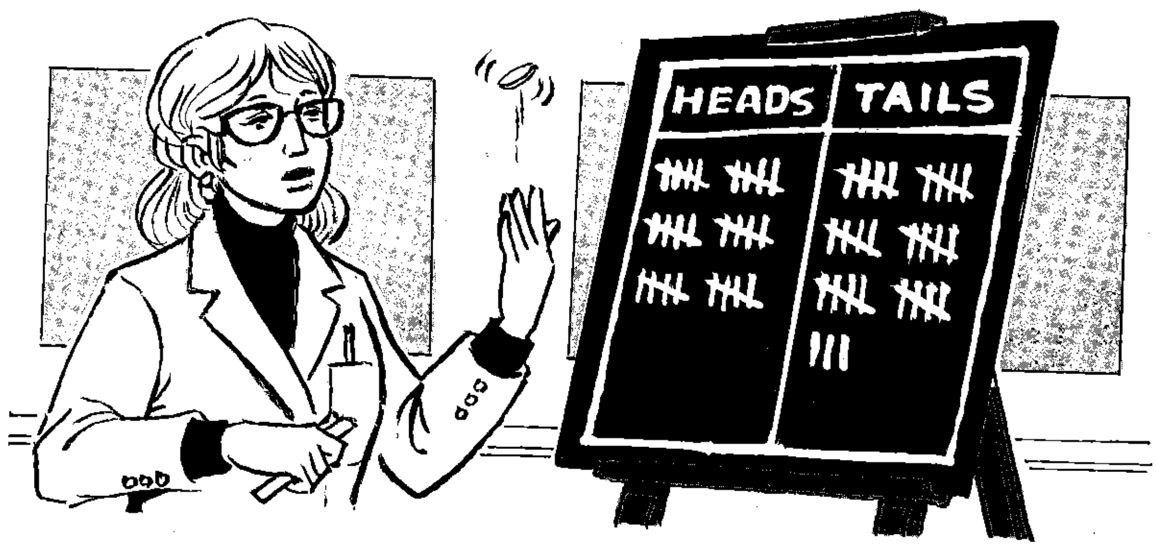
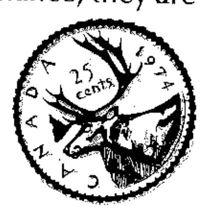
UNIT 15
Probability and Statistics



Outcomes

An **outcome** is the result of an action such as the flipping of a coin. When a coin is flipped, it will land either heads up or tails up. Thus there are two possible outcomes which are *equally likely* to occur. Since the outcomes are strictly a matter of chance, they are said to occur *randomly*.



Two possible outcomes: {heads, tails}

The **probability** of each outcome is 1 chance in 2: $\frac{1}{2}$ $\frac{1}{2}$

We write each probability: $P(\text{heads}) = \frac{1}{2}$ or 0.5 $\quad P(\text{tails}) = \frac{1}{2}$ or 0.5

In any situation where any one of n different outcomes is equally likely, the probability that any one of them will occur is $\frac{1}{n}$.

The probability of an outcome ranges from 0 (when it is certain not to happen) to 1 (when it is certain to happen).

P (The sun sets in the west.) $= 1$

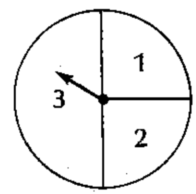
P (The sun is up.) $\approx \frac{1}{2}$ or 0.5

P (The sun sets in the east.) $= 0$

EXERCISES

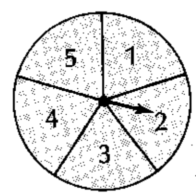
1. There are 3 marbles in a jar; a red one, a green one, and a purple one.
   a. List the possible outcomes for taking a marble from the jar.
   b. P (red marble) = ■
   c. P (green marble) = ■
   d. P (white marble) = ■

2. The spinner to the right is spun.
   a. Is it equally likely that either a 2 or a 3 will be obtained? Why?
   b. How could the spinner be changed so that each outcome would be equally likely?



PRACTICE

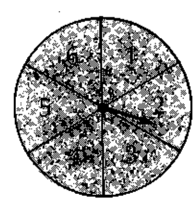
1. For each experiment given, list the set of possible outcomes. Then write the probability of each given outcome as a fraction and as a decimal.
   a. rolling a die
   b. spinning the spinner shown at the right
   c. choosing a pool ball from a bag containing 15 differently-numbered balls.
   d. selecting a student from a group of 10 students
   e. choosing a card from a deck of 52



2. Which of the following are likely to be random outcomes and which are not? Explain.
   a. choosing a raffle ticket number from a barrel of ticket stubs
   b. electing a class president
   c. choosing the winner of a race
   d. selecting a number by spinning a roulette wheel
   e. choosing a coin from a bag

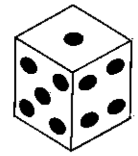
3. If a coin is chosen from a bag containing 2 dimes and a penny, is it just as likely that a penny will be chosen as a dime? Why or why not?

4. The spinner to the right is spun. Write each probability as a fraction and as a decimal.
   a. P (2) = ■
   b. P (9) = ■
   c. P (1, 2, 3, 4, 5, or 6) = ■
   d. Is it just as likely that a 3 is obtained as any number less than 4? Why or why not?



Events

25¢ For Each Roll
Roll a 2 and win a dollar.
Roll a 3 and win a penny.



A roll of a 2 or 3 on the die would win a cash prize. Thus there are 2 favourable outcomes out of the 6 likely outcomes.

A set of one or more favourable outcomes is called an **event.**

The event is { ⚁ or ⚂ }.

The probability of an event P (E) is the sum of the probabilities of the favourable outcomes.

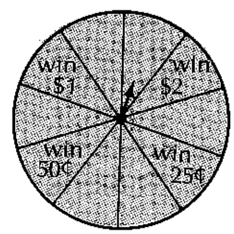
Six possible outcomes: { ⚀, ⚁, ⚂, ⚃, ⚄, ⚅ }

P (E) or P (⚁ or ⚂): $\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$ or $\frac{1}{3}$ or $0.\overline{3}$

If an experiment can result in n different equally likely outcomes and an event consists of f favourable outcomes, the probability the event will occur is:

$$P\,(\text{E}) = \frac{f \leftarrow \text{number of favourable outcomes}}{n \leftarrow \text{number of possible outcomes}}$$

EXERCISES



1. Refer to the spinner at the right.
   Write the probability as a fraction and as a decimal.
   a. P (win \$2) $= \frac{■}{■}$ or ■.■
   b. P (win 25¢ or win 50¢) $= \frac{■}{■} + \frac{■}{■} = \frac{■}{■}$ or ■.■
   c. P (not to win a cash prize) $= \frac{■}{■}$ or ■.■
   d. P (win \$3) = ■
   e. P (win a cash prize) $= \frac{■}{■}$ or ■.■
2. A set of pool balls (numbered 1 to 15) is put in a bag.
   One ball is drawn from the bag at random.
   Find the given probability as a simplest terms fraction and as a decimal.
   a. P (2)
   b. P (even number)
   c. P (odd number or 10)
   d. P (number >10)
3. A bag is filled with 3 blue marbles and 2 red marbles.
   One marble is drawn from the bag.
   a. What is an event that has probability 1?
   b. What is an event that has probability 0?

PRACTICE

1. Toss a die 50 times and keep a tally of the given events. Write each probability as a fraction and as a decimal.
   a. P (3) b. P (2) c. P (8) d. P (2 or 3) e. P (5, 3, or 1)
   f. P (even number) g. P (not a 3)
   h. How do the probabilities compare to the actual results?

2. Draw one card from a deck of 52 cards 50 times. Shuffle the cards between each draw. Keep a tally of the results of the given events. Count aces as ones.
   a. What is the probability?
   (i) P (4) (ii) P (heart) (iii) P (red queen) (iv) P (even number)
   b. How do the probabilities compare to the actual results?

3. Tiles numbered from 1 to 100 are placed in a bin. A tile is drawn from the bin. What is the probability of each event?
   a. P (odd number) b. P (even number) c. P (factor of 12)
   d. P (multiple of 5) e. P (prime number) f. P (even number > 65)

4. In an experiment, the probability of a favourable outcome is 0.4. What is the probability of an unfavourable outcome?

5. A bag contains poker chips of various colours. There are 5 red chips, 3 blue chips, 7 white chips, and 10 black chips. A chip is drawn from the bag.
   What is the probability, as a fraction and as a decimal, of each event?
   a. P (red) b. P (blue) c. P (orange) d. P (black or white)
   e. P (not red) f. P (not yellow) g. P (white, blue, red, or black)

A BASIC Probability Program

The BASIC program below conducts a probability experiment. The program simulates the throw of a die and determines whether or not the outcome is 3 or more.

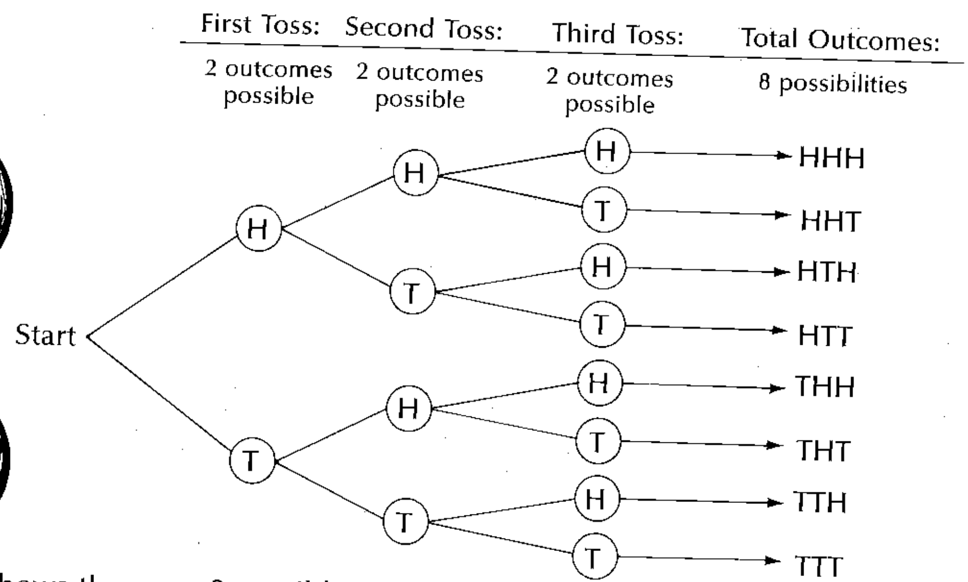
```
10 LET X = INT (6 * RND(1)) + 1
20 IF X >= 3 THEN PRINT "YES"
30 IF X <  3 THEN PRINT "NO"
40 END
```

1. Describe how line 10 simulates the throw of a die.

2. Write a program that simulates 50 throws of a die and prints the results.

Tree Diagrams

A **tree diagram** helps you count all possible outcomes or combinations. The tree diagram below was made for three successive coin tosses.

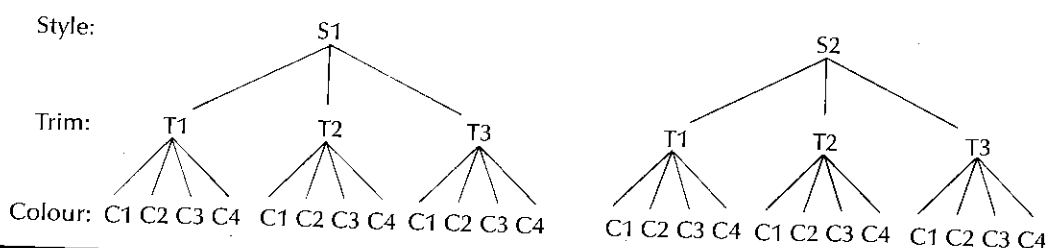


The diagram shows there are 8 possible outcomes.
This is the product of the outcomes at each stage: $2 \times 2 \times 2 = 8$

EXERCISES

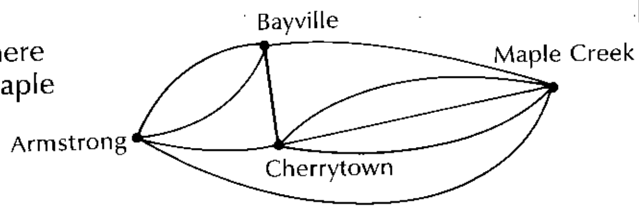
1. Refer to the example above.
   a. What is the probability of tossing three tails in succession?
   b. Extend the tree diagram to show all possible outcomes for four successive coin tosses.
   c. By multiplying, find the number of outcomes for five successive coin tosses.

2. The tree diagram shows the different exteriors on a car you could buy if you had two choices of body style, three choices of trim, and four choices of colour.
   a. How many combinations are possible?
   b. How many combinations would be possible if there were:
   (i) 6 colours? (ii) 3 body styles?

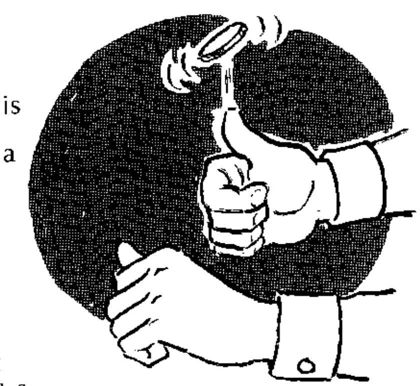


PRACTICE

1. Draw a tree diagram to show all possible outcomes or combinations.
   a. Three different kinds of pies; plain or with ice cream.
   b. Five types of doughnuts with or without frosting.
   c. Jackets in three styles; with or without buttons; in 5 different colours.
   d. Pizzas in three different sizes; one of seven toppings; with either thin or thick crust.
2. Calculate the total number of outcomes or combinations.
   a. Two-tone paint finishes on cars with six colour choices for the top and 8 colour choices for the bottom.
   b. Ice cream cones in any of 15 flavours; with or without nuts and with or without chocolate dip.
   c. Throwing two dice and recording the numbers that show up.
3. Lisa has three different sweaters and four blouses.
   a. How many different combinations can she wear?
   b. If she chooses her combinations randomly, what is the probability that she will dress in any one particular outfit?
4. When Henry goes to the drive-in, he orders either a hot-dog or a hamburger. He must decide whether or not to have mustard, mayonnaise, and onions. Henry never has onions with his hot-dog and never has onions with mayonnaise. All other combinations are acceptable to him. How many different combinations could he order?
5. Look at the map on the right. How many different ways are there of getting from Armstrong to Maple Creek?



The Thousandth Toss



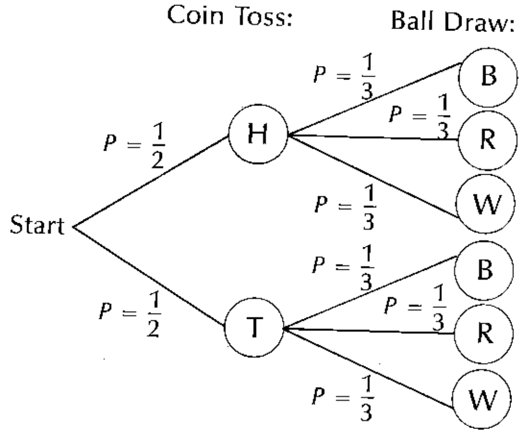
The probability of tossing a coin and having it turn up heads is $\frac{1}{2}$ or 0.5. One would expect to get approximately 500 heads if a coin were tossed 1000 times.

a. Say a coin is tossed 999 times and 499 heads show up. What would be the probability of the next toss landing heads?

b. The coin is tossed 999 times and 999 tails show up. What would be the probability of the next toss landing heads?

Independent Events

John tosses a coin and draws a ball out of a bag containing a red ball, a blue ball, and a white ball. Below is a tree diagram showing all the possible outcomes.



Outcomes:	Probability:
H, B	$P\,(HB) = \frac{1}{6}$ $\frac{1}{2} \times \frac{1}{3}$
H, R	$P\,(HR) = \frac{1}{6}$
H, W	$P\,(HW) = \frac{1}{6}$
T, B	$P\,(TB) = \frac{1}{6}$
T, R	$P\,(TR) = \frac{1}{6}$
T, W	$P\,(TW) = \frac{1}{6}$

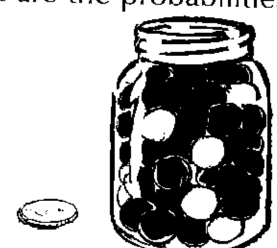
The result of the coin toss does not affect the result of drawing the ball. Thus, the two events are said to be **independent.**

Because the results are independent, the probability that any combination occurs is the product of the individual probabilities.

For independent events A and B, $P\,(A \text{ and } B) = P\,(A) \times P\,(B)$.

EXERCISES

1. Two coins are tossed. Draw a tree diagram to find the number of possible outcomes and the given probabilities.
   - a. number of possible outcomes
   - b. *P* (first coin heads and second coin tails)
   - c. *P* (two heads)
   - d. *P* (two tails)
   - e. *P* (exactly one head)
   - f. *P* (at least one head)
2. The probability of drawing a red marble from a jar is $\frac{1}{4}$.
   The probability of drawing a blue marble is $\frac{1}{2}$.
   If you draw a marble and toss a coin, what are the probabilities?
   - a. *P* (red **and** heads)
   - b. *P* (red **and** tails)
   - c. *P* (blue **and** heads)
   - d. *P* (blue **or** red)
   - e. *P* (*blue or red* **and** tails)
   - f. *P* (*not red or blue* **and** heads)
   - g. *P* (not blue **and** tails)

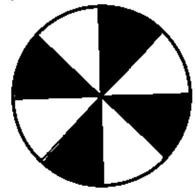


PRACTICE

What are the given probabilities?

1. A coin is tossed and a die is thrown.
   - a. P (heads and 3)
   - b. P (tails and 2)
   - c. P (heads and >4)
   - d. P (heads and <3)
   - e. P (heads and <7)
   - f. P (heads or tails and 1)
   - g. P (heads or tails and >0)
   - h. P (heads and 3, 4, or 5)

2. A die is tossed and the spinner at the right is spun.



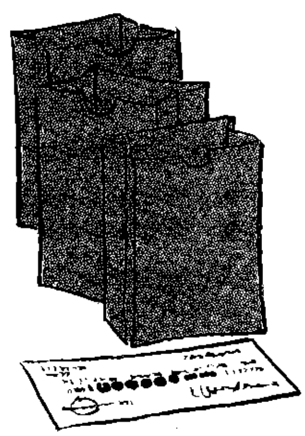
   - a. P (3 and red)
   - b. P (5 and yellow)
   - c. P (3 and white)
   - d. P (>4 and green)
   - e. P (even and blue)
   - f. P (odd or 2 and yellow)
   - g. P (<5 and red or green)
   - h. P (<7 and yellow)
   - i. P (odd and green or red)
   - j. P (<5 and red or blue or green)

3. A coin is tossed three times.
   - a. P (H T H)
   - b. P (T T H)
   - c. P (two heads in a row)
   - d. P (at least two heads)
   - e. P (H on the second toss)
   - f. P (H on last two tosses)
   - g. P (different results on the last two tosses)

4. Doug feels that the probability that he will be elected as treasurer is $\frac{3}{4}$. Trudy feels that the probability that she will be elected vice-president is $\frac{2}{3}$. What is the probability that Doug and Trudy will serve together?

5. In a community lottery, 20 names are chosen randomly from a barrel and put in a smaller container. From that container, the names of the three winners are drawn. If there are 1000 names in the barrel, what is the probability that any particular name will be chosen as one of the 3 winners?

The Eccentric Millionaire



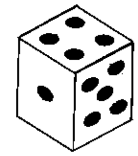
An eccentric millionaire made Alvin a strange offer. He would give Alvin one million dollars if, blindfolded, Alvin could draw a white ball out of each of three bags containing a total of 200 balls: 100 white and 100 black.

Alvin was a good math student and asked if he could arrange the balls in the bags. The millionaire agreed and Alvin arranged the balls in such a way that he had the maximum chance of drawing a white ball even after the balls in the bags had been thoroughly shaken.

How do you think Alvin arranged the balls? What was the probability that he would draw a white ball?

Odds

The chances of an event can be given in the form of **odds.** Odds are similar to probabilities, but not the same.



The **odds against** an event is the ratio of unfavourable outcomes to favourable outcomes.

Example: When rolling a die, what are the odds against rolling a four?

There are six possible outcomes.

unfavourable outcomes: favourable outcomes = 5:1

5 + 1 = 6 possible outcomes

Probabilities can be used to calculate odds.

For the above example: $\frac{P\text{ (not 4)}}{P\text{ (4)}} = \frac{\frac{5}{6}}{\frac{1}{6}} = \frac{5}{6} \div \frac{1}{6} = \frac{5}{6} \times \frac{6}{1} = \frac{5}{1}$

The odds against rolling a 4 are 5:1 or "5 to 1".

The second term of the ratio does not have to be 1.

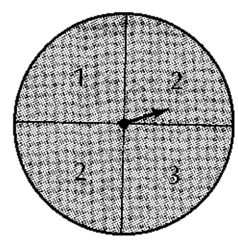
Example: In picking a marble from a jar containing 2 red ones and 3 blue ones, what are the odds against getting a red marble?

unfavourable outcomes: favourable outcomes = 3:2

3 + 2 = 5 possible outcomes

The odds are 3 to 2 against picking a red marble.
This could also be expressed as 1.5 to 1, but it is usual to give odds in whole numbers.

EXERCISES



1. What are the *odds* against each event?
   a. flipping a coin and having heads show
   b. drawing a club from a deck of 52
   c. rolling a die and having a 2 show
   d. drawing a king from a deck of 52
   e. spinning a 3 on the spinner at the right
2. The *probability* of drawing a red ball from a jar is $\frac{1}{4}$.
   a. What is the *probability* of not drawing a red ball?
   b. What are the *odds against* drawing a red ball?
3. The *odds* against the home team winning a ball game are given as 2 to 3.
   a. What is the *probability* of the home team winning?
   b. What is the *probability* of the home team not winning?

PRACTICE

1. What are the odds against each event?
   a. flipping tails with a coin
   b. rolling a 3 or a 4 with a die
   c. drawing a jack from a deck of 52
   d. drawing a black queen from a deck of 52
   e. drawing a card other than a jack from a deck of 52
   f. drawing a card that is not a club but is a face card from a deck of 52

2. The odds against drawing a red marble from a bag of differently-coloured marbles are 8 to 3.
   a. How many red marbles might be in the bag?
   b. How many marbles of other colours might be in the bag?
   c. What total number of marbles could be in the bag?
   d. What is P (red)? e. What is P (not red)?

3. The odds against winning a card game are 7 to 3.
   a. What is the probability of winning the game?
   b. What is the probability of losing?

4. A die is tossed and a coin is flipped.
   Give the probabilities and the odds against each event.
   a. heads and 6
   b. heads and a number <4
   c. heads or tails and 5
   d. tails and 3 or 4
   e. tails and an even number

5. Two dice are thrown. What are the probabilities and the odds against each?
   a. two threes
   b. a two and a five
   c. the first die 4 and the second die 6
   d. two numbers with a sum of 2
   e. two numbers with a sum of 8
   f. the first die less than 4 and the second 5
   g. the first die 2, 3 or 5 and the second die even
   h. the sum of the numbers is 7

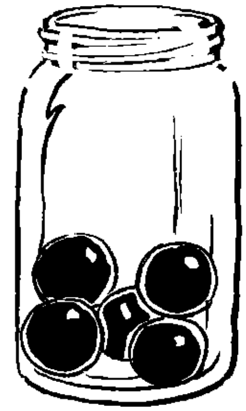
The Track Meet

In order for Gerald's school to place first in a track meet, he must win the hammer throw and the girls' relay team must place first or second.

The probability that Gerald will win is 0.6. The probability that the relay team will place first or second is 0.8.

What are the odds against the school winning the track meet?

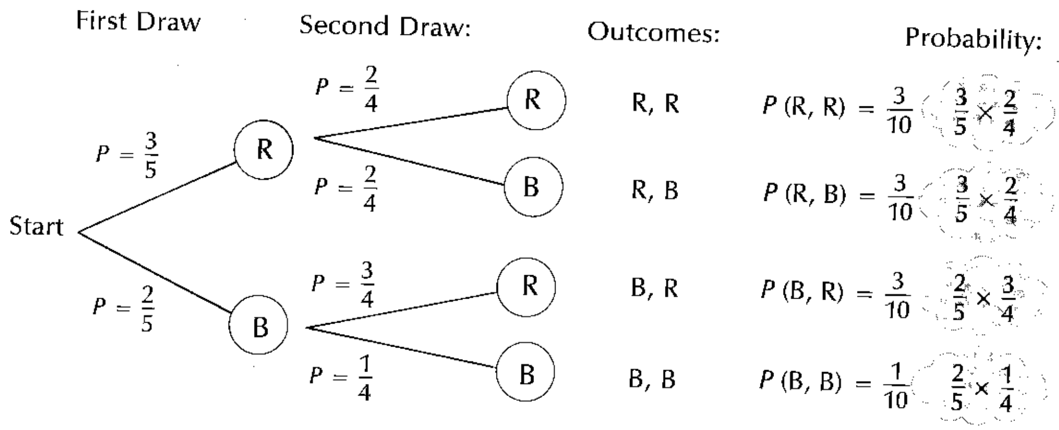
Dependent Events



When one event influences the result of a second event, the events are said to be **dependent.**

Two draws from the jar at the right without replacement are dependent events.

Suppose a blue ball is picked on the first draw and is not put back in the jar.
What is the probability of picking a red ball on the second draw?



The probability of picking a red ball after first picking a blue one is $\frac{3}{10}$ or 0.3.

When an event can result from more than one outcome, the probability of the event is the sum of the probabilities of the outcomes.

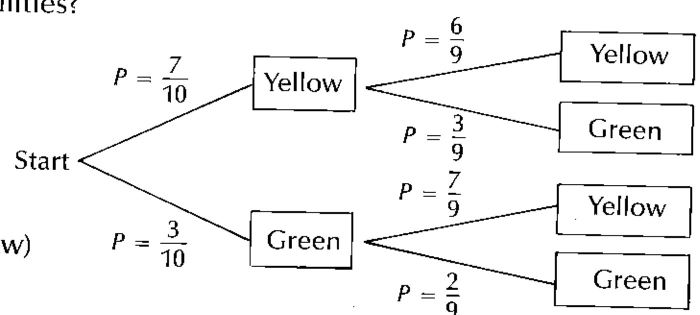
What is the probability of picking at least one blue ball in the same experiment?

$$P\text{ (at least one blue ball)} = P\text{ (R, B)} + P\text{ (B, R)} + P\text{ (B, B)} = \frac{3}{10} + \frac{3}{10} + \frac{1}{10} = \frac{7}{10}$$

EXERCISES

At the right is a tree diagram for an experiment in which two coloured marbles are picked from a jar without replacement.
What are the given probabilities?

1. *P* (yellow, green)
2. *P* (yellow, yellow)
3. *P* (green, green)
4. *P* (no green)
5. *P* (green on the first draw)
6. *P* (at least one yellow)

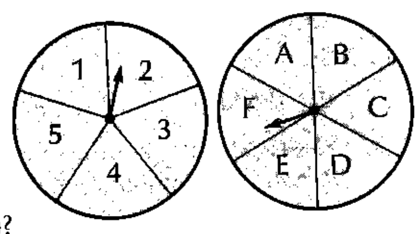


PRACTICE

1. Draw a tree diagram showing two draws out of a jar containing 5 black marbles and 3 white marbles. List all possible outcomes and their probabilities.
2. An experiment has 4 possible outcomes. The probabilities for the first three outcomes are $\frac{1}{10}$, $\frac{2}{5}$, and $\frac{1}{5}$. What must be the probability of the fourth outcome?
3. A jar contains 10 pennies, 5 nickels, and 3 dimes. Coins are drawn and not replaced. Calculate the probability of drawing:
   a. 15 cents in two draws;
   b. 15 cents in three draws;
   c. more than 25 cents in two draws;
   d. more than 25 cents in three draws;
   e. more than 2 cents in two draws.
4. Michael has all his socks loose in his drawer. He has 8 black socks, 4 brown socks, and 6 blue socks. What is the probability that he will randomly choose a matching pair of socks? What is the probability that he will choose a pair of blue socks?

REVIEW

1. A dime and a quarter are tossed. What are all possible outcomes?
2. A die is tossed. What is the given probability?
   a. $P(2)$ b. $P(5 \text{ or } 4)$ c. $P(\text{odd number})$ d. $P(\text{number} > 6)$
3. Sarah packs four blouses, two skirts, and two sweaters on her trip to Ottawa. How many different combinations of these clothes can she wear?
4. The two spinners at the right are spun in succession. What are the probabilities?



   a. $P(3, C)$ b. $P(\text{odd number}, A)$
   c. $P(1, Z)$ d. $P(\text{even number, vowel})$
   e. What are the odds against spinning a 5?
   f. What are the odds against spinning (1, A)?
5. During a *Scrabble* game, Janet must choose two letter tiles at random from a box. The letters left for her to choose are shown below. Find the probabilities.
   a. $P(\text{A and S})$ b. $P(\text{two As})$
   c. $P(\text{two vowels})$ d. $P(\text{A and R})$

$\boxed{A_1}$ $\boxed{R_1}$ $\boxed{A_1}$ $\boxed{S_1}$ $\boxed{S_1}$ $\boxed{A_1}$

Probability and Percent

Probabilities can be expressed as percents simply by converting the fraction or decimal probability to a percent.

Flip a coin.

$P\text{ (heads)} = \frac{1}{2}$ or 0.5 or 50%

The coin has a 50% chance of landing heads up.

When probabilities are expressed as percents, they may be *added* or *multiplied* just like fraction and decimal probabilities.

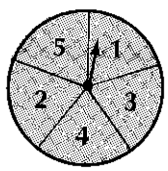
Example 1: Adding Probabilities

Try your luck on the lottery.		
1st prize — $10 —	**5% chance to win**	
2nd prize — $ 5 —	**10% chance to win**	

$$\begin{aligned} P\text{ (first or second)} &= P\text{ (first)} + P\text{ (second)} \\ &= 5\% + 10\% \\ &= 15\% \end{aligned}$$

Example 2: Multiplying Probabilities

Spin a 2 and there is a prize for you!

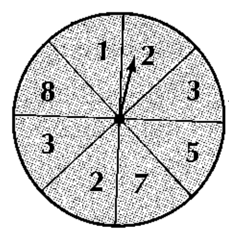


$P\text{ (win)} = 20\%$

$$\begin{aligned} P\text{ (win, win)} &= P\text{ (win)} \times P\text{ (win)} \\ &= 20\% \times 20\% \\ &= 0.2 \times 0.2 \\ &= 0.04 \\ &= 4\% \end{aligned}$$

EXERCISES

1. What is the percent probability of an event that is certain to occur?
2. The spinner at the right is spun. Find the percent probability.
   a. P (2) b. P (5) c. P (prime)
   d. P (odd) e. P (number >2)



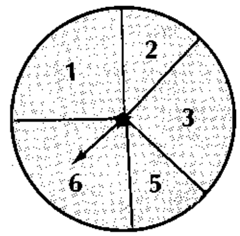
3. There are three white balls and two black balls in a jar. A ball is drawn and replaced and another is drawn. What are the percent probabilities for each event?
   a. P (white, white) b. P (black, white) c. P (white, red)
4. Three white balls and two black balls are drawn from a jar without replacement. What is the percent probability?
   a. P (white, black) b. P (black, black)
   c. P (black or white, black or white)

PRACTICE

1. Suppose only the face cards and the aces of a regular deck of cards are spread out face down on a table. What is the probability of drawing each?

   a. a club
   b. a queen or a king
   c. the queen of hearts
   d. a spade face card
   e. an ace
   f. a black card
   g. a red face card
   h. a black ace
   i. a red 10

2. The spinner at the right is spun twice. What are the percent probabilities?



   a. $P\ (3, 5)$
   b. $P\ (1, 2)$
   c. P (even, even)
   d. P (even, odd)
   e. P (3, odd)
   f. P (sum of 8)
   g. P (product of 6)
   h. P (difference of 3)

3. Danielle has six dimes, two nickels, and seven quarters in her piggy bank. What is the percent probability of her drawing two coins and getting each amount?

   a. 15 cents
   b. 30 cents
   c. more than 5 cents
   d. less than 50 cents
   e. 10 cents
   f. 35 cents
   g. more than 50 cents
   h. 20 cents
   i. 25 cents

4. The probability of the boys basketball team winning the district championship is 60%. The probability of the girls winning the championship is 75%. The probability of the track team winning the championship is 30%. Find the percent probability for each.

   a. the boys win and the girls win
   b. the girls win and the track team wins
   c. the boys do not win
   d. the boys do not win and the track team wins

A Birthday Program

How many people would you have to ask before the probability is greater than 50% that 2 people have the same birthday?

Use the computer program to try out different group sizes to answer the question.

a. What do the variables *N*, *PROB*, and *PERCENT* stand for?

b. What is the purpose of the equation in line 160?

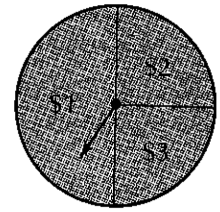
c. How many people did you find needed to be in a group to satisfy the conditions of the question?

```
100 INPUT "HOW MANY PEOPLE IN THE GROUP ";N
110 P = 1
120 FOR I = 2 TO N
130 P = P * ( ( 366 - I ) / 365 )
140 NEXT I
150 PROB = 1 - P
160 PERCENT = ( INT ( PROB * 10000 ) ) / 100
170 PRINT "THE PROBABILITY OF TWO PEOPLE"
180 PRINT "IN A GROUP OF ";N;" PEOPLE"
190 PRINT "HAVING THE SAME BIRTHDAY IS ";PERCENT;"%"
```

Making Predictions

When the probability of an event is known, it is possible to make a prediction about the number of times that the event *is expected to occur* after many trials.

After many trial spins, the *expected results* can be calculated by multiplying the probabilities of each outcome by the number of trials.



$P(\$1) = 0.5$

$P(\$2) = 0.25$

$P(\$3) = 0.25$

Outcome	Probability × Number of Trials	Expected Result
$1	0.5 × 1000	Spinner lands on 1 500 times.
$2	0.25 × 1000	Spinner lands on 2 250 times.
$3	0.25 × 1000	Spinner lands on 3 250 times.

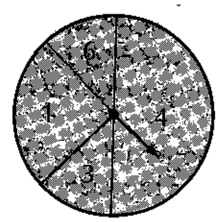
The experiment above has three possible outcomes: $1, $2, and $3. After 1000 trials, the total *expected* result is $1750, for an *average* of $1.75 per trial.
An expected *average* outcome can also be calculated by adding the products of each outcome and its probability.

Expected *average* outcome value $= (1 \times 0.5) + (2 \times 0.25) + (3 \times 0.25)$
$= 0.5 + 0.5 + 0.75$
$= 1.75$

After many trial spins, the expected *average* outcome per spin is $1.75.

EXERCISES

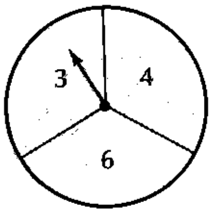
1. There are 3 white balls, 2 black balls, and 5 blue balls in a bag. A ball is drawn out and replaced. How many times is each event expected to occur after 100 trials?
   a. a blue ball is drawn
   b. a white ball is drawn
   c. a black or blue ball is drawn
   d. a yellow ball is drawn
   e. How many times is each event expected to occur after 840 trials?
2. Refer to the spinner at the right.
   a. List all possible outcomes and their probabilities after one spin.
   b. How many times is each outcome expected to occur after 360 trials?
   c. What is the expected average outcome per spin after many trials?



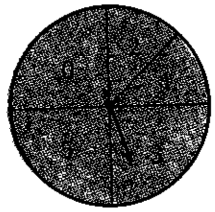
PRACTICE

Answer the questions for each spinner below.

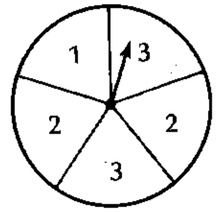
1.



2.



3.



a. How many times is the number 1 expected to occur after 420 trials?
b. What is the expected average outcome value after many trials?

4. Half of the time, Lolita spends \$3.80 at the drive-in. Three-tenths of the time she spends only \$1.20, and $\frac{1}{5}$ of the time she spends \$3.20.

a. After 20 visits to the drive-in, how many times is Lolita expected to spend \$3.80?
b. On the average, how much can Lolita expect to spend at the drive-in on a single visit?

5. Brian scores 3 goals in 5% of his hockey games. In 10% of the games, he scores 2 goals and in 20% of his games, he scores 1 goal. In other games, he doesn't score any goals.

a. After 40 games are played, in how many of them is Brian expected to score 3 goals?
b. On the average, how many goals can he expect to score per game?

6. Each time the Tigers football team has possession of the ball, the coach estimates that there is a 30% chance of a touchdown (6 points) and a 20% chance of a field goal (3 points). He also expects that 80% of the touchdowns will be converted for an extra point.

a. What is the expected number of points the Tigers would score if they had possession of the ball 15 times?
b. What is the expected score on each possession?

7. Two dice are thrown and the sum of the dots is computed.

a. What is the most probable sum?
b. What is the average expected outcome value?
c. Does the most probable sum have to be the expected outcome value?

Letter Sequence

Find the next four letters that would continue the sequence.

O T T F F S S ___ ___ ___ ___

Population Samples

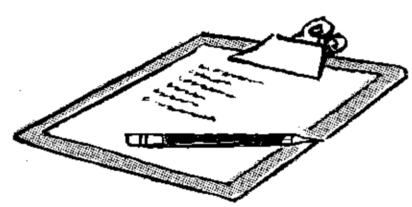
A soft drink company performed a taste test on a *large* group of people to find out which of two of their drinks is preferred and should be produced in larger quantities.

Since it would have been too costly and time-consuming to poll an entire city or country, the company polled a *sample group* of 100 people. These 100 were chosen *at random* and were a *representative* mix of young and old, high-income and low-income, male and female. Thus, the sample group had a minimum chance of being *biased.*

Seventy of the 100 people chose the orange soda over the grape. The soft drink company decided to produce 70% orange soda to 30% grape.

EXERCISES

1. To find out students' opinions on the school assembly, the president of the Student Council asks the opinions of his 5 best friends. Is this a *representative* sample of the student body?
2. The town of Maple Creek was surveyed during the Summer Olympic Games. Of the 400 people surveyed, 175 said that they watched the Games on TV.
   a. Based on the survey, what percent of the population of Maple Creek watched the Olympic Games?
   b. If the population of Maple Creek is 15 000, about how many people would have watched the Games?
3. Suppose you wish to poll the students at your grade level. Explain how you would get a sample group that is *randomly* chosen and is *representative* of the total population.



4. At a pottery, plates are fired in a large furnace. To check for even firing, plates are chosen from the middle of the furnace and at the eight corners. Is this a good method for taking the sample?

PRACTICE

1. Explain why each of the samples may not be *representative* of their populations.
   a. Population: citizens of a large city.
      Sample: 1000 people stopped on the corner of a busy street.
   b. Population: citizens of Canada.
      Sample: 10 000 people chosen randomly from the phone book in Halifax.
   c. Population: students in Grade 8.
      Sample: five students chosen at random from the school roster.
   d. Population: all the students in the school.
      Sample: all the students on the basketball team.
   e. Population: Alberta voters.
      Sample: 10 000 people chosen at random from the Saskatchewan voters list.

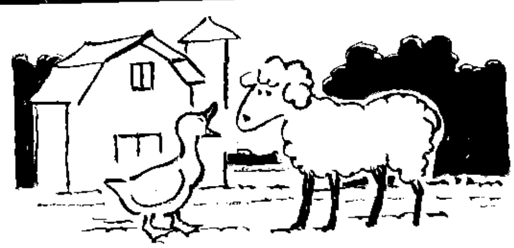
2. A quality control inspector at a cereal packaging factory measured the mass of 100 boxes of cereal at random and found that the average mass of the boxes was 397 g. The mass of the cereal boxes is supposed to be 400 g. When he took a second sample, he found the average mass to be 401 g.
   Is it reasonable to expect that the whole population of cereal boxes was, on the average, 400 g?

3. In a random sample of 200 runners, a clothing manufacturer found that the runners had the preferences shown in the table for sizes and colours in shorts.

Size/Colour	Red	Blue	Green
S	19	18	13
M	32	24	22
L	17	13	10
XL	13	10	9

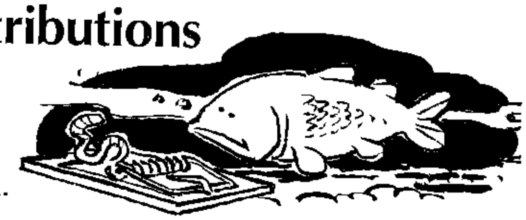
   a. If the store expects to sell shorts to 300 runners, how many of each size and colour should the store order?
   b. It takes 0.5 m of material to make one pair of shorts. The manufacturer wishes to produce 50 000 pairs of shorts. How much of each colour of material should the manufacturer order?
   c. A store wants to stock T-shirts as well. If the store wants to stock 500 shirts, how many of each size should it order if the owner assumes that people take the same size shirt as shorts?

Sheep and Ducks



A farmer has sheep and ducks in the barnyard. The farmer can count 100 heads and 280 feet. How many of each animal are there in the barnyard?

Frequency and Percent Distributions



A conservation officer placed traps to determine the population of fish in a stream. After trapping 75 fish, the officer made a tally and frequency table to draw some conclusions from the data.

Type of Fish	Tally	Frequency	Percent $= \frac{\text{Frequency}}{\text{Total}}$	
Rainbow trout	卌卌卌卌卌	25	33.3%	$25 \div 75$
Steelhead trout	卌卌卌 III	18	24.0%	$18 \div 75$
Dolly Varden trout	卌卌 I	11	14.7%	$11 \div 75$
Coho salmon	卌卌卌卌 I	21	28.0%	$21 \div 75$
	Total	75	100.0%	

The conservation officer made various conclusions, such as:

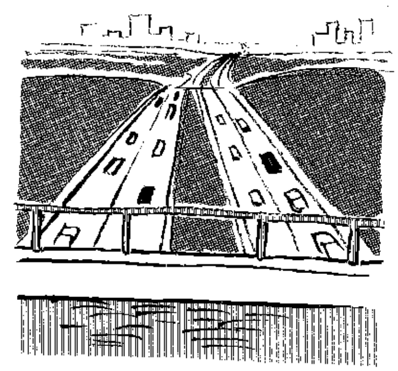
- If a fisherman catches a fish, there is a 28% chance that it is a coho salmon.
- The percent of Dolly Varden trout in the stream is up (or down) from that of a previous date.

EXERCISES

Copy and complete the tally and frequency table.

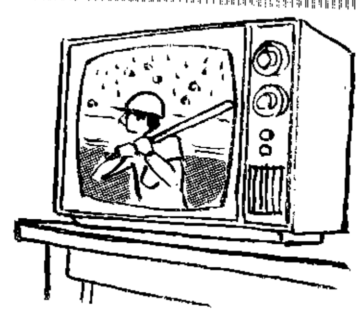
1.

Survey of Automobiles on the Expressway			
Model	Tally	Frequency	Percent
Sedan		28	
Convertible		9	
Pick-up		7	
Truck		6	
	Total		



2.

Survey of Favourite Sports			
Sport	Tally	Frequency	Percent
Football			14%
Baseball		38	
Soccer		22	
Hockey		39	
Other			20%
	Total	150	100%



EXERCISES

Construct a tally and frequency table for each set of data. Include the percent frequency in the table.

1. Marks on a 10-point quiz: 7, 8, 9, 8, 7, 6, 5, 7, 8, 6, 9, 7, 5, 4, 8, 6, 10, 7, 7, 6, 9, 8, 7, 6, 7, 6.
2. Ages of students in years: 13, 13, 14, 12, 13, 14, 13, 13, 14, 12, 15, 14, 14, 15, 14, 13, 15, 14, 15, 14, 13, 14, 13, 14, 13, 15, 13, 14, 15, 14, 12, 15, 15, 14, 13, 13, 14, 16, 14.
3. Number of cars in the family: 1, 2, 1, 3, 1, 1, 3, 1, 2, 0, 2, 1, 4, 1, 2, 3, 2, 3, 1, 2, 1, 0, 2, 2, 1, 3, 1, 2, 1, 2, 1, 2, 3, 5, 1, 0, 1.
4. Record of wins (W), losses (L), and ties (T): W, W, W, T, W, L, W, W, L, T, L, W, W, W, L, W, W, W, L, T.

Copy and complete the tally and frequency table. Then answer the question assuming the sample is random and representative.

5.

<table>
<tr><th colspan="4">Survey of Favourite Colours</th></tr>
<tr><th>Colour</th><th>Tally</th><th>Frequency</th><th>Percent</th></tr>
<tr><td>Red</td><td></td><td>8</td><td></td></tr>
<tr><td>Blue</td><td>𝍸𝍸||||</td><td></td><td></td></tr>
<tr><td>Green</td><td></td><td>6</td><td></td></tr>
<tr><td>Yellow</td><td></td><td></td><td>10%</td></tr>
<tr><td>Orange</td><td>𝍸|||</td><td></td><td></td></tr>
<tr><td></td><td>Total</td><td>40</td><td></td></tr>
</table>

a. If the sample group surveyed came from a student population of 600, how many students of the total population are expected to choose blue as a favourite colour?

b. How many students of the 600 are expected to prefer red, yellow, or orange?

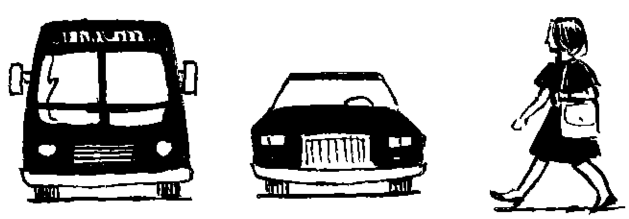
Canadian Facts

Probably the most well-known activity of Statistics Canada is the Census taken every 10 years. The 1981 Census counted 24 343 181 people. The information in the table was also gathered by Statistics Canada.

a. What percent of the Canadian population use public transportation to go to work?

b. What percent of the Canadian population walk, travel by car, or travel by public transportation to get to work?

How we get to work



6.5 million people **use public transportation**

5.6 million people **by car**
4.6 drive a car
1.0 as a passenger

1.3 million people **walk**

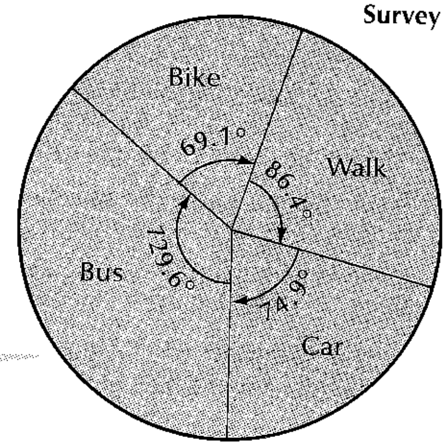
Source: Statistics Canada, Dec. 1983

Percent Distributions and Circle Graphs

A **circle graph** is often used to display the data in a frequency and percent distribution table.

Survey of Transportation to School		
Type	Frequency	Percent
Walk	30	24.0%
Bike	24	19.2%
Bus	45	36.0%
Car	26	20.8%
Total	125	100.0%

School Transportation Survey



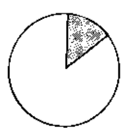
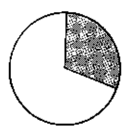
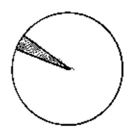
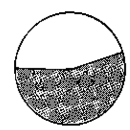
Each sector of the graph sweeps out an angle proportional to the percent distribution.

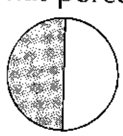
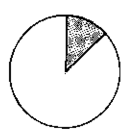
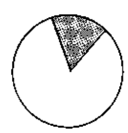
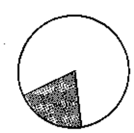
To find the size of the angle to use for each kind of transportation in the circle graph, multiply the percent by 360 (total degrees in a circle).

Sector	Angle size
Walk	$24\% \times 360 = 86.4°$
Bus	$36\% \times 360 = 129.6°$
Bike	$19.2\% \times 360 = 69.1°$
Car	$20.8\% \times 360 = 74.9°$
	Total: 360°

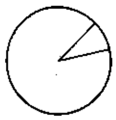
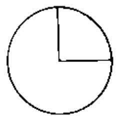
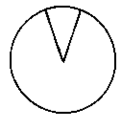
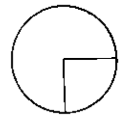
EXERCISES

1. How large a sector (in degrees) would represent each percentage?
   a. 10% **b.** 50% **c.** 90% **d.** 60% **e.** 35%
2. Measure the size of each sector with a protractor.

   a.
   
   b.
   
   c.
   
   d.
   
3. What percentage of a whole circle does each sector represent?

   a.
   
   b.
   
   c.
   
   d.
   

PRACTICE

1. How large a sector (in degrees) would represent each percentage?
   a. 25% **b.** 30% **c.** 1% **d.** 57% **e.** 89%
2. Measure the size of each sector with a protractor.
   a.  **b.**  **c.**  **d.** 
3. What percent of the whole circle does each sector in question 2 represent?
4. Construct circle graphs of the following information.

a.

Mineral Production in Canada	
Metals	30%
Non-metals	8%
Fuel	57%
Other metals	5%

b.

Population by Age Group in Canada	
Under 15	24%
15–24	20%
25–44	28%
45–64	19%
65 and over	9%

5. Take a survey of the eye colour of the students in your classroom. Construct a tally and frequency table with a column for percent distribution. Then show the results in a circle graph.

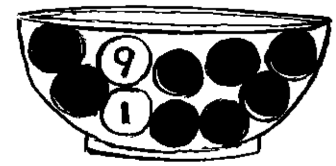
REVIEW

1. A die is rolled. What are the percent probabilities of rolling each event?
   a. an even number **b.** a 6 **c.** any number but 4
2. Darrel does gardening to earn money. He estimates that 60% of his jobs take 3 hours, 25% of the jobs take 5 hours, and 15% of the jobs take 8 hours. On the average, how long should he expect to work on each job?
3. The student council took a survey to determine whether students preferred athletics (A), dances (B), or concerts (C). The results of the survey are shown at the right.
   a. Construct a tally and frequency table of the data. Include the percent distributions for the data.
   b. Construct a circle graph of the data.
   c. If the total student body is 800 students, about how many are expected to prefer athletics?

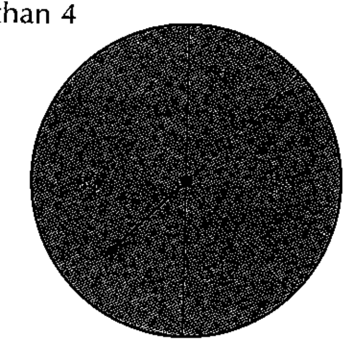
C, A, C, A, A, B, B, A, C, B,
B, A, B, C, A, B, A, C, B, A,
C, A, B, A, B, B, C, C, A, A,
A, B, C, A, A, B, A, A, B, C,
A, A, B, C, B.

TEST — UNIT 15

1. One ball is drawn at random. What is the probability of each event?
   a. P (red) b. P (even)
   c. P (yellow or odd)



2. A ball is drawn and replaced and then another ball is drawn from the set above. What are the probabilities?
   a. P (red, yellow) b. P (red, red) c. P (blue, even)
   d. What are the odds against drawing blue on the first draw?
   e. What are the odds against drawing (blue, 3)?
3. Two balls are drawn from the set of balls above and not replaced. What are the probabilities?
   a. P (red, red) b. P (5, blue) c. P (4, blue)
4. A paint company advertises that it has one line of paint on sale. The paint comes in 7 shades of 10 colours with flat, semi-gloss, and gloss finishes. How many different kinds of paint are on sale?
5. What are the percent probabilities of each?
   a. flipping a coin and getting heads
   b. rolling a die and getting a number bigger than 4
   c. choosing a heart from a deck of 52
6. The spinner at the right is spun.
   a. List all possible outcomes and their probabilities.
   b. How many times is each outcome expected to occur after 640 trials?
   c. What is the average expected outcome per spin?



7. A theatre manager took a poll of his customers to determine their favourite *Star Wars* movie. Of the 40 people he polled, he found that 18 of them preferred *Return of the Jedi*, 12 liked *The Empire Strikes Back*, and 10 liked *Star Wars*.
   a. The manager is going to order 500 T-shirts featuring the 3 movies. How many of each should he buy?
   b. Complete a tally chart with percent distributions to show this data.
   c. Construct a circle graph to show this data.

Graphs and Relations

List the solution set.
Replacement Set = $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$.

1. $x < 2$
2. $x + (-1) = 2$
3. $-x + (-3) = 0$
4. $4x - 4 = -24$
5. $-3x + (-2) = 1$
6. $x + (-1) > 5$

Write the relation rule.
Then write two more ordered pairs for the relation.

7. {(Newfoundland, 12), (British Columbia, 15), (Saskatchewan, 12)}
8. $\left\{\left(25\%, \frac{1}{4}\right), \left(90\%, \frac{9}{10}\right), \left(45\%, \frac{9}{20}\right)\right\}$

Copy and complete the table.

9.

Rule: $y = 5x$	
x	y
-2	
-1	
0	
1	

10.

Rule: $y = -7x$	
x	y
-2	
-1	
0	
1	

11.

Rule: $y = -x + 4$	
x	y
-1	
0	
1	
2	

Identify the quadrant or axis in which the ordered pair is located.

12. $(-2, 4)$
13. $(6, -5)$
14. $(4, 0)$
15. $(-7, -2)$

Make a table to find some solutions to each equation.
Use $\{-2, -1, 0, 1, 2\}$ as the possible values for x.

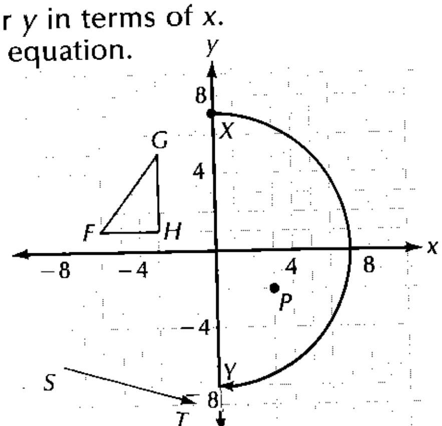
16. $y = -3x$
17. $y = 8x + 1$
18. $y = -x + 7$

Find three solutions for the given equation and then draw its graph.

19. $y = x + 2$
20. $y = -3x + 2$
21. $y = -x + (-1)$

For each equation, write a similar equation for y in terms of x.
Then write one ordered pair that satisfies the equation.

22. $-x + y = 15$
23. $-4x + y = 0$



Refer to the diagram at the right.
Copy and complete the ordered pairs for the transformation images of point P and $\triangle FGH$.

24. translation under arrow ST
25. reflection in y-axis
26. rotation under arc XY