) + \left(-\frac{1}{6}\right)$
31. $2\frac{7}{10} + \left(-5\frac{3}{5}\right)$
32. $\left(-\frac{9}{10}\right) - \frac{9}{10}$
33. $2\frac{7}{8} - \left(-\frac{1}{2}\right)$
34. $\left(-3\frac{5}{6}\right) - \left(-2\frac{2}{9}\right)$
35. $\left(-\frac{5}{6}\right) \times \frac{8}{9}$
36. $\left(-3\frac{3}{4}\right) \times \left(-\frac{1}{3}\right)$
37. $\left(-6\frac{1}{3}\right) \times \left(-2\frac{1}{4}\right)$
38. $\left(-\frac{3}{11}\right) \div \frac{6}{7}$
39. $(-12) \div \left(-2\frac{2}{3}\right)$
40. $6\frac{1}{5} \div \left(-2\frac{9}{10}\right)$

LOOKING BACK

Number Theory and Fractions

List the set of factors for each.

1. 28 **2.** 60 **3.** 17 **4.** 145 **5.** 205

Is the number *prime* or *composite*?

6. 9 **7.** 13 **8.** 2 **9.** 39 **10.** 61

Write each as a product of prime factors.

11. 36 **12.** 63 **13.** 850 **14.** 120 **15.** 420

Evaluate the square root.

16. $\sqrt{64}$ **17.** $\sqrt{81}$ **18.** $\sqrt{100}$ **19.** $\sqrt{121}$ **20.** $\sqrt{225}$

Find the GCF.

21. 24, 32 **22.** 48, 96 **23.** 64, 100, 88

Find the LCM.

24. 12, 16 **25.** 15, 9 **26.** 24, 36, 40

Write each fraction in simplest terms.

27. $\frac{6}{16}$ **28.** $\frac{9}{12}$ **29.** $\frac{24}{36}$ **30.** $\frac{42}{56}$ **31.** $\frac{75}{175}$

Write each fraction as a mixed numeral in simplest terms.

32. $\frac{21}{4}$ **33.** $\frac{16}{12}$ **34.** $\frac{31}{7}$ **35.** $\frac{65}{9}$ **36.** $\frac{48}{15}$

Compare using $<$ or $>$.

37. $\frac{5}{6} \bullet \frac{3}{4}$ **38.** $\frac{3}{5} \bullet \frac{2}{3}$ **39.** $\frac{3}{5} \bullet \frac{7}{12}$

40. $2\frac{9}{10} \bullet 3\frac{1}{10}$ **41.** $3\frac{2}{3} \bullet \frac{10}{3}$ **42.** $1\frac{6}{7} \bullet 1\frac{11}{13}$

Convert to a decimal.

43. $\frac{3}{8}$ **44.** $1\frac{7}{8}$ **45.** $\frac{4}{25}$ **46.** $2\frac{2}{3}$ **47.** $\frac{7}{9}$

Convert to a fraction in simplest terms.

48. 0.64 **49.** 0.625 **50.** 4.375 **51.** 0.3 **52.** 1.6

53. Palindromic numbers read the same forwards or backwards. For example, 373 and 48 384 are palindromes. Find the 5 palindromic primes between 100 and 200.