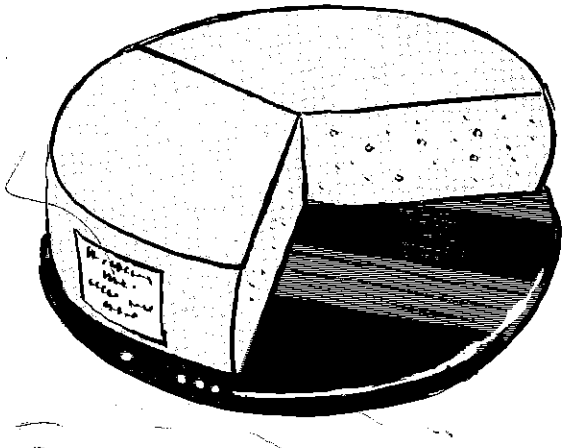
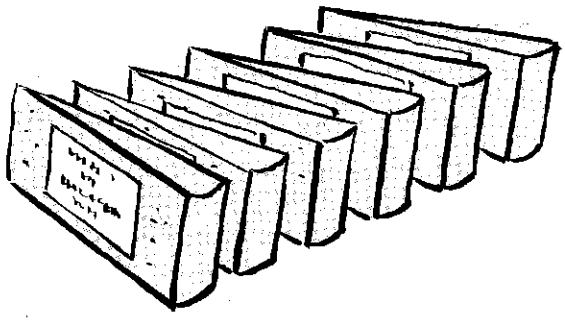
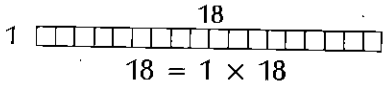
# UNIT 6
# Number Theory and Fractions



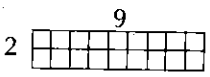


## Factors and Divisibility

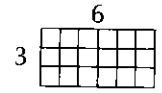
There are three ways to write 18 as a **product** so that its factor pairs are whole numbers.




$18 = 1 \times 18$




$18 = 2 \times 9$




$18 = 3 \times 6$

The **set of factors** of 18 is $\{1, 2, 3, 6, 9, 18\}$.

Each number in the set divides evenly into 18, leaving no remainder.

$1\overline{)18}$ $2\overline{)18}$ $3\overline{)18}$ $6\overline{)18}$ $9\overline{)18}$ $18\overline{)18}$

These divisibility rules will help you find the factors of a number.

A number is evenly divisible by:
2. if it is even.
3. if the sum of the digits is divisible by 3.
4. if the last two digits are divisible by 4.
5. if the last digit is 0 or 5.
6. if the number is divisible by 2 and 3.
9. if the sum of the digits is divisible by 9.
10. if the last digit is 0.

**Examples**

150 is evenly divisible by 2, 3, 5, 6, and 10.

All whole numbers are evenly divisible by 1.

The set of factors of 150 is:
{1, 2, 3, 5, 6, 10, 15, 25, 30, 50, 75, 150}

## EXERCISES

1. Is 487 596 divisible by the number?

   a. 2 b. 3 c. 4 d. 5 e. 6 f. 9 g. 10

2. Copy and complete each set of factors.

   a. 20: {1, ■, 4, ■, 10, 20}

   b. 28: {1, 2, ■, 7, ■, ■}

   c. 84: {■, 2, ■, 4, ■, ■, 12, ■, ■, 28, ■, 84}

   d. 132:{■, ■, 3, ■, ■, 11, ■, ■, 33, ■, 66, ■}

3. Write the set of factors for each.

   a. 50: {■, ■, ■, ■, ■, ■} b. 60: {■, ■, ■, ■, ■, ■, ■, ■, ■, ■, ■, ■}

   c. 153 d. 200 e. 101 f. 225

## PRACTICE

1. Is the second number evenly divisible by the first?

   a. 2; 80 386 b. 3; 57 217 c. 4; 415 728 d. 5; 28 386

   e. 6; 357 492 f. 9; 44 208 g. 3; 116 303 h. 6; 327 513

2. Write the set of factors for each.

   a. 90 b. 32 c. 38 d. 75

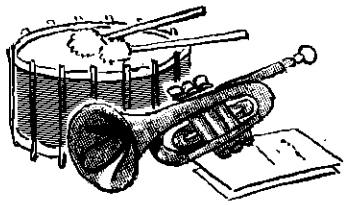
   e. 110 f. 42 g. 51 h. 135

   i. 103 j. 144 k. 250 l. 363

Solve.

3. What are the different ways that $12 can be divided into equal amounts of dollars?

4. The Maple Creek Band has 36 members. For the July 1 parade, the band leader wants the members to march in equal rows. In how many different ways can the band members be arranged?



5. Paula earned $57 one afternoon for planting seedlings in flats at a nursery. She received a whole number of dollars for each flat she planted. If Paula planted more than 10 flats, how much money was she paid for each?

# Prime Numbers and Prime Factors

A number that has *exactly 2 factors*, itself and 1, is a **prime number.**

23 is a prime number.

A number that has *more than 2 factors* is a **composite number.**

25 is a composite number.

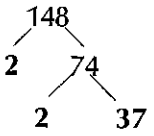
Every composite number can be written as a *product of prime factors* by using a factor tree or by dividing by primes.

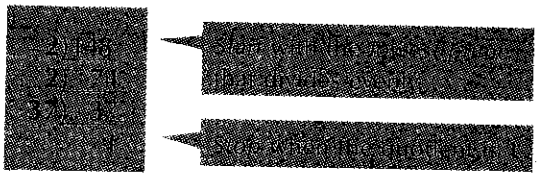
| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | 24 | | | | | | 30 |
| | | | | 12 | | | | | | 15 |
| 20 | | | | 8 | | | | 28 | | 10 |
| 10 | | | | 6 | | | | 14 | | 6 |
| 5 | 21 | 22 | | 4 | | 26 | 27 | 7 | | 5 |
| 4 | 7 | 11 | | 3 | 25 | 13 | 9 | 4 | | 3 |
| 2 | 3 | 2 | | 2 | 5 | 2 | 3 | 2 | | 2 |
| 1 | 1 | 1 | | 1 | 1 | 1 | 1 | 1 | | 1 |
| | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |

Factors

Whole Numbers $<31$ and $>19$.

$148 = 2 \times 2 \times 37$
$= 2^2 \times 37$



The set of factors of a number can be found by multiplying its prime factors.

$148 = 2 \times 2 \times 37 = 1 \times 148$

$= 2 \times 2 \times 37 = 2 \times 74$ The set of prime factors of 148 is {2, 37}.

$= 2 \times 2 \times 37 = 4 \times 37$ The set of factors of 148 is {1, 2, 4, 37, 74, 148}.

## EXERCISES

**1.** Write the set of factors for each number.
Then state whether the number is *prime* or *composite*.

**a.** 11: {■, ■} **b.** 9: {■, ■, ■} **c.** 15 **d.** 19

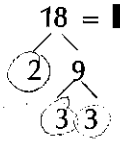
**e.** 41 **f.** 57 **g.** 69 **h.** 107

**2.** Write the prime factors in each set of factors.

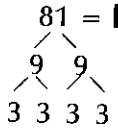
**a.** 30: {1, 2, 3, 5, 6, 10, 15, 30} **b.** 44: {1, 2, 4, 11, 22, 44}

**3.** Write each as a product of prime factors using exponents.

**a.** $18 = ■$




**b.** $81 = ■$




**c.** 90 **d.** 140 **e.** 160

**f.** 108 **g.** 112 **h.** 575

## PRACTICE

1. Which are square numbers?
   Find the square root of each square number.

   a. $2025 = 3^4 \times 5^2$ b. $200 = 2^3 \times 5^2$ c. $1080 = 2^3 \times 3^3 \times 5$

   d. $375 = 3 \times 5^3$ e. $484 = 2^2 \times 11^2$ f. $72 = 2^3 \times 3^2$

   g. $30\,625 = 5^4 \times 7^2$ h. $1800 = 2^3 \times 3^2 \times 5^2$ i. $1521 = 3^2 \times 13^2$

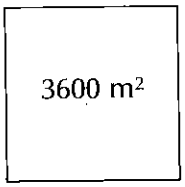
2. Write each number as a product of prime factors.
   Find the square root of each square number.

   a. 48 b. 81 c. 27 d. 400 e. 576

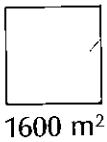
   f. 132 g. 270 h. 10 000 i. 171 j. 2500

3. Find the width of each square.

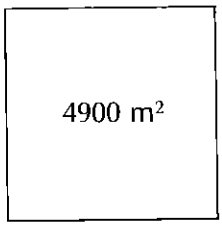
   a.

   b.

   c.

4. Evaluate.

   a. $\sqrt{441}$ b. $\sqrt{121}$ c. $\sqrt{729}$ d. $\sqrt{1936}$ e. $\sqrt{1089}$

   f. $\sqrt{1764}$ g. $\sqrt{3025}$ h. $\sqrt{1024}$ i. $\sqrt{19\,600}$ j. $\sqrt{1\,000\,000}$

# Negative Square Roots

A positive number has two different square roots which are *opposites* of each other.

For example, the square roots of 4 are 2 and $-2$.

$\sqrt{4} = 2$ because $2^2 = 4$

The *positive square root* is denoted by the symbol $\sqrt{\ }$.

$-\sqrt{4} = -2$ because $(-2)^2 = 4$

The *negative square root* is denoted by the symbol $-\sqrt{\ }$.

Simplify.

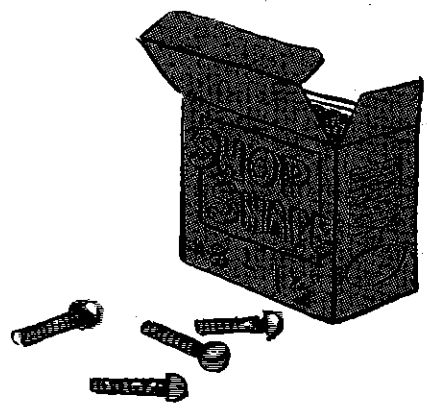
a. $-\sqrt{100}$ b. $-\sqrt{121}$ c. $-\sqrt{400}$ d. $-\sqrt{225}$

e. $-\sqrt{1}$ f. $-\sqrt{10\,000}$ g. $-\sqrt{900}$ h. $-\sqrt{196}$

i. $-\sqrt{625}$ j. $-\sqrt{324}$ k. $-\sqrt{1\,000\,000}$ l. $-\sqrt{484}$

# Greatest Common Factor

Beth and David bought some machine screws for their metalwork projects. Beth spent $1.62 and David spent $1.26. What is the maximum possible cost of one machine screw?



**Decide.** Find the GCF of 162 and 126.

*Method 1:* List all factors.

162: {1, 2, 3, 6, 9, 18, 27, 54, 81, 162}
126: {1, 2, 3, 6, 7, 9, 14, 18, 21, 42, 63, 126}

Common factors: {1, 2, 3, 6, 9, 18}
GCF = 18

*Method 2:* Write products of prime factors.

$162 = 2 \times 3 \times 3 \times 3 \times 3$
$126 = 2 \times 3 \times 3 \times 7$

Common prime factors: $2 \times 3 \times 3$
GCF = 18

The maximum cost of one screw is $0.18.

## EXERCISES

Use Method 1 to find the GCF.

1. 35 and 40
   35: {■, ■, ■, ■}
   40: {■, ■, ■, ■, ■, ■, ■, ■}
   GCF = ■

2. 15, 25, and 10
   15: {■, ■, ■, ■}
   25: {■, ■, ■}
   10: {■, ■, ■, ■}
   GCF = ■

3. 24 and 27
4. 16 and 20
5. 36 and 54
6. 16, 24, and 36
7. 8, 20, and 14
8. 60, 90, and 75

Use Method 2 to find the GCF.

9. 24 and 60
   $24 = ■ \times ■ \times ■ \times ■$
   $60 = ■ \times ■ \times ■ \times ■$
   GCF = ■

10. 36, 18, and 27
    $36 = ■ \times ■ \times ■ \times ■$
    $18 = ■ \times ■ \times ■$
    $27 = ■ \times ■ \times ■$
    GCF = ■

11. 50 and 75
12. 76 and 66
13. 63 and 28
14. 98, 42, and 56
15. 300, 450, and 750
16. 252, 108, and 144

## PRACTICE

Find the GCF.

1. 70, 35
2. 36, 96
3. 39, 52
4. 48, 35
5. 140, 110
6. 105, 140
7. 17, 25, 50
8. 12, 48, 40
9. 80, 100, 150
10. 800, 300, 500, 700
11. 18, 27, 36, 25
12. 72, 20, 28, 48

Solve.

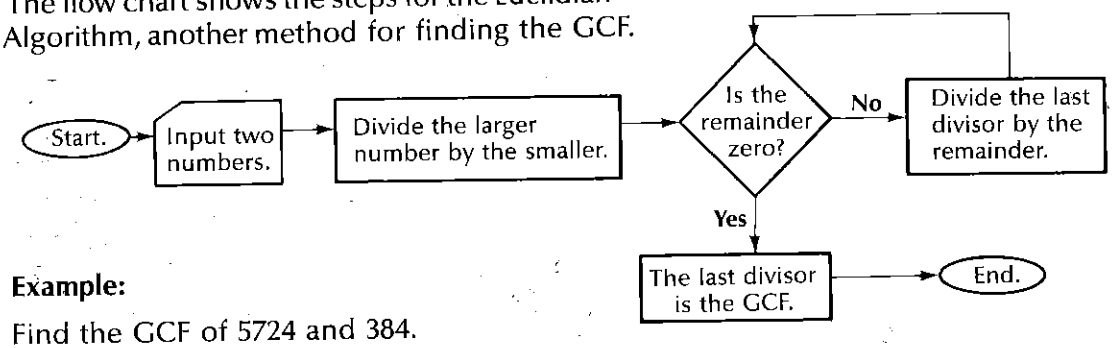
13. What is the maximum number of people who can share 42 cookies, 70 apples, and 56 chocolate bars equally?
14. What are the largest square tiles that could be used to tile an area 105 cm by 280 cm?
15. Two numbers with a GCF of 1 are said to be *relatively prime.* Which of the following pairs of numbers are relatively prime?

a. 21, 35 b. 42, 28 c. 47, 32 d. 38, 15

e. 87, 53 f. 60, 93 g. 25, 36 h. 75, 40

# The Euclidian Algorithm

The flow chart shows the steps for the Euclidian Algorithm, another method for finding the GCF.




**Example:**

Find the GCF of 5724 and 384.

$$\begin{array}{r} 14 \\ 384\overline{)5724} \\ -384 \\ \hline 1884 \\ -1536 \\ \hline 348 \end{array} \qquad \begin{array}{r} 1 \\ 348\overline{)384} \\ -348 \\ \hline 36 \end{array} \qquad \begin{array}{r} 9 \\ 36\overline{)348} \\ -324 \\ \hline 24 \end{array} \qquad \begin{array}{r} 1 \\ 24\overline{)36} \\ -24 \\ \hline 12 \end{array} \qquad \begin{array}{r} 2 \\ 12\overline{)24} \\ -24 \\ \hline 0 \end{array}$$

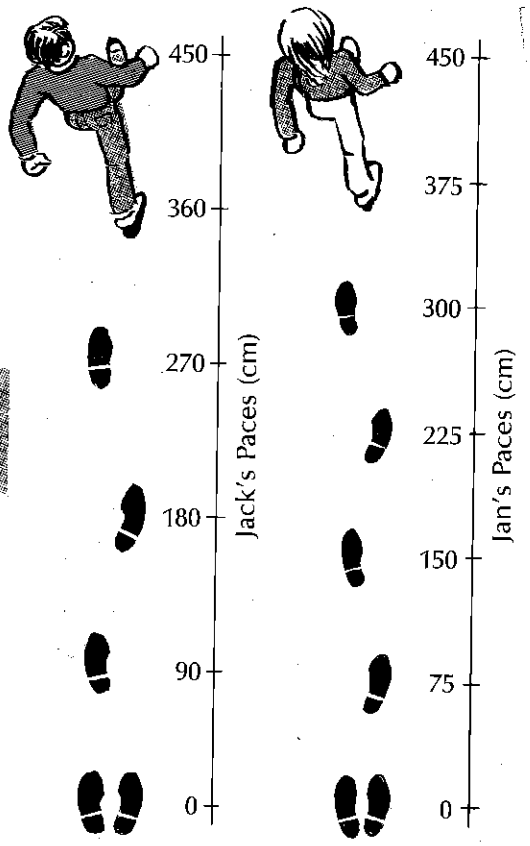
The GCF of 5724 and 384 is 12.

Use the Euclidian Algorithm to find the GCF.

1. 1092, 798
2. 1110, 345
3. 2919, 896
4. 918, 1314
5. 3075, 2162

# Least Common Multiple

Jack's average pace is 90 cm long.
Jan's average pace is 75 cm long.
What is the shortest distance they could pace and finish with both going exactly the same distance?




**Decide.** Find the LCM of 90 and 75.

**The least common multiple or LCM is the smallest non-zero number that is a multiple of each of 2 or more given numbers.**

*Method 1:* List the multiples.

90: {90, 180, 270, 360, 450, 540, . . .}
75: {75, 150, 225, 300, 375, 450, . . .}
LCM = 450

*Method 2:* Write products of prime factors.

$90 = 2 \times 3 \times 3 \times 5$
$75 = \quad\quad 3 \times 5 \times 5$
LCM $= 2 \times 3 \times 3 \times 5 \times 5$
LCM = 450

They could both pace 450 cm.

## EXERCISES

Use Method 1 to find the LCM.

**1.** 60 and 80
60: {■, ■, ■, ■, ■, ■, ■, ■, . . .}
80: {■, ■, ■, ■, ■, ■, ■, ■, . . .}
LCM = ■

**2.** 6, 4, and 10
6: {■, ■, ■, . . .}
4: {■, ■, ■, . . .}
10: {■, ■, ■, . . .}
LCM = ■

**3.** 15 and 12

**4.** 14 and 28

**5.** 18 and 12

**6.** 10, 12, and 8

**7.** 14, 21, and 35

**8.** 30, 50, and 60

Use Method 2 to find the LCM.

**9.** 24 and 20
24 = ■ × ■ × ■ × ■
20 = ■ × ■ × ■
LCM = ■

**10.** 9, 15, and 25
9 = ■ × ■
15 = ■ × ■
25 = ■ × ■
LCM = ■

**11.** 90 and 150

**12.** 16 and 114

**13.** 216 and 156

## PRACTICE

Find the LCM.

1. 6 and 15
2. 10 and 16
3. 12 and 20
4. 50 and 75
5. 28 and 52
6. 33 and 44
7. 21 and 28
8. 36 and 45
9. 42 and 35
10. 490 and 470
11. 306 and 270
12. 315 and 242
13. 5, 9, and 7
14. 6, 8, and 14
15. 12, 36, and 40
16. 2, 15, and 12
17. 14, 21, 10, and 12
18. 15, 20, 25, and 40

Find the GCF and LCM.

19. 24 and 32
20. 72 and 40
21. 98 and 35
22. 490 and 700

Solve.

23. It takes Donald 35 s to pack and label a box of electronic components. The same job takes Melissa 42 s and Sandy 28 s. If they all start at the same time, after how many seconds will they all want to use the labelling machine at once?

24. Cinema I and Cinema II start their movies at 6:00 P.M. The movie at Cinema I takes 60 min. The movie at Cinema II takes 90 min. When will the two movies start together again?

25. Spring-flowering bulbs are on sale at the garden shop. Steve wants to buy equal amounts of all four kinds. What is the least number of each kind of bulb he can buy?

| Spring-Flowering Bulb Sale | |
|---|---|
| crocus | 6 for $1.65 |
| tulip | 4 for $3.50 |
| daffodil | 3 for $1.95 |
| hyacinth | 5 for $2.25 |

# Relatively Prime Numbers

The pairs of numbers below are **relatively prime.**
Find the LCM of each pair.

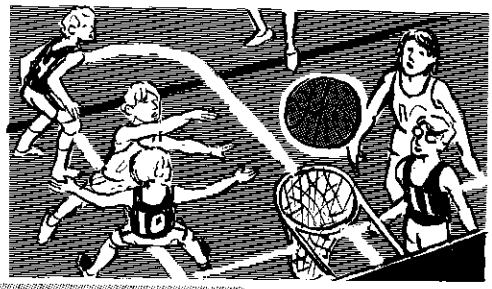
1. 8, 3
2. 5, 9
3. 15, 8
4. 10, 21
5. 14, 25

Find the GCF and LCM of each pair of **consecutive numbers.**

6. 2, 3
7. 4, 5
8. 7, 8
9. 11, 12
10. 20, 21

State a rule for calculating the LCM of relatively prime numbers.

# Equivalent Fractions



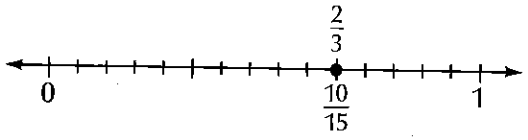
The Brookwood Bobcats won 10 of the 15 games they played. This is $\frac{10}{15}$ of their games.

A fraction *equivalent* to $\frac{10}{15}$ can be written using smaller numbers.

To write an **equivalent fraction** in "simpler terms", divide both the numerator and the denominator by a common factor.

numerator → $\frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$
denominator →

$\frac{10}{15}$ is *equivalent* to $\frac{2}{3}$.



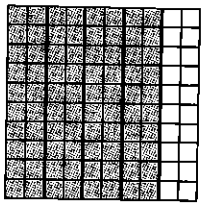

For the 15 games, Tom averaged scoring a free-shot point in 4 out of 5 attempts. This is $\frac{4}{5}$ of his attempts. What is Tom's free-shot average per hundred?

To write an **equivalent fraction**, multiply both the numerator and denominator by the same number.

numerator → $\frac{4}{5} = \frac{4 \times 20}{5 \times 20} = \frac{80}{100}$
denominator →

$\frac{4}{5}$ is equivalent to $\frac{80}{100}$.

Tom averaged 80 free shots per hundred.

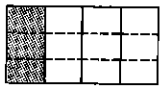


## EXERCISES

Find two equivalent fractions for each diagram.

**1.**  $\frac{1}{2} = \frac{\blacksquare}{6}$

**2.**  $\frac{3}{9} = \frac{\blacksquare}{3}$

**3.**  $\frac{1}{4} = \frac{\blacksquare}{12}$

Complete the equivalent fraction.

**4.** $\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{\blacksquare}{\blacksquare}$

**5.** $\frac{10}{35} = \frac{10 \div 5}{35 \div 5} = \frac{\blacksquare}{\blacksquare}$

**6.** $\frac{3}{10} = \frac{3 \times 2}{10 \times 2} = \frac{\blacksquare}{\blacksquare}$

**7.** $\frac{15}{21} = \frac{\blacksquare}{7}$

**8.** $\frac{2}{3} = \frac{20}{\blacksquare}$

**9.** $\frac{4}{24} = \frac{\blacksquare}{6}$

## PRACTICE

Complete the equivalent fraction.

1. $\frac{3}{5} = \frac{\blacksquare}{10}$
2. $\frac{8}{12} = \frac{4}{\blacksquare}$
3. $\frac{3}{8} = \frac{9}{\blacksquare}$
4. $\frac{16}{20} = \frac{4}{\blacksquare}$
5. $\frac{8}{16} = \frac{\blacksquare}{4}$
6. $\frac{2}{3} = \frac{\blacksquare}{18}$
7. $\frac{\blacksquare}{13} = \frac{28}{52}$
8. $\frac{12}{14} = \frac{6}{\blacksquare}$
9. $\frac{16}{25} = \frac{48}{\blacksquare}$
10. $\frac{21}{39} = \frac{7}{\blacksquare}$
11. $\frac{12}{15} = \frac{\blacksquare}{45}$
12. $\frac{36}{112} = \frac{\blacksquare}{56}$
13. $\frac{7}{9} = \frac{\blacksquare}{63}$
14. $\frac{13}{18} = \frac{\blacksquare}{90}$
15. $\frac{72}{162} = \frac{12}{\blacksquare}$
16. $\frac{\blacksquare}{31} = \frac{48}{62}$
17. $\frac{36}{120} = \frac{\blacksquare}{40}$
18. $\frac{27}{35} = \frac{\blacksquare}{105}$
19. $\frac{38}{\blacksquare} = \frac{190}{200}$
20. $\frac{\blacksquare}{46} = \frac{18}{23}$

Draw a diagram illustrating that the fractions are equivalent.

21. $\frac{5}{10} = \frac{1}{2}$
22. $\frac{2}{3} = \frac{6}{9}$
23. $\frac{10}{16} = \frac{5}{8}$
24. $\frac{5}{20} = \frac{1}{4}$

Write each fraction in *simpler* terms.

25. $\frac{4}{6}$
26. $\frac{8}{12}$
27. $\frac{5}{10}$
28. $\frac{9}{15}$
29. $\frac{10}{16}$
30. $\frac{15}{18}$

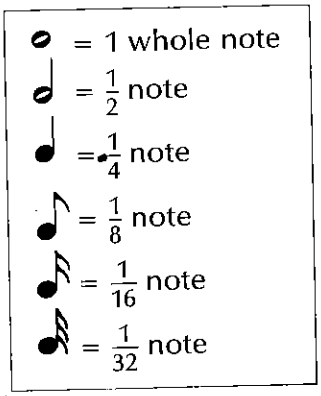
Solve by finding an equivalent fraction.

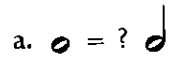
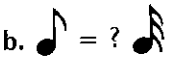
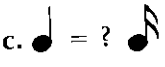
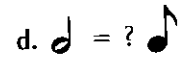
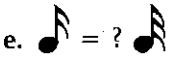
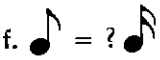
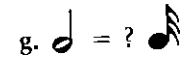
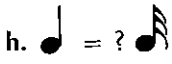
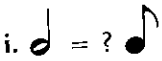
31. Marta finished $\frac{3}{5}$ of her math homework at school. There were 20 questions assigned. How many questions did Marta have left to do at home?

32. Peter needs $\frac{3}{4}$ of a piece of fabric for his model airplane. The piece is 120 cm long. What length of fabric will he use?

## Musical Interlude

Musical notation is based on fractions. Use the table at the right to complete note comparison statements.




a. 
b. 
c. 
d. 
e. 
f. 
g. 
h. 
i. 

One quarter-note (♩) is referred to as one beat. How many beats are there in the following?

j. 

k. 

l. 

# Simplest Terms

A recent survey indicated that 45 out of 75 commuters bought their tickets by the month. What is $\frac{45}{75}$ in simplest terms?

*Method 1*

To write a fraction in **simplest terms**, divide both the numerator and the denominator by the GCF.

$$\frac{45}{75} = \frac{45 \div 15}{75 \div 15} = \frac{3}{5}$$

The GCF of 45 and 75 is 15.

*Method 2*

To simplify a fraction, write the numerator and denominator as products of prime numbers.

**Example:** What is $\frac{126}{210}$ in simplest terms?

$$\frac{126}{210} = \frac{2 \times 3 \times 7 \times 3}{2 \times 3 \times 7 \times 5} = \frac{3}{5}$$

$\frac{126}{210} = \frac{3}{5}$ in simplest terms.

The GCF is $2 \times 3 \times 7$ or 42.

The fraction $\frac{3}{5}$ is in simplest terms because the GCF of the numerator and the denominator is 1.

Two numbers with a GCF of 1 are *relatively prime*.

## EXERCISES

Write the fraction in simplest terms using Method 1.

1. GCF (16, 20) = ■
   $\frac{16}{20} = \frac{■}{■}$
2. GCF (12, 36) = ■
   $\frac{12}{36} = \frac{■}{■}$
3. GCF (75, 100) = ■
   $\frac{75}{100} = \frac{■}{■}$

4. $\frac{36}{81} = \frac{■}{■}$
5. $\frac{60}{45} = \frac{■}{■}$
6. $\frac{56}{84} = \frac{■}{■}$
7. $\frac{40}{140} = \frac{■}{■}$
8. $\frac{300}{75} = \frac{■}{■}$

Write each fraction in simplest terms using Method 2.

9. $\frac{6}{16} = \frac{2 \times 3}{2 \times 2 \times 2 \times 2}$
10. $\frac{15}{9}$
11. $\frac{21}{28}$
12. $\frac{45}{75}$
13. $\frac{200}{10}$
14. $\frac{150}{750}$
15. $\frac{18}{42}$
16. $\frac{25}{500}$
17. $\frac{24}{15}$
18. $\frac{36}{78}$
19. $\frac{110}{22}$

## PRACTICE

Write each fraction in simplest terms.

| | | | | |
|---|---|---|---|---|
| **1.** $\frac{12}{28}$ | **2.** $\frac{55}{11}$ | **3.** $\frac{6}{10}$ | **4.** $\frac{12}{9}$ | **5.** $\frac{15}{75}$ |
| **6.** $\frac{48}{54}$ | **7.** $\frac{120}{72}$ | **8.** $\frac{28}{42}$ | **9.** $\frac{46}{69}$ | **10.** $\frac{28}{70}$ |
| **11.** $\frac{500}{175}$ | **12.** $\frac{240}{400}$ | **13.** $\frac{375}{1000}$ | **14.** $\frac{750}{225}$ | **15.** $\frac{625}{1000}$ |

Write the first fraction in simplest terms. Then complete the missing number in the second equivalent fraction.

| | | | |
|---|---|---|---|
| **16.** $\frac{3}{6} = \frac{5}{\blacksquare}$ | **17.** $\frac{5}{20} = \frac{7}{\blacksquare}$ | **18.** $\frac{24}{20} = \frac{\blacksquare}{30}$ | **19.** $\frac{18}{27} = \frac{\blacksquare}{12}$ |
| **20.** $\frac{15}{24} = \frac{40}{\blacksquare}$ | **21.** $\frac{15}{12} = \frac{100}{\blacksquare}$ | **22.** $\frac{35}{40} = \frac{56}{\blacksquare}$ | **23.** $\frac{81}{90} = \frac{\blacksquare}{1000}$ |

## REVIEW

List the set of factors for each.

| | | | | |
|---|---|---|---|---|
| **1.** 40 | **2.** 63 | **3.** 84 | **4.** 300 | **5.** 288 |

Is the number prime or composite?

| | | | | |
|---|---|---|---|---|
| **6.** 17 | **7.** 9 | **8.** 57 | **9.** 41 | **10.** 51 |

Write each as a product of prime factors.

| | | | | |
|---|---|---|---|---|
| **11.** 45 | **12.** 60 | **13.** 63 | **14.** 1000 | **15.** 360 |

Evaluate the square root.

| | | | | |
|---|---|---|---|---|
| **16.** $\sqrt{36}$ | **17.** $\sqrt{81}$ | **18.** $\sqrt{225}$ | **19.** $\sqrt{10\,000}$ | **20.** $\sqrt{441}$ |

Find the GCF.

| | | | | |
|---|---|---|---|---|
| **21.** 24, 16 | **22.** 30, 45 | **23.** 27, 50 | **24.** 72, 96 | **25.** 210, 140 |

Find the LCM.

| | | | | |
|---|---|---|---|---|
| **26.** 15, 35 | **27.** 40, 24 | **28.** 8, 10, 15 | **29.** 180, 150 | **30.** 21, 14, 30 |

Complete an equivalent fraction.

| | | | | |
|---|---|---|---|---|
| **31.** $\frac{16}{25} = \frac{\blacksquare}{100}$ | **32.** $\frac{35}{85} = \frac{\blacksquare}{17}$ | **33.** $\frac{3}{8} = \frac{\blacksquare}{1000}$ | **34.** $\frac{25}{40} = \frac{\blacksquare}{8}$ | **35.** $\frac{63}{180} = \frac{\blacksquare}{60}$ |

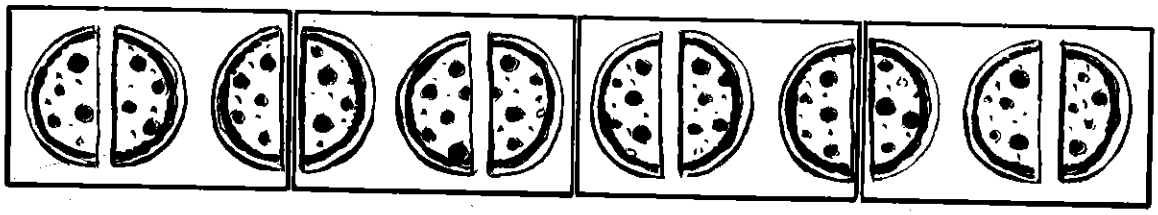
Write each fraction in simplest terms.

| | | | | |
|---|---|---|---|---|
| **36.** $\frac{15}{20}$ | **37.** $\frac{36}{27}$ | **38.** $\frac{30}{54}$ | **39.** $\frac{96}{56}$ | **40.** $\frac{625}{1000}$ |

# Mixed Numerals

The 6 mini-pizzas are to be equally shared by 4 people.

Each person gets $\frac{6}{4}$ or $6 \div 4$ of the pizzas.



The *fraction* $\frac{6}{4}$ can also be written as the **mixed numeral** $1\frac{1}{2}$.

[redacted]

[redacted]

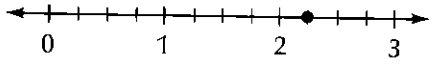
$$\begin{array}{r} 1 \\ 4\overline{)6} \\ -4 \\ \hline 2 \end{array} \qquad \frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}$$

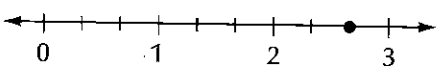
[redacted]

$$4\frac{2}{3} = \frac{3 \times 4 + 2}{3} = \frac{14}{3}$$

## EXERCISES

Write each point as a mixed numeral and as a fraction.

1. 


2. 


Write each as a mixed numeral in simplest terms.

3. $5\frac{3}{6}$ 4. $4\frac{15}{20}$ 5. $3\frac{6}{8}$ 6. $5\frac{20}{24}$ 7. $10\frac{6}{20}$

Write the fraction as a mixed numeral in simplest terms.

8. $\frac{9}{4}$ 9. $\frac{5}{2}$ 10. $\frac{13}{5}$ 11. $\frac{14}{8}$ 12. $\frac{35}{8}$

13. $\frac{44}{6}$ 14. $\frac{65}{10}$ 15. $\frac{90}{25}$ 16. $\frac{72}{52}$ 17. $\frac{118}{50}$

Write the mixed numeral as a fraction in simplest terms.

18. $2\frac{1}{4}$ 19. $1\frac{5}{8}$ 20. $2\frac{2}{7}$ 21. $3\frac{4}{5}$ 22. $10\frac{7}{10}$

23. $6\frac{7}{11}$ 24. $4\frac{3}{4}$ 25. $5\frac{2}{6}$ 26. $11\frac{8}{10}$ 27. $9\frac{6}{8}$

## PRACTICE

Write each as a mixed numeral in simplest terms.

| | | | | |
|---|---|---|---|---|
| **1.** $\frac{12}{5}$ | **2.** $\frac{18}{4}$ | **3.** $\frac{9}{2}$ | **4.** $\frac{17}{8}$ | **5.** $\frac{19}{7}$ |
| **6.** $\frac{28}{10}$ | **7.** $\frac{14}{8}$ | **8.** $\frac{35}{15}$ | **9.** $\frac{50}{12}$ | **10.** $\frac{100}{21}$ |
| **11.** $\frac{21}{16}$ | **12.** $\frac{78}{15}$ | **13.** $\frac{49}{28}$ | **14.** $\frac{808}{100}$ | **15.** $\frac{63}{14}$ |

Write each as a fraction in simplest terms.

| | | | | |
|---|---|---|---|---|
| **16.** $3\frac{9}{12}$ | **17.** $2\frac{10}{12}$ | **18.** $4\frac{3}{7}$ | **19.** $5\frac{6}{8}$ | **20.** $7\frac{6}{9}$ |
| **21.** $8\frac{5}{10}$ | **22.** $3\frac{13}{20}$ | **23.** $12\frac{11}{15}$ | **24.** $15\frac{10}{14}$ | **25.** $2\frac{9}{12}$ |
| **26.** $7\frac{7}{10}$ | **27.** $8\frac{15}{27}$ | **28.** $9\frac{1}{11}$ | **29.** $14\frac{14}{20}$ | **30.** $6\frac{37}{100}$ |

Solve.

**31.** Harold fertilizes his lawn and garden 4 times a year. He uses 10 bags of fertilizer during the year. How many bags of fertilizer does he use each time?

**32.** Doris and her 3 friends were given 10 cakes left over from the carnival for helping to clean up. How much cake did each person get?

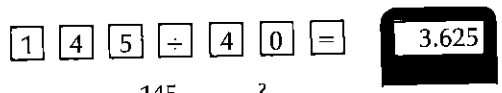
# Calculator Changes

A calculator can be used to change a fraction to a mixed numeral.

The example below shows how $\frac{145}{40}$ could be changed.

*Step 1:*

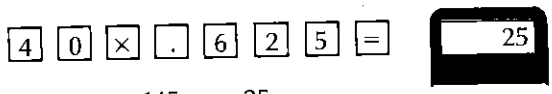
Divide to find the whole number.




$\frac{145}{40} = 3\frac{?}{40}$

*Step 2:*

Multiply the denominator by the decimal part of the quotient to find the numerator of the fraction.

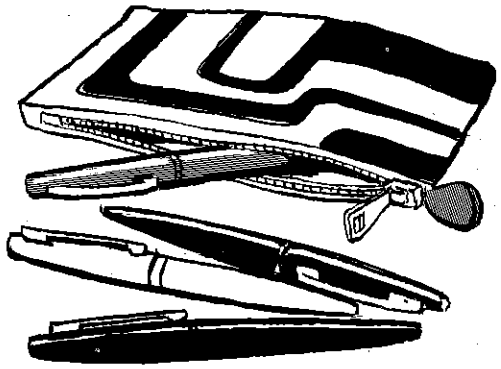



$\frac{145}{40} = 3\frac{25}{40}$

Use a calculator to change each fraction to a mixed numeral.

| | | | | | |
|---|---|---|---|---|---|
| **1.** $\frac{357}{8}$ | **2.** $\frac{246}{16}$ | **3.** $\frac{975}{40}$ | **4.** $\frac{311}{125}$ | **5.** $\frac{477}{64}$ | **6.** $\frac{508}{40}$ |

# Comparing Fractions



Tom, Patsy, and Gina have the same number of felt-tip pens for their art project. Tom uses $\frac{2}{5}$ of his pens. Patsy uses $\frac{3}{5}$, and Gina uses $\frac{4}{7}$. Does Gina use more pens than Patsy?

To compare fractions with *different denominators*, find equivalent fractions with the same denominator.

Gina: $\frac{4}{7} = \frac{4 \times 5}{7 \times 5} = \frac{20}{35}$ Patsy: $\frac{3}{5} = \frac{3 \times 7}{5 \times 7} = \frac{21}{35}$

$\frac{21}{35} > \frac{20}{35}$ $\frac{3}{5} > \frac{4}{7}$

Gina uses fewer pens than Patsy.

## EXERCISES

Copy and compare. Use $<$ or $>$.

1. $\frac{6}{7} \bullet \frac{2}{7}$ 2. $\frac{5}{11} \bullet \frac{4}{11}$ 3. $\frac{15}{8} \bullet \frac{13}{8}$ 4. $2\frac{1}{6} \bullet 2\frac{5}{6}$ 5. $\frac{2}{50} \bullet \frac{1}{50}$

6. $\frac{3}{4} = \frac{\blacksquare}{12}$ $\frac{2}{3} = \frac{\blacksquare}{12}$ $\frac{3}{4} \bullet \frac{2}{3}$

7. $\frac{1}{2} = \frac{\blacksquare}{8}$ $\frac{3}{8} = \frac{\blacksquare}{8}$ $\frac{1}{2} \bullet \frac{3}{8}$

8. $\frac{5}{7} = \frac{\blacksquare}{63}$ $\frac{5}{9} = \frac{\blacksquare}{63}$ $\frac{5}{7} \bullet \frac{5}{9}$

9. $\frac{17}{15} = \frac{\blacksquare}{30}$ $\frac{14}{10} = \frac{\blacksquare}{30}$ $\frac{17}{15} \bullet \frac{14}{10}$

10. $\frac{5}{7} \bullet \frac{2}{3}$ 11. $\frac{7}{12} \bullet \frac{3}{4}$ 12. $\frac{12}{5} \bullet \frac{10}{3}$ 13. $\frac{8}{16} \bullet \frac{3}{5}$

14. $\frac{7}{15} \bullet \frac{4}{10}$ 15. $\frac{7}{5} \bullet 1\frac{1}{2}$ 16. $\frac{7}{12} \bullet \frac{2}{3}$ 17. $\frac{5}{2} \bullet 2\frac{1}{4}$

Rewrite the fractions in each set with a common denominator. Order the fractions from least to greatest.

18. $\left\{\frac{3}{10}, \frac{4}{5}, \frac{1}{2}, \frac{3}{4}\right\}$ 19. $\left\{\frac{2}{3}, \frac{5}{6}, \frac{3}{4}, \frac{1}{3}\right\}$ 20. $\left\{\frac{1}{3}, \frac{2}{5}, \frac{1}{6}, \frac{1}{2}\right\}$

21. $\left\{\frac{3}{4}, \frac{8}{6}, \frac{1}{2}, \frac{7}{12}, \frac{2}{3}\right\}$ 22. $\left\{\frac{3}{10}, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{1}{4}\right\}$ 23. $\left\{1\frac{1}{4}, \frac{9}{10}, \frac{5}{8}, \frac{3}{2}, \frac{3}{4}\right\}$

## PRACTICE

Copy and compare. Use $<$ or $>$.

1. $\frac{2}{3} \bullet \frac{5}{6}$
2. $\frac{3}{2} \bullet 1\frac{3}{4}$
3. $\frac{5}{7} \bullet \frac{7}{5}$
4. $\frac{6}{10} \bullet \frac{6}{8}$
5. $\frac{6}{5} \bullet \frac{9}{7}$
6. $\frac{7}{15} \bullet \frac{3}{10}$
7. $2\frac{2}{5} \bullet \frac{8}{3}$
8. $\frac{5}{6} \bullet \frac{7}{10}$
9. $\frac{11}{16} \bullet \frac{19}{24}$
10. $\frac{25}{30} \bullet \frac{42}{50}$
11. $\frac{12}{21} \bullet \frac{9}{14}$
12. $\frac{17}{25} \bullet \frac{7}{10}$
13. $\frac{1}{6} \bullet \frac{2}{13}$
14. $\frac{4}{5} \bullet \frac{5}{6}$
15. $\frac{12}{23} \bullet \frac{13}{24}$
16. $\frac{10}{11} \bullet \frac{9}{10}$
17. $\frac{5}{33} \bullet \frac{3}{13}$
18. $\frac{4}{9} \bullet \frac{9}{20}$
19. $\frac{3}{8} \bullet \frac{11}{20}$
20. $\frac{5}{9} \bullet \frac{17}{30}$

Order the fractions in each set from least to greatest.

13. $\left\{\frac{1}{2}, \frac{2}{3}, \frac{1}{6}, \frac{1}{3}\right\}$
14. $\left\{\frac{2}{5}, \frac{3}{10}, \frac{1}{2}, \frac{4}{5}\right\}$
15. $\left\{\frac{3}{5}, \frac{1}{6}, \frac{5}{3}, \frac{2}{3}, 1\frac{1}{3}, \frac{1}{2}\right\}$
16. $\left\{\frac{1}{2}, \frac{1}{4}, \frac{2}{3}, \frac{5}{6}\right\}$
17. $\left\{\frac{7}{8}, 1\frac{1}{4}, \frac{2}{3}, \frac{5}{6}, \frac{6}{4}\right\}$
18. $\left\{\frac{7}{15}, \frac{5}{6}, \frac{9}{10}, \frac{6}{5}, 1\frac{1}{10}, \frac{4}{5}\right\}$

Solve.

21. At one lunchroom table, a pizza was cut into 6 pieces and 5 of these were eaten. At another table, a pizza was cut into 15 pieces and 11 were eaten. Was more pizza eaten at the first table or the second?

# Circle Graphs

Students at Riel High School were polled on their favourite sports. The results were as follows:

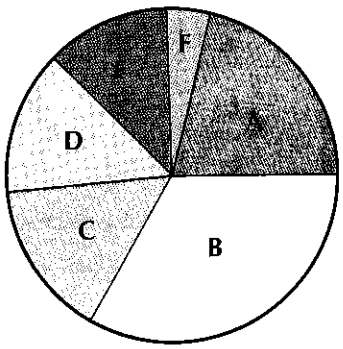
$\frac{1}{3}$ like hockey.

$\frac{5}{24}$ like baseball.

$\frac{1}{8}$ like football.

$\frac{5}{36}$ like swimming.

$\frac{11}{72}$ like tennis.




The rest like other sports.

Rank the sports by writing each fraction using 360 as the denominator.
Why did we use 360?
Match the sports to the regions of the graph.
What fraction (in simplest terms) like other sports?

# Fractions to Decimals

We divide to convert a fraction to a decimal.

$\frac{3}{8}$ means 3 divided by 8.

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -24 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline \end{array} \quad \frac{3}{8} = 0.375$$

The quotient 0.375 is called a **terminating decimal**.

$\frac{5}{6}$ means 5 divided by 6.

$$\begin{array}{r} 0.833 \\ 6\overline{)5.000} \\ -48 \\ \hline 20 \\ -18 \\ \hline 20 \\ -18 \\ \hline 2 \end{array} \quad \frac{5}{6} = 0.8333\ldots = 0.8\overline{3}$$

The quotient $0.8\overline{3}$ is called a **repeating decimal**. The bar over the 3 shows that it repeats forever.

Some fractions can be converted to decimals by finding an equivalent fraction with a denominator that is a power of 10.

$$\frac{3}{8} = \frac{3}{8}\frac{\times 125}{\times 125} = \frac{375}{1000} = 0.375$$

## EXERCISES

1. Write as an equivalent fraction with a denominator of 10, 100, or 1000. Then rewrite as a decimal.

   a. $\frac{3}{4}$ b. $\frac{7}{20}$ c. $3\frac{1}{2}$ d. $5\frac{4}{5}$ e. $\frac{7}{50}$

   f. $\frac{9}{250}$ g. $\frac{5}{8}$ h. $1\frac{3}{8}$ i. $\frac{21}{125}$ j. $6\frac{43}{50}$

2. Divide to convert the fractions to *terminating* decimals.

   a. $\frac{1}{4} = \blacksquare$ b. $\frac{2}{4} = \blacksquare$ c. $\frac{3}{4} = \blacksquare$ d. $\frac{4}{4} = \blacksquare$ e. $\frac{5}{4} = \blacksquare$

   f. $\frac{1}{2} = \blacksquare$ g. $\frac{2}{2} = \blacksquare$ h. $\frac{3}{2} = \blacksquare$ i. $\frac{4}{2} = \blacksquare$ j. $\frac{5}{2} = \blacksquare$

   k. $\frac{1}{8} = \blacksquare$ l. $\frac{2}{8} = \blacksquare$ m. $\frac{3}{8} = \blacksquare$ n. $\frac{4}{8} = \blacksquare$ o. $\frac{5}{8} = \blacksquare$

3. Rewrite each decimal repeating pattern using a bar.

   a. $\frac{7}{48} = 0.145\,833\,333\ldots$ b. $\frac{2}{7} = 0.285\,714\,285\,714\,285\ldots$

4. Divide to convert the fraction to a *repeating* decimal. Place a bar over the repeating pattern.

   a. $\frac{1}{6} = \blacksquare$ b. $\frac{2}{6} = \blacksquare$ c. $\frac{4}{6} = \blacksquare$ d. $\frac{5}{6} = \blacksquare$ e. $\frac{7}{6} = \blacksquare$

   f. $\frac{1}{9} = \blacksquare$ g. $\frac{2}{9} = \blacksquare$ h. $\frac{3}{9} = \blacksquare$ i. $\frac{4}{9} = \blacksquare$ j. $\frac{5}{9} = \blacksquare$

## PRACTICE

Convert the fraction to a decimal. Use a bar to show repeating patterns.

| | | | | |
|---|---|---|---|---|
| **1.** $\frac{79}{100}$ | **2.** $6\frac{3}{10}$ | **3.** $\frac{755}{1000}$ | **4.** $\frac{8}{100}$ | **5.** $\frac{2}{1000}$ |
| **6.** $\frac{7}{9}$ | **7.** $\frac{5}{8}$ | **8.** $\frac{7}{32}$ | **9.** $\frac{2}{45}$ | **10.** $\frac{1}{3}$ |
| **11.** $\frac{9}{16}$ | **12.** $\frac{7}{18}$ | **13.** $\frac{13}{20}$ | **14.** $\frac{9}{37}$ | **15.** $\frac{16}{25}$ |
| **16.** $\frac{13}{22}$ | **17.** $\frac{11}{50}$ | **18.** $\frac{3}{4}$ | **19.** $\frac{19}{36}$ | **20.** $\frac{29}{9}$ |

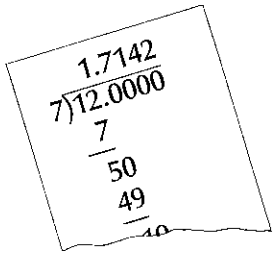
Compare using $<$, $=$, or $>$.

**21.** $0.762 \bullet \frac{21}{32}$ **22.** $\frac{6}{9} \bullet 0.\overline{6}$ **23.** $0.36 \bullet \frac{8}{21}$ **24.** $\frac{51}{99} \bullet 0.53$

Solve.

**25.** Darcy has 15 hits in 42 times at bat while Stacy has a batting average of 0.356. Who has the better average?

# Computer Conversion




The BASIC program converts a fraction to a decimal. The output of the program for N = 12 and D = 7 is

$$\frac{12}{7} = 1.7142.$$

1. Trace the value of the program variables N, D, and A in a table until I = 4.

| | | | |
|---|---|---|---|
| 1 | 50 | 7 | 7 |
| 2 | 10 | 7 | 1 |
| 3 | | | |
| 4 | | | |

```
100 REM A FRACTION TO DECIMAL CONVERSION
110 REM ACCURATE TO 4 DECIMAL PLACES
120 PRINT: PRINT
130 INPUT"WHAT IS THE NUMERATOR";N
140 INPUT"WHAT IS THE DENOMINATOR";D
150 LET A = INT(N/D)
160 PRINT: PRINT N;"/";D;"=";A;".";
170 FOR I = 1 TO 4
180 LET N = (N-A*D)*10
190 LET A = INT(N/D)
200 PRINT A;
210 NEXT I
220 GOTO 120
230 END
```

2. Relate the values of N, D, A, and I in the table to the numbers in the division problem above.

# ecimals to Fractions

To convert a terminating decimal to a fraction, write the decimal in fraction form and convert it to simplest terms.

$0.8 = \frac{8}{10} = \frac{4}{5}$ (in simplest terms) — 8 tenths

$0.045 = \frac{45}{1000} = \frac{9}{200}$ (in simplest terms) — 45 thousandths

**Unit fractions** can be used to convert decimals to fractions.

$0.2 = \boxed{\frac{1}{5}}$

$0.4 = \frac{2}{5}$

$0.6 = \frac{3}{5}$ and so on.

$0.05 = \boxed{\frac{1}{20}}$

$0.10 = \frac{2}{20}$

$0.15 = \frac{3}{20}$ and so on.

Unit fractions can also be used to convert repeating decimals to fractions.

$0.\overline{3} = \boxed{\frac{1}{3}}$

$0.\overline{6} = \frac{2}{3}$ and so on.

$0.\overline{09} = \boxed{\frac{1}{11}}$

$0.\overline{18} = \frac{2}{11}$

$0.\overline{27} = \frac{3}{11}$ and so on.

## EXERCISES

Convert each decimal to a fraction or mixed numeral in simplest terms.

1. $0.2 = \frac{\blacksquare}{10} = \frac{\blacksquare}{\blacksquare}$
2. $0.7$
3. $3.4$
4. $6.8$
5. $0.45 = \frac{\blacksquare}{100} = \frac{\blacksquare}{\blacksquare}$
6. $0.82$
7. $0.96$
8. $5.24$
9. $0.375 = \frac{\blacksquare}{1000} = \frac{\blacksquare}{\blacksquare}$
10. $0.654$
11. $32.002$
12. $1.045$
13. $0.2463 = \frac{\blacksquare}{10\ 000}$
14. $0.0002$
15. $4.0025$
16. $0.0050$
17. $0.\overline{1} = \frac{\blacksquare}{9}$
18. $0.\overline{2} = \frac{\blacksquare}{9}$
19. $0.\overline{3} = \frac{\blacksquare}{\blacksquare}$
20. $0.\overline{4} = \frac{\blacksquare}{\blacksquare}$
21. $0.\overline{01} = \frac{\blacksquare}{99}$
22. $0.\overline{02} = \frac{\blacksquare}{99}$
23. $0.\overline{03} = \frac{\blacksquare}{\blacksquare}$
24. $0.\overline{04} = \frac{\blacksquare}{\blacksquare}$
25. $0.\overline{142\ 857} = \frac{\blacksquare}{7}$
26. $0.\overline{285\ 714} = \frac{\blacksquare}{7}$
27. $0.\overline{428\ 571} = \frac{\blacksquare}{7}$
28. $0.\overline{571\ 428} = \frac{\blacksquare}{7}$

## PRACTICE

Convert each decimal to a fraction or mixed numeral in simplest terms.

| | | | | |
|---|---|---|---|---|
| **1.** 0.6 | **2.** 0.2 | **3.** 8.15 | **4.** 0.35 | **5.** 0.24 |
| **6.** 0.16 | **7.** 0.012 | **8.** 0.055 | **9.** 4.0008 | **10.** 0.0004 |
| **11.** 2.875 | **12.** 1.0125 | **13.** 1.38 | **14.** 2.46 | **15.** 35.07 |
| **16.** 12.625 | **17.** 41.01 | **18.** 9.35 | **19.** 0.0025 | **20.** 32.755 |
| **21.** $0.\overline{3}$ | **22.** $0.\overline{6}$ | **23.** $0.\overline{7}$ | **24.** $0.\overline{5}$ | **25.** $0.\overline{2}$ |
| **26.** $0.\overline{05}$ | **27.** $0.\overline{07}$ | **28.** $0.\overline{10}$ | **29.** $0.\overline{14}$ | **30.** $0.\overline{23}$ |

**31.** **a.** Continue the pattern:
0.5, 0.25, 0.125, 0.0625, ■, ■, ■

**b.** Write the pattern using fractions.

**c.** Continue the fraction pattern for 3 more terms.
How many decimal places will the last term of the fraction pattern have?

**32.** Write the repeating decimal for $\frac{1}{999}$.

Write a fraction for each decimal.

**a.** $0.\overline{003}$ **b.** $0.\overline{023}$ **c.** $0.\overline{101}$ **d.** $0.\overline{303}$

**33.** Write the repeating decimal for $\frac{1}{9999}$.

Write a fraction for each decimal:

**a.** $0.\overline{0004}$ **b.** $0.\overline{0028}$ **c.** $0.\overline{0753}$ **d.** $1.\overline{4762}$

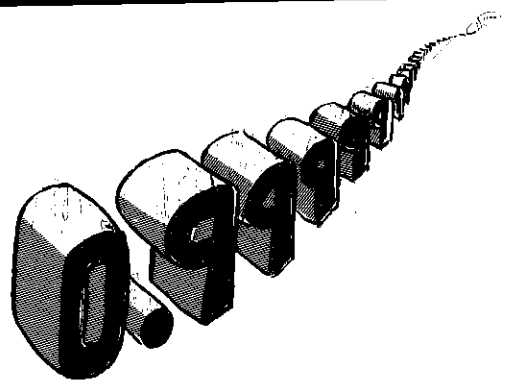
# Terminations

We have seen that $\frac{1}{9} = 0.111\,11\ldots$

So $9 \times \frac{1}{9} = 9 \times 0.111\,11\ldots$
$= 0.999\,99\ldots$
$= 0.\overline{9}$

But $9 \times \frac{1}{9} = 1$

So $1 = 0.\overline{9}$




Is it true that any fraction can be written as a non-terminating decimal? Write a terminating decimal and a fraction for each of these non-terminating decimals.

**1.** $0.4\overline{9}$ **2.** $0.24\overline{9}$ **3.** $0.1\overline{9}$ **4.** $0.5\overline{9}$ **5.** $0.2\overline{9}$

# Sets

A **set** is a collection of objects or **elements**.
A set may be specified by a description or by a listing of its elements.

$A$ is the set of factors of 80.
$A = \{1, 2, 4, 5, 8, 10, 16, 20, 40, 80\}$

$B$ is the set of factors of 48.
$B = \{1, 2, 3, 4, 6, 8, 12, 16, 24, 48\}$

The symbols $\in$ and $\notin$ are used to show the relationship of an element and a set.

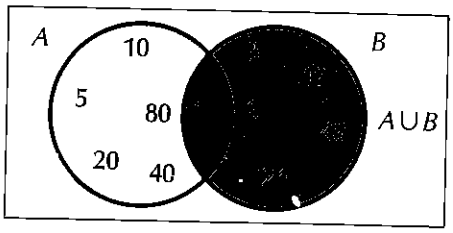
$24 \notin A$ means:
24 *is not an element of* set $A$.

**Set Union:**

[redacted]

$A \cup B$ is the set of all factors of either 80 or 48.

$A \cup B = \{1, 2, 3, 4, 5, 6, 8, 10, 12, 16, 20, 24, 40, 48, 80\}$



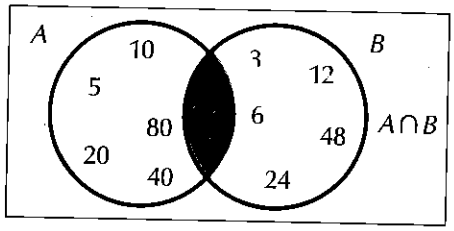

These illustrations are called **Venn diagrams.**

**Set Intersection:**

[redacted]

$A \cap B$ is the set of factors common to 80 and 48.

$A \cap B = \{1, 2, 4, 8, 16\}$




The **empty set** ($\phi$ or $\{\ \}$) is the set containing no elements.
If the set of flying horses is set $F$, then $F = \phi$ or $F = \{\ \}$.

## EXERCISES

List the elements of each set.

$A = \{1, 3, 5, 7\}$
$B = \{2, 5, 9\}$
$C = \{2, 4, 6\}$

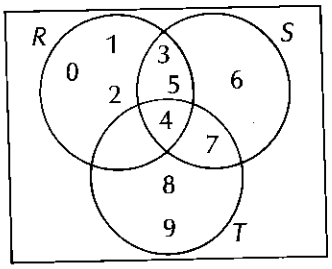
1. $A \cup B$
2. $A \cap B$
3. $A \cup C$
4. $A \cap C$
5. $B \cup C$
6. $B \cap C$
7. $A \cup B \cup C$
8. $A \cap B \cap C$
9. $A \cap (B \cup C)$
10. $(A \cap C) \cup B$

## PRACTICE

1. List the elements of each set.
   a. $M$ = the set of multiples of 11 up to 165
   b. $Y$ = the set of factors of 60.
   c. $P$ = the set of prime numbers between 5 and 20.
   d. $E$ = the set of even factors of 27.
   e. $F$ = the set of fractions equivalent to $\frac{14}{21}$, expressed in *simpler terms*.
   f. $C$ = the set of common factors of 252 and 462.
   g. $B$ = the set of fractions with denominator 10 between $\frac{1}{3}$ and $\frac{5}{6}$.
   h. $R$ = the set of repeating digits in the decimal form for $\frac{1}{7}$.

2. Use $\in$ or $\notin$ to complete each statement.
   $P = \{2, 4, 6, 8\}$ $Q = \{\bigcirc, \star, \square, \triangle\}$ $R = \{\ \}$
   a. $5 \bullet P$ b. $\square \bullet Q$ c. $6 \bullet R$ d. $\star \bullet P$
   e. $4 \bullet P$ f. $3 \bullet Q$ g. $P \bullet Q$ h. $\bigcirc \bullet R$

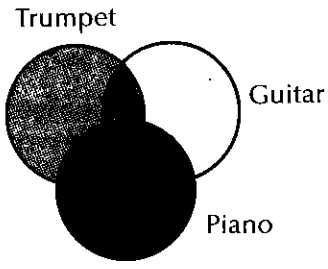
3. Use the Venn Diagram at the right to list the elements of each set.
   a. $R \cup S$ b. $S \cup T$
   c. $R \cap S \cap T$ d. $R \cup S \cup T$
   e. $(R \cap S) \cup T$ f. $(R \cap S) \cap T$
   g. $(R \cup S) \cap (R \cup T)$ h. $(R \cap S) \cup (R \cap T)$




4. Draw a Venn Diagram. Then list the set members.
   $X$ = {the factors of 54} $Y$ = {the factors of 12} $Z$ = {the even factors of 24}
   a. $X \cup Y$ b. $X \cap Y$ c. $Y \cup Z$ d. $Y \cap Z$
   e. $X \cup Z$ f. $X \cap Z$ g. $X \cup Y \cup Z$ h. $X \cap Y \cap Z$

## A Set Solution

Complete the Venn Diagram to solve the problem.
Forty-four Grade 8 students play musical instruments.
20 play trumpet, 20 play guitar, and 20 play piano.
3 play trumpet, guitar, and piano.
7 play trumpet and guitar only.
8 play guitar and piano only.
4 play trumpet and piano only.




How many students play just trumpet? just guitar? just piano?

# Computer Factor Pairs

The program below lists the factor pairs of numbers.

```
10 PRINT "THIS PROGRAM PRINTS THE FACTOR PAIRS"
20 PRINT "OF A NUMBER."
30 PRINT:PRINT
40 PRINT "ENTER A NUMBER.";
50 INPUT N
60 PRINT:PRINT
70 PRINT "THE FACTOR PAIRS OF ";N;" ARE ..."
80 PRINT
90 REM LINE 110 CHECKS TO SEE IF F DIVIDES N EVENLY
100 FOR F = 1 TO N
110 IF N/F <> INT(N/F) THEN GOTO 130
120 PRINT F;N/F
130 NEXT F
140 PRINT:PRINT
150 GOTO 30
160 END
```

1. What is the output for $N = 193$? for $N = 529$?
2. How could the program be used to find a *square number*?
3. Use the program to find the squares.

   **a.** 841 **b.** 1764 **c.** 4356 **d.** 2718 **e.** 1521 **f.** 2304

# The Factorial Symbol

Three persons can stand in line in 6 different ways.

— Any of the $\boxed{3}$ persons can stand in the first place.
— Once someone is standing in first place, either of the remaining $\boxed{2}$ persons can stand in the second place.
— Once the first two places are filled, there is only $\boxed{1}$ person left to stand in the third place.

Therefore, the number of possible arrangements is $\boxed{3 \times 2 \times 1}$ or $\boxed{6}$.
$3 \times 2 \times 1$ can be written as **3!** or **three factorial.**

**a.** In how many different ways can 4 people line up?

**b.** What is 4! ? **c.** What is 5! ? **d.** What is 6! ?

**e.** In how many different ways can 5 people line up?

# REVIEW

Write each fraction as a mixed numeral in simplest terms.

**1.** $\frac{5}{3}$ **2.** $\frac{3}{2}$ **3.** $\frac{6}{5}$ **4.** $\frac{12}{8}$ **5.** $\frac{17}{2}$

**6.** $\frac{27}{8}$ **7.** $\frac{66}{10}$ **8.** $\frac{34}{16}$ **9.** $\frac{42}{28}$ **10.** $\frac{18}{6}$

Write each mixed numeral as a fraction in simplest terms.

**11.** $3\frac{5}{8}$ **12.** $1\frac{1}{4}$ **13.** $2\frac{1}{5}$ **14.** $5\frac{1}{6}$ **15.** $21\frac{7}{9}$

**16.** $1\frac{2}{12}$ **17.** $5\frac{4}{8}$ **18.** $3\frac{4}{6}$ **19.** $2\frac{3}{9}$ **20.** $5\frac{5}{10}$

Compare using $<$, $>$, or $=$.

**21.** $\frac{3}{8} \bullet \frac{4}{9}$ **22.** $\frac{6}{5} \bullet 1\frac{2}{9}$ **23.** $\frac{3}{8} \bullet \frac{4}{7}$ **24.** $\frac{27}{50} \bullet \frac{11}{20}$

**25.** $\frac{2}{3} \bullet \frac{3}{12}$ **26.** $\frac{3}{10} \bullet \frac{6}{20}$ **27.** $\frac{4}{3} \bullet \frac{6}{5}$ **28.** $\frac{9}{12} \bullet \frac{15}{20}$

**29.** $\frac{14}{16} \bullet \frac{10}{12}$ **30.** $2\frac{1}{3} \bullet 1\frac{7}{8}$ **31.** $3\frac{2}{3} \bullet \frac{10}{3}$ **32.** $\frac{17}{2} \bullet 8\frac{2}{3}$

Convert to a decimal.

**33.** $\frac{17}{20}$ **34.** $\frac{5}{18}$ **35.** $\frac{2}{3}$ **36.** $\frac{4}{9}$ **37.** $\frac{7}{8}$

**38.** $3\frac{5}{8}$ **39.** $2\frac{5}{6}$ **40.** $\frac{16}{3}$ **41.** $2\frac{1}{9}$ **42.** $4\frac{3}{5}$

Convert to a fraction in simplest terms.

**43.** 0.4 **44.** 1.6 **45.** 0.38 **46.** 1.62 **47.** 0.125

**48.** 2.375 **49.** $0.\overline{3}$ **50.** $0.\overline{6}$ **51.** $2.\overline{3}$ **52.** $0.0\overline{3}$

List the elements of each set.

**53.** $F$ = the set of factors of 48.

**54.** $G$ = the set of factors of 36.

**55.** $H = F \cup G$

**56.** $K = F \cap G$

**57.** $P$ = the set of prime numbers less than 15.

**58.** $Q$ = the set of fractions equivalent to $\frac{20}{24}$, expressed in *simpler terms.*

**59.** $M$ = the common multiples of 3 and 5, up to 105.

# TEST UNIT 6

List the set of factors for each.

**1.** 15 **2.** 90 **3.** 156 **4.** 300

Is the number prime or composite?

**5.** 31 **6.** 57 **7.** 79 **8.** 99

Write each as a product of prime factors. Use exponents.

**9.** 150 **10.** 208 **11.** 630 **12.** 1000

Evaluate.

**13.** $\sqrt{16}$ **14.** $\sqrt{225}$ **15.** $\sqrt{961}$ **16.** $\sqrt{10\,000}$

Find the GCF.

**17.** 72, 27 **18.** 110, 44 **19.** 29, 57 **20.** 28, 42, 126

Find the LCM.

**21.** 16, 18 **22.** 95, 25 **23.** 160, 240 **24.** 35, 21, 14

Complete an equivalent fraction.

**25.** $\frac{27}{40} = \frac{\blacksquare}{200}$ **26.** $\frac{\blacksquare}{9} = \frac{16}{36}$ **27.** $\frac{50}{75} = \frac{2}{\blacksquare}$ **28.** $\frac{31}{72} = \frac{155}{\blacksquare}$

Write each fraction in simplest terms.

**29.** $\frac{18}{20}$ **30.** $\frac{27}{63}$ **31.** $\frac{42}{105}$ **32.** $\frac{375}{1000}$

Write each fraction as a mixed numeral in simplest terms.

**33.** $\frac{25}{8}$ **34.** $\frac{94}{20}$ **35.** $\frac{366}{75}$ **36.** $\frac{1108}{25}$

Write each mixed numeral as a fraction in simplest terms.

**37.** $3\frac{5}{7}$ **38.** $2\frac{4}{6}$ **39.** $17\frac{5}{9}$ **40.** $12\frac{16}{20}$

Compare using $<$ or $>$.

**41.** $\frac{3}{8} \bullet \frac{2}{5}$ **42.** $\frac{15}{22} \bullet \frac{19}{33}$ **43.** $\frac{15}{6} \bullet 2\frac{3}{4}$ **44.** $4\frac{1}{4} \bullet \frac{21}{5}$

Convert to a decimal.

**45.** $\frac{5}{12}$ **46.** $3\frac{4}{5}$ **47.** $1\frac{5}{16}$ **48.** $\frac{28}{66}$

Convert to a fraction in simplest terms.

**49.** 0.65 **50.** 0.700 **51.** 0.125 **52.** $0.\overline{3}$

Solve.

1. Name two ways that a computer could be used in a school.
2. Name two peripheral devices for a computer.
3. Convert each number from base 2 to base 10.

   a. 11 b. 101 c. 1011 d. 1111

   e. 10110 f. 111000 g. 11001100 h. 11000011

4. Convert each number from base 10 to base 2.

   a. 7 b. 12 c. 16 d. 27

   e. 38 f. 94 g. 155 h. 255

Describe the output for each program.

5.
```
10 LET B = 1
20 PRINT B
30 LET B = B + 1
40 GOTO 20
50 END
```

6.
```
10 READ P
20 IF P = 0 THEN 60
30 PRINT P^2
40 GOTO 10
50 DATA 0.1,1,10,100,1000,0
60 END
```

7.
```
200 INPUT "TYPE THE FIRST NUMERATOR";N1
400 INPUT "TYPE THE FIRST DENOMINATOR";D1
450 PRINT
600 INPUT "TYPE THE SECOND NUMERATOR";N2
800 INPUT "TYPE THE SECOND DENOMINATOR";D2
850 PRINT
900 IF (N1*D2) > (N2*D1) THEN PRINT
    "THE FIRST FRACTION IS BIGGER!!"
1000 IF (N1*D2) < (N2*D1) THEN PRINT
     "THE SECOND FRACTION IS BIGGER!!"
1100 IF (N1*D2) = (N2*D1) THEN PRINT
     "THE FRACTIONS ARE EQUAL!!"
1200 END
```

   a. What is the output if $N1 = 5$, $D1 = 7$, $N2 = 8$, and $D2 = 11$?

   b. What new line would simplify the task of comparing many fractions?

8. Write a program that will make the computer print the odd numbers from 1111 to 1143.