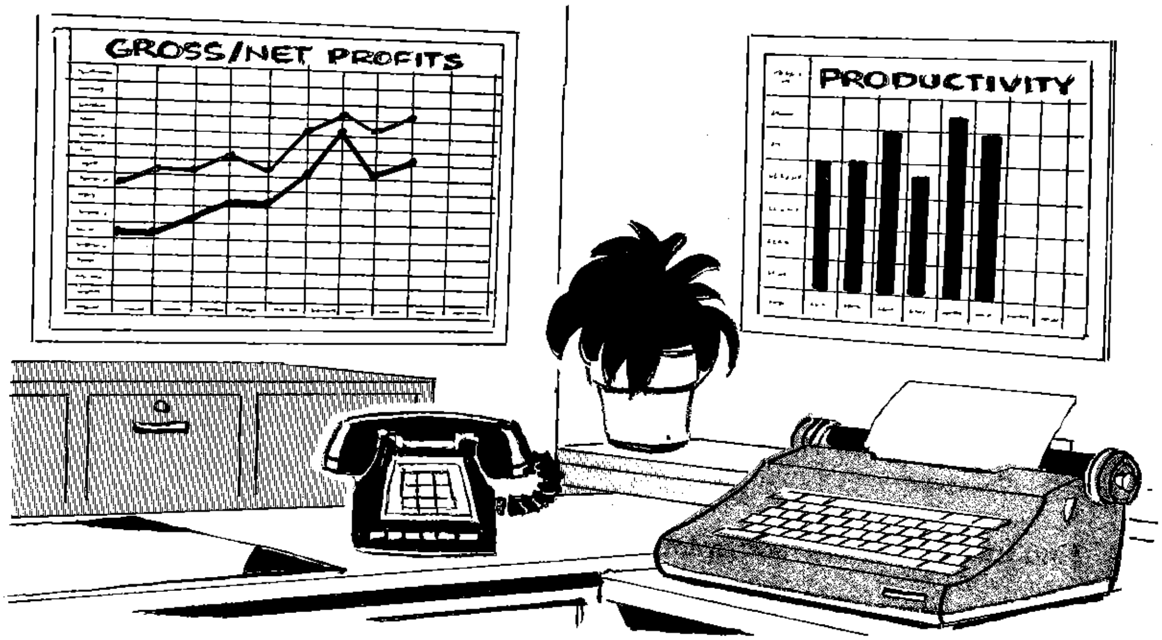
UNIT 14 Graphs and Relations



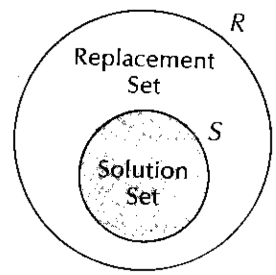
The Solution Set of An Open Sentence

An equation containing one or more variables is called an **open sentence**.

The set of numbers used to try to solve an open sentence is called the **replacement set**.

The **solution set** of an open sentence is the set of values from the replacement set that make the sentence true.

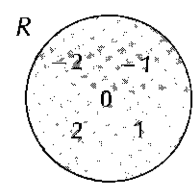
$\therefore$ The solution set *is a subset of* the replacement set.



S is a subset of R.
We write: $S \subset R$.

Examples: Assume that the replacement set R is $\{-2, -1, 0, 1, 2\}$.

Open sentence:	*Solution set:*
1. $x - 4 = -3$	$S = \{1\}$
2. $2x - 8 = 9$	$S = \phi$ or $\{\ \}$



$1 \in R$ $\quad 8\frac{1}{2} \notin R$

In Example 2, there is no solution in the replacement set R. If the replacement set were the set of rational numbers, the solution set would be $\left\{8\frac{1}{2}\right\}$.

EXERCISES

Using the replacement set $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$, list the solution set.

1. $x < 0$ 2. $x > 3$ 3. $x < 3$
4. $x > 4$ 5. $x + 2 = 3$ 6. $x - (-3) = 7$
7. $x + (-5) = -2$ 8. $x + (-6) = -12$ 9. $-2x + 3 = 5$
10. $-5x + 4 = -1$ 11. $3x - (-1) = 7$ 12. $7(x + 4) = 0$

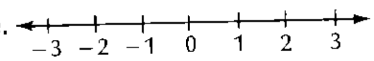
PRACTICE

What is the solution set if the replacement set is the set of integers?

1. $x + 8 = -2$ 2. $x - (-45) = 57$ 3. $6x = 0.6$ 4. $x + 7 = 10\frac{1}{2}$
5. $7x + 3 = 2$ 6. $8x + 6x = 42$ 7. $3x + 4x = 15$ 8. $3(x + 5) = 3$

What is the solution set if the replacement set is the set of rational numbers?

9. $x + 8 = -1.2$ 10. $3.6x = 1.44$ 11. $x - (-4.5) = 10$ 12. $9x + 6.5 = 20$
13. $x = 20 + \sqrt{2}$ 14. $x + 2\frac{1}{5} = 5\frac{13}{15}$ 15. $12x - 7 = 2.6$ 16. $x = \sqrt{3} \times 5$

List each solution set and graph it on a number line.


The replacement set is $\{-3, -2, -1, 0, 1, 2, 3\}$.

17. $x < 1$ 18. $2(x + 7) = 8$ 19. $x > 4$ 20. $8x + 3x = -11$
21. $x < -1$ 22. $5x + 3x = -11$ 23. $(2x)(3x) = 6$ 24. $x^2 = 1$

"And" and "Or" Inequalities

The inequality $2 < x < 5$ means $x > 2$ **and** $x < 5$.
The inequality $x \leq 6$ means $x < 6$ **or** $x = 6$.

Examples: Replacement set $= \{0, 1, 2, 3, 4, 5, 6, 7\}$

Open sentence:	*Solution Set:*	*Open sentence:*	*Solution Set:*
$2 < x$	$A = \{3, 4, 5, 6, 7\}$	$x < 6$	$A = \{5, 4, 3, 2, 1, 0\}$
$x < 5$	$B = \{4, 3, 2, 1, 0\}$	$x = 6$	$B = \{6\}$
$2 < x < 5$	$\{4, 3\}$	$x \leq 6$	$\{6, 5, 4, 3, 2, 1, 0\}$

Determine the solution set for the inequality with replacement set $\{1, 2, 3, \ldots, 10\}$.

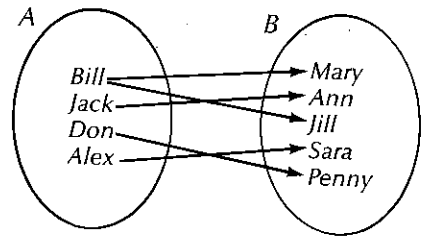
1. $3 < x < 8$ 2. $x \leq 4$ 3. $0 < x < 9$
4. $x \geq 5$ 5. $2 < x < 11$ 6. $x \leq 8$

Relations and Ordered Pairs

A **relation** maps the elements of one set onto another set.

An *arrow diagram* shows a relation between two sets.

The two sets are related by a rule.
Relation Rule: is the brother of



The relation of two sets can also be shown as a set of *ordered pairs.*

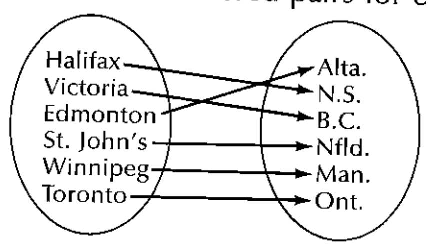
{(Bill, Mary), (Bill, Jill), (Jack, Ann), (Don, Penny), (Alex, Sara)}

The *first* item in each ordered pair is an element of Set *A*.
The *second* item in each ordered pair is an element of Set *B*.

EXERCISES

Write the set of ordered pairs for each relation.

1.



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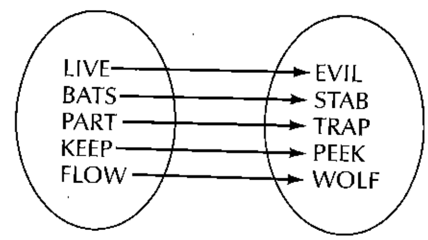
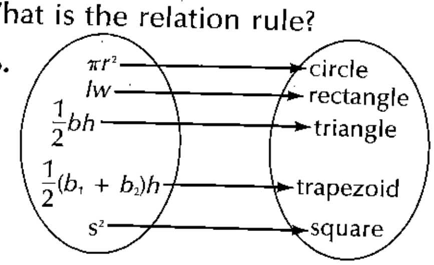


Draw the arrow diagram for each relation.

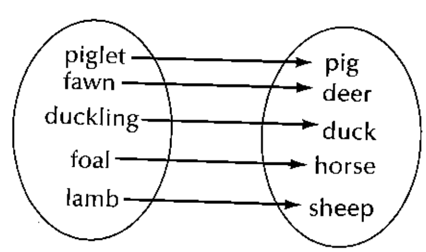
3. {(centimetre, cm), (millimetre, mm), (micrometre, μm), (nanometre, nm)}
4. {(fish, scales), (dog, hair), (raccoon, fur), (turtle, shell)}
5. {(le pont, bridge), (la roue, wheel), (la chanson, song)}

What is the relation rule?

6.



7.



8. {(triangle, 3), (quadrilateral, 4), (pentagon, 5), (hexagon, 6)}

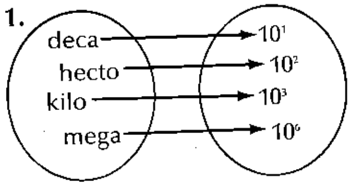
Write another ordered pair for each relation.

9. {(rooster, hen), (gander, goose), (stallion, mare)}
10. {(Winnipeg, Jets), (Calgary, Flames), (Vancouver, Canucks)}
11. {(Ottawa, Canada), (London, England), (Paris, France)}

PRACTICE

Write each relation as a set of ordered pairs.
Then write the relation rule.

1.



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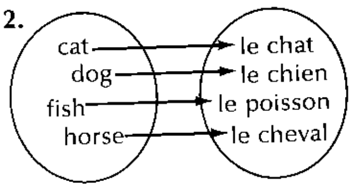

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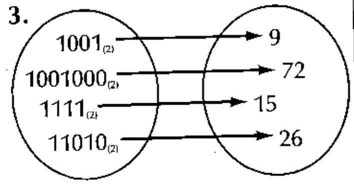

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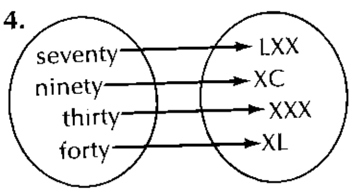

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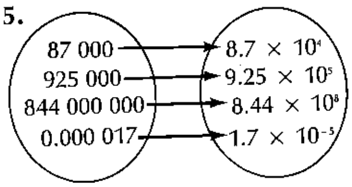

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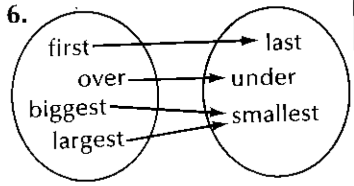

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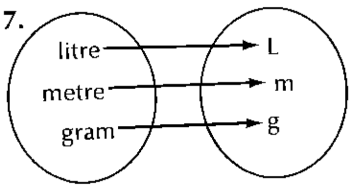

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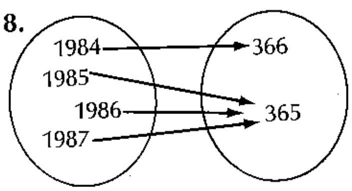

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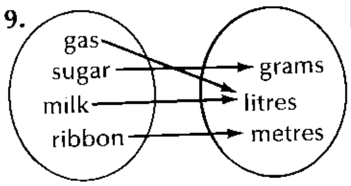


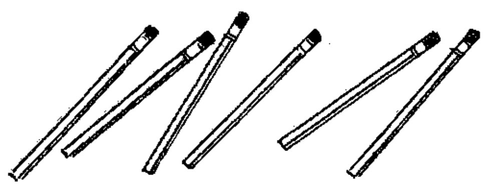
Write the relation rule.
Then write two more ordered pairs for each relation.

10. {(1 P.M., 13:00), (4 P.M., 16:00), (6 P.M., 18:00)}
11. {(January, 31), (February, 28), (March, 31)}
12. {(nose, 1), (ear, 2), (eye, 2)}
13. {(orange, fruit) (carrot, vegetable) (tomato, fruit)}

Research the information needed.
Then draw an arrow diagram of the relation.

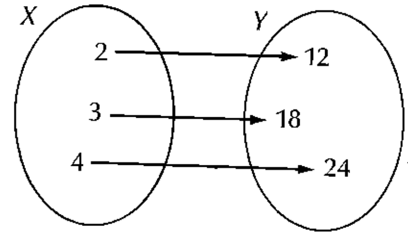
14. important rivers in 5 Canadian provinces
15. 5 classmates and the streets on which they live
16. 5 cities and their professional baseball teams
17. 3 planets and their number of moons
18. 4 European cities (which are not capitals) and their countries

Triangle Challenge



Use 6 equal-sized pencils to make a figure that is composed of 4 congruent triangles.

Numerical Relations



Rule: $y = 6x$

The numbers in the arrow diagram are *related* to each other by the same rule.

Each number in set Y is *6 times greater* than its related number in set X.

The **relation rule** for the two sets of numbers can be expressed as $y = 6x$.

A numerical relation can also be shown as a set of *ordered pairs*.

The first number in the ordered pair is an element of set X.
The second number is the related element of set Y.

$\{(2, 12)\ (3, 18)\ (4, 24)\}$

Other numbers related by the same rule can be found by making a table.

Rule: $y = 6x$	
x	y
2	12
3	18
4	24
5 →	30
6 →	36
7 →	42

EXERCISES

Copy and complete each table.

1. Rule: $y = 2x$

x	y
0	
1	
2	
3	

2. Rule: $y = -5x$

x	y
0	
1	
2	
3	

3. Rule: $y = x + 8$

x	y
−2	
−1	
0	
1	

4. Rule: $y = 5x + 2$

x	y
−2	
−1	
0	
1	

5. Rule: $y = -x + 3$

x	y
−1	
0	
1	
2	

6. Rule: $y = -6x - 5$

x	y
−2	
−1	
0	
1	

Use the table to find the missing values.

7. Rule: $y = 2x + ■$

x	y
1	8
2	10
3	12

8. Rule: $y = -■x + 1$

x	y
1	0
−1	2
−3	4

9. Rule: $y = ■x + ■$

x	y
1	7
2	11
3	15

PRACTICE

Copy and complete each table.

1.

Rule: $y = 3x$	
x	y
−1	■
0	■
1	■
2	■

2.

Rule: $y = x - 1$	
x	y
−2	■
−1	■
0	■
1	■

3.

Rule: $y = x + 4$	
x	y
−2	■
−1	■
0	■
1	■

4.

Rule: $y = -x + 3$	
x	y
2	■
1	■
0	■
−1	■

5.

Rule: $y = -7x + 2$	
x	y
−2	■
−1	■
0	■
1	■

6.

Rule: $y = 5x + (-3)$	
x	y
−2	■
−1	■
0	■
1	■

Use the table to find the missing values.

7.

Rule: $y = ■x + ■$	
x	y
−2	−4
−1	−1
0	2
1	5

8.

Rule: $y = ■x + ■$	
x	y
−2	−10
−1	−5
0	0
1	5

9.

Rule: $y = -■x + ■$	
x	y
−2	9
−1	5
0	1
1	−3

Find the relation rule.
Then complete the set of ordered pairs.

10. {(5, −25), (7, −35), (9, −45), (11, ■), (15, ■), (17, ■)}

11. {(1, −2), (2, −1), (3, 0), (4, ■), (5, ■), (6, ■)}

12. {(1, 3), (2, 5), (3, 7), (4, ■), (5, ■), (6, ■)}

A Formula for a Relation

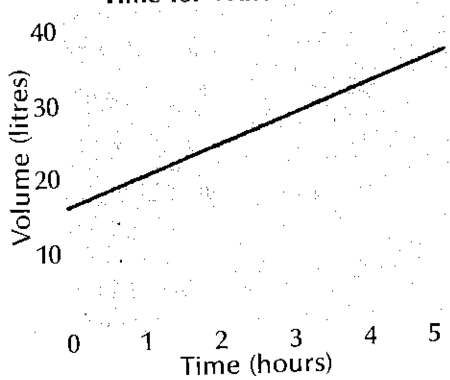
The graph shows the amount of water in a tank at different times.

a. How much water was already in the tank at 0 hours?

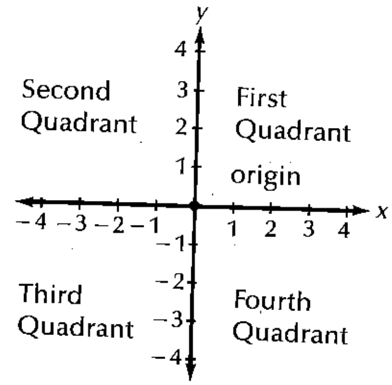
b. How many litres per hour flowed into the tank?

c. Write a formula to show the relation between the amount of water in the tank, *a*, after a number of hours, *h*.

Plot of Volume vs. Time for Water in a Tank



The Coordinate Plane

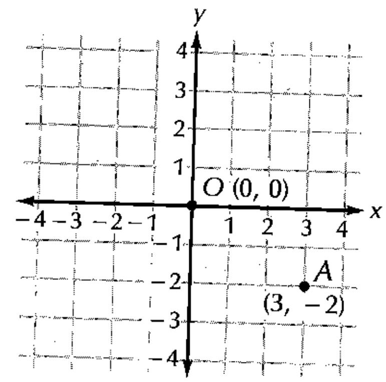


A **plane** is a flat surface stretching infinitely in all directions.

A **coordinate system** for a plane can be made by drawing two perpendicular number lines that intersect at a point called the **origin.**

The horizontal number line (the ***x*-axis**) and the vertical number line (the ***y*-axis**) separate the plane into four *quadrants.*

A point anywhere in the coordinate plane can be located by an *ordered pair,* which describes the horizontal and vertical distance from the origin (0, 0).



x-coordinate → (3, −2) ← y-coordinate

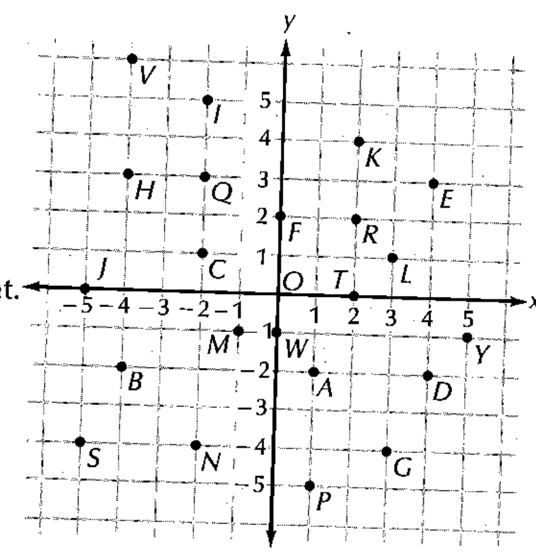
horizontally, 3 right — vertically, 2 down

Point *A* (3, −2) is located in the fourth quadrant.
Point *O* (0, 0) is not in any quadrant.

EXERCISES

1. Copy and complete the table.

Quadrant	x-coordinate	y-coordinate
First	positive	positive
Second	?	?
Third	?	?
Fourth	?	?



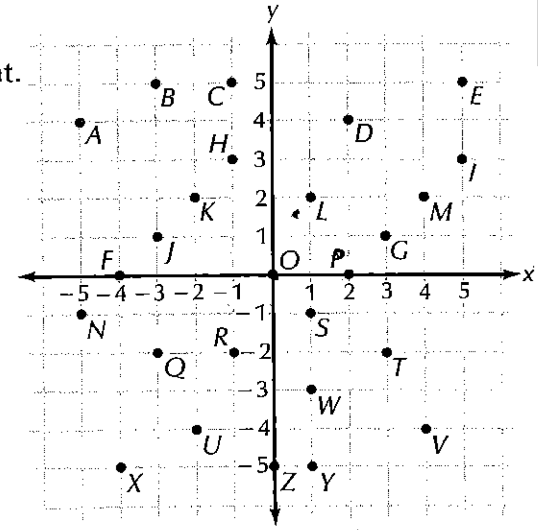
2. Write the ordered pair for each point.

a. *L*	**b.** *S*	**c.** *Q*
d. *P*	**e.** *F*	**f.** *M*
g. *W*	**h.** *O*	**i.** *C*

3. List the ordered pairs for all points located in the second quadrant.

PRACTICE

Use the grid to answer the questions.



1. Write the ordered pair for each point.

   a. *B*   b. *X*   c. *T*

   d. *O*   e. *W*   f. *N*

   g. *A*   h. *G*   i. *P*

2. List all ordered pairs for points in the first quadrant.
3. List all ordered pairs for points in the second quadrant.
4. List all ordered pairs for points in the third quadrant.
5. List all ordered pairs for points in the fourth quadrant.

Identify the quadrant or axis in which the ordered pair is located.

6. (3, 5)   7. (−4, 5)   8. (5, −4)   9. (−4, −5)

10. (0, 4)   11. (5, 0)   12. (−100, 87)   13. (35, −72)

Plot the given points in order.
Then identify the geometric figure made.

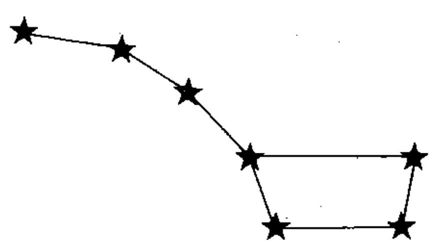
14. *A* (−4, −4)   *B* (4, −4)   *C* (0, 5)   *A* (−4, −4)

15. *P* (−4, 0)   *Q* (0, 4)   *R* (4, 0)   *S* (2, −4)   *T* (−2, −4)   *P* (−4, 0)

For Star Gazers

In ancient times, astronomers named constellations that formed outlines of familiar objects.

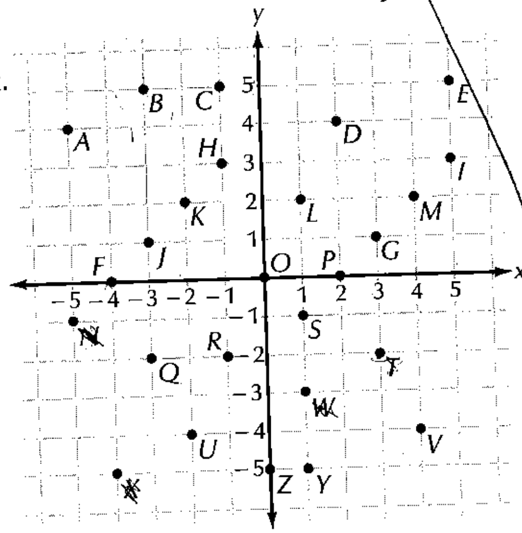
By joining the points listed below in order, you can draw some well known constellations. Try to identify them.



1. (−9, 3) (−4, 3) (−1.5, 1) (2, −1) (2, −4) (8.5, −5) (9.5, −1) (2, −1)
2. (−4, 5) (−3, 2) (−1, 0) (2.5, −1) (2.5, −3) (6.5, −3) (6, −0.5) (2.5, −1)
3. (−11, 8) (−15, 9) (−13, 12) (−11, 13) (−8, 12) (−11, 8) (−10, 6) (−6, 3) (−1, −2) (3, −1) (15, −2) (10, −8) (1, −6) (10, −8) (−2, −9) (−10, 6)
   The eye is point (5, −3).

PRACTICE

Use the grid to answer the questions.



1. Write the ordered pair for each point.

   a. B b. X c. T

   d. O e. W f. N

   g. A h. G i. P

2. List all ordered pairs for points in the first quadrant.
3. List all ordered pairs for points in the second quadrant.
4. List all ordered pairs for points in the third quadrant.
5. List all ordered pairs for points in the fourth quadrant.

Identify the quadrant or axis in which the ordered pair is located.

6. (3, 5) 7. (−4, 5) 8. (5, −4) 9. (−4, −5)

10. (0, 4) 11. (5, 0) 12. (−100, 87) 13. (35, −72)

Plot the given points in order.
Then identify the geometric figure made.

14. A (−4, −4) B (4, −4) C (0, 5) A (−4, −4)

15. P (−4, 0) Q (0, 4) R (4, 0) S (2, −4) T (−2, −4) P (−4, 0)

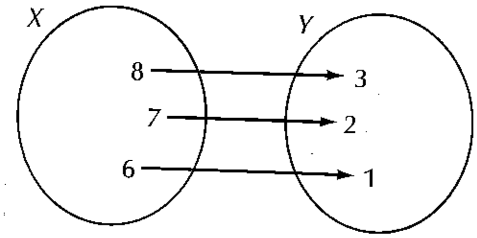
For Star Gazers

In ancient times, astronomers named constellations that formed outlines of familiar objects.

By joining the points listed below in order, you can draw some well known constellations. Try to identify them.

1. (−9, 3) (−4, 3) (−1.5, 1) (2, −1) (2, −4) (8.5, −5) (9.5, −1) (2, −1)
2. (−4, 5) (−3, 2) (−1, 0) (2.5, −1) (2.5, −3) (6.5, −3) (6, −0.5) (2.5, −1)
3. (−11, 8) (−15, 9) (−13, 12) (−11, 13) (−8, 12) (−11, 8) (−10, 6) (−6, 3) (−1, −2) (3, −1) (15, −2) (10, −8) (1, −6) (10, −8) (−2, −9) (−10, 6)
   The eye is point (5, −3).

Equations in Two Variables



The numbers in the arrow diagram are *related* by the same rule. Each number in set Y is *5 less than* its related number in set X.

The *relation rule* can be expressed as $y = x - 5$.

The relation $y = x - 5$ is called an **equation in two variables.** A *solution* to the equation would consist of two numbers, one for the unknown x value and one for the unknown y value.

Below are the steps for solving an equation in two variables.

	Choose a value for x.	Substitute the x value in the equation.	Solve for y.
$y = x - 5$	$x = 8$	$y = 8 - 5$	$y = 3$

A solution to the above equation can be found for *any* value of x.

Other possible solutions can be found by making a table and looking for patterns.

$y = x - 5$		
x	y	(x, y)
8	3	(8, 3)
7	2	(7, 2)
6	1	(6, 1)
5	0	(5, 0)
4	-1	(4, -1)

It is easier to solve an equation in two variables if the y value is expressed *in terms of* x, as in $y = x + 2$ or $y = 5x + (-1)$.

The y value can be isolated in an equation by writing an equivalent equation.

Examples:

$$\begin{aligned} 6x + y &= 2 \\ 6x - 6x + y &= 2 - 6x \\ 0 + y &= 2 - 6x \\ y &= -6x + 2 \end{aligned}$$

Isolate y by *subtracting* $6x$ from both sides.

$6x + y = 2$ and $y = -6x + 2$ are equivalent equations.

$$\begin{aligned} -x + y &= 3 \\ -x + x + y &= 3 + x \\ 0 + y &= 3 + x \\ y &= x + 3 \end{aligned}$$

Isolate y by *adding* x to both sides.

$-x + y = 3$ and $y = x + 3$ are equivalent equations.

EXERCISES

Find the missing y value so the ordered pair is a solution to the given equation.

1. $y = -x + 7$; $(5, ■)$
2. $y = 3x + 8$; $(7, ■)$
3. $y = x + 2$; $(4, ■)$
4. $y = -9x$; $(2, ■)$
5. $y = -x + 4$; $(1, ■)$
6. $y = -4x + 2$; $(0, ■)$
7. $y = 3x + (-2)$; $(5, ■)$
8. $y = 9x + 7$; $(-5, ■)$
9. $y = -6x + 13$; $(-1, ■)$

Copy and complete the table to find some solutions to the given equation.

10.

$y = x + 4$		
x	y	(x, y)
-1		
0		
1		
2		
3		

11.

$y = -x + 2$		
x	y	(x, y)
3		
2		
1		
0		
-1		

12.

$y = 5x + 3$		
x	y	(x, y)
-2		
-1		
0		
1		
2		

13.

$y = -x + 8$		
x	y	(x, y)
2		
1		
0		
-1		
-2		

14.

$y = 7x + (-2)$		
x	y	(x, y)
2		
1		
0		
-1		
-2		

15.

$y = -3x + 1$		
x	y	(x, y)
2		
1		
0		
-1		
-2		

For each equation, write an equivalent equation for y in terms of x.

16.
$$\begin{aligned} x + y &= 3 \\ x - ■ + y &= 3 - ■ \\ 0 + y &= 3 - ■ \\ y &= -■ + ■ \end{aligned}$$

17.
$$\begin{aligned} x + y &= -8 \\ x - ■ + y &= -8 - ■ \\ 0 + y &= -8 - ■ \\ y &= -■ - ■ \end{aligned}$$

18. $-x + y = 1$

19.
$$\begin{aligned} 4x + y &= 16 \\ 4x - ■ + y &= 16 - ■ \\ 0 + y &= 16 - ■ \\ y &= -■ + ■ \end{aligned}$$

20.
$$\begin{aligned} -2x + y &= -5 \\ -2x + ■ + y &= -5 + ■ \\ 0 + y &= -5 + ■ \\ y &= ■ - ■ \end{aligned}$$

21. $x + y = -1$

Solve the equation by writing an equivalent equation for y in terms of x.
Use $\{-2, -1, 0, 1, 2\}$ as the values for x.

22. $-x + y = 3$
23. $x + y = -2$
24. $-6x + y = -9$
25. $2x + y = 13$
26. $y - 5x = 0$
27. $y + 1 = 4x$

PRACTICE

Write the ordered pairs that are solutions to each given equation.

1. $y = -2x + 7$ **a.** $(3, 1)$ **b.** $(4, 3)$ **c.** $(4, -1)$
2. $y = -3x + 8$ **a.** $(4, 2)$ **b.** $(2, 2)$ **c.** $(3, -1)$
3. $y = x - 5$ **a.** $(5, 1)$ **b.** $(6, 1)$ **c.** $(7, 1)$

Copy and complete the table to find some solutions to the equation.

4. $y = x - 3$

x	y	(x, y)
1		
2		
3		
4		
5		

5. $y = x + 5$

x	y	(x, y)
0		
2		
4		
6		
8		

6. $y = 3x + 1$

x	y	(x, y)
−1		
0		
1		
2		
3		

7. $y = -x + 11$

x	y	(x, y)
−3		
−2		
−1		
0		
1		

8. $y = -3x + 10$

x	y	(x, y)
−2		
−1		
0		
1		
2		

9. $y = 8x + 7$

x	y	(x, y)
−3		
−2		
−1		
0		
1		

10. $y = 6x - 4$

x	y	(x, y)
−2		
−1		
0		
1		
2		

11. $y = -3x + 7$

x	y	(x, y)
−2		
−1		
0		
1		
2		

12. $y = x + (-2)$

x	y	(x, y)
−2		
−1		
0		
1		
2		

Make a table to find some solutions to each equation.
Use $\{-2, -1, 0, 1, 2\}$ as the possible values for x.

13. $y = x + 4$ **14.** $y = -x + 1$ **15.** $y = -x + 5$

16. $y = 7x + 2$ **17.** $y = -3x + 12$ **18.** $y = 8x - 5x$

19. $y = -6x + 4$ **20.** $y = -4x$ **21.** $y = -6x + 7$

For each equation, write an equivalent equation for y in terms of x.

22. $x + y = 9$
23. $x + y = -6$
24. $y - 3 = x$
25. $4 + y = x$
26. $2 = -x + y$
27. $10 = x + y$
28. $5x + y = 2$
29. $3x + y = 4$
30. $x + y = -11$
31. $x + y = -2$
32. $2x + y = 3$
33. $x = -2 - y$

Solve the equation by writing an equivalent equation for y in terms of x. Use $\{-2, -1, 0, 1, 2\}$ as the possible values for x.

34. $x + y = 4$
35. $x - y = 12$
36. $-x + y = 1$
37. $12x = 6y$
38. $-2x + y = -2$
39. $x - y = 4$
40. $x + y = -5$
41. $-x + y = -9$
42. $9x + y = 6$

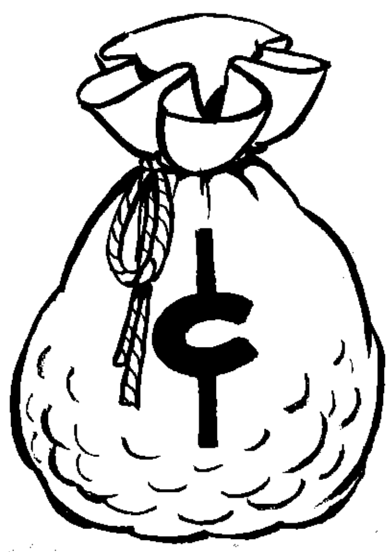
Write the equation for y in terms of x for the given solution set.

43. $\{\ldots, (0, 4), (1, 3), (2, 2), (3, 1), \ldots\} \rightarrow y = \blacksquare$
44. $\{\ldots, (6, 0), (5, 1), (4, 2), (3, 3), \ldots\} \rightarrow y = \blacksquare$
45. $\{\ldots, (5, 0), (6, 1), (7, 2), (8, 3), \ldots\} \rightarrow y = \blacksquare$
46. $\{\ldots, (1, 1), (2, 2), (3, 3), (4, 4), \ldots\} \rightarrow y = \blacksquare$

Coin Problems

Find the solutions to the problem by making a table of the possible values for the two unknowns.

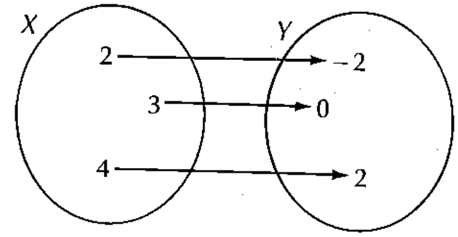
1. In how many different ways can you make 35¢ using both pennies and nickels?
2. Scott has 60 cents made up of nickels and dimes. What are the possible combinations of coins he could have?
3. Mandy has 85 cents made up of nickels and dimes. What are the possible combinations of coins she could have?
4. Reed has $1.25 in quarters and nickels. What combinations of coins could he have?
5. Wade has $1.75 in dimes and quarters. What combinations of coins could he have?
6. In how many different ways can you make $4.00 using both dimes and quarters?



Graphing Equations in Two Variables

The numbers in set Y are *related* to the numbers in set X by the rule: $y = 2x - 6$.

Each number in set Y is the double of its related number in set X minus 6.



The **graph** of the equation $y = 2x - 6$ consists of all the points in the coordinate plane whose x- and y-coordinates are solutions to the equation. Since the number of solutions is *infinite*, it is impossible to graph all solutions. Yet, by graphing a few solutions, one can detect a *pattern* in the graph.

Below are the steps for graphing an equation in two variables.

Step 1:

Make a table of some solutions. Two of the easiest solutions to find are in the form $(0, y)$ and $(x, 0)$.

For $y = 2x - 6$

If $x = 0$, then $y = -6$ $(0, -6)$.
If $y = 0$, then $x = 3$ $(3, 0)$.

$y = 2x - 6$		
x	y	(x, y)
3	0	(3, 0)
2	−2	(2, −2)
1	−4	(1, −4)
0	−6	(0, −6)
−1	−8	(−1, −8)
−2	−10	(−2, −10)

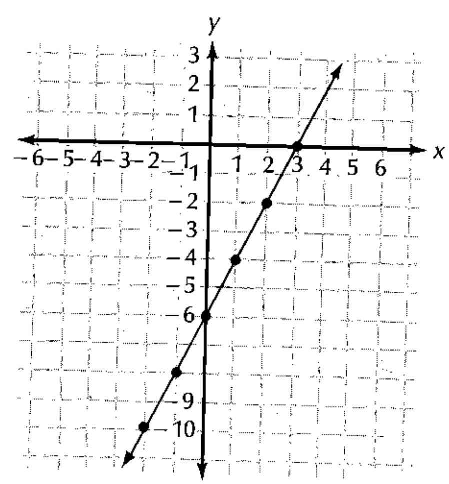
Solution set: $\{\ldots, (3, 0), (2, -2), (1, -4), (0, -6), (-1, -8), (-2, -10), \ldots\}$

Step 2:

Graph the solutions on a coordinate grid.

You can see that all the points lie in a *straight line*. Therefore, the relation between x and y is called **linear**.

Equations that have a straight line graph are called **linear equations**.

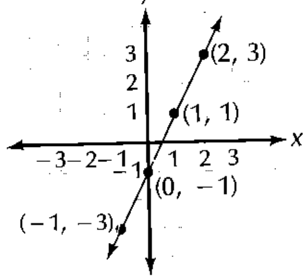


EXERCISES

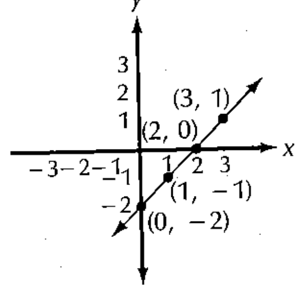
Match the number of the equation with the letter of its graph.

1. $y = x - 2$ 2. $y = 3x$ 3. $y = 2x - 1$

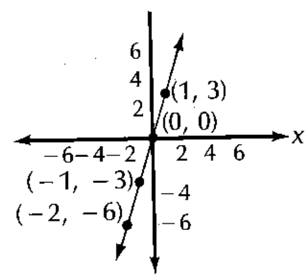
a.



b.



c.



Copy and complete the table.
Then draw a graph of the equation.

4.

$y = 3x$		
x	y	(x, y)
−1	■	■
0	■	■
1	■	■

5.

$y = x + 2$		
x	y	(x, y)
−2	■	■
−1	■	■
0	■	■

6.

$y = x$		
x	y	(x, y)
−3	■	■
−2	■	■
−1	■	■

7.

$y = 3x - 3$		
x	y	(x, y)
−1	■	■
0	■	■
1	■	■

8.

$y = -2x - 4$		
x	y	(x, y)
0	■	■
1	■	■
2	■	■

9.

$y = -x + 2$		
x	y	(x, y)
1	■	■
2	■	■
3	■	■

Copy and complete the solution set for the equation.
Then draw a graph of the equation.

10. $y = x + 4$ {. . ., (0, ■), (1, ■), (2, ■), . . .}
11. $y = -x + 1$ {. . ., (1, ■), (2, ■), (3, ■), . . .}
12. $y = 2x - 1$ {. . ., (−1, ■), (0, ■), (1, ■), . . .}
13. $y = -3x + 2$ {. . ., (−1, ■), (0, ■), (1, ■), . . .}

Find three solutions to the equation and then draw its graph.
When needed, write an equivalent equation for y in terms of x.

14. $y = x + 5$ 15. $y = 9 - x$ 16. $x + y = 5$
17. $3x + y = 2$ 18. $-3x + y = 22$ 19. $-6x + y = -9$

PRACTICE

Copy and complete. Then draw the graph of the equation.

1.

$y = 5x$		
x	y	(x, y)
-1		
0		
1		
2		

2.

$y = x + 6$		
x	y	(x, y)
-6		
-5		
-4		
-3		

3.

$y = x - 4$		
x	y	(x, y)
0		
1		
2		
3		

4.

$y = -5x + 6$		
x	y	(x, y)
0		
1		
2		
3		

5.

$y = 3x - 1$		
x	y	(x, y)
-1		
0		
1		
2		

6.

$y = 8x$ [illegible]		
x	y	(x, y)
-2		
-1		
0		
1		

7. $y = 3x$ {. . ., (−1, ■), (0, ■), (1, ■), . . .}
8. $y = x - 1$ {. . ., (0, ■), (1, ■), (2, ■), . . .}
9. $y = -3x - 4$ {. . ., (0, ■), (−1, ■), (−2, ■), . . .}
10. $y = -2x + 5$ {. . ., (−2, ■), (−1, ■), (0, ■), . . .}

Find three solutions to the equation and then draw its graph. When needed, write an equivalent equation for y in terms of x.

11. $2x + y = 15$
12. $y = 6x - 5$
13. $x - y = 2$
14. $4x - 4y = 0$
15. $x + 2y = 13$
16. $y = -x - 4$
17. $x + y = -4$
18. $x + y = 9$
19. $-x + y = -7$

20. Graph the given equations. Use $\{-3, -2, -1, 0, 1, 2, 3\}$ as the x values.
    - **a.** $y = x^2$ **b.** $y = x^2 - 4$ **c.** $y = x^2 + 2$
    - **d.** Which of the above equations are not linear?

21. Graph the given equations. Use $\{0, 1, 2, 3, 4\}$ as the x-values.
    - **a.** $y = \frac{1}{2}x$ **b.** $y = x$ **c.** $y = 2x$ **d.** $y = 4x$
    - **e.** Make a statement about the *steepness* (or slope) of the graph of each line.
    - **f.** Predict how the steepness of the graph for $y = \frac{1}{3}x$ would compare to that of $y = \frac{1}{2}x$.

Using the Computer to Produce Tables of Values

Read the program listing below that constructs a table of values for a linear equation in two variables.

```
10  PRINT TAB(5);"-----------------------"
20  PRINT TAB(5);"       X            Y"
30  PRINT TAB(5);"-----------------------"
40  FOR X = 1 TO 10
50  LET Y = 3 * X + 1
60  PRINT TAB(9);X; TAB(20);Y
70  NEXT X
80  PRINT TAB(5);"-----------------------"
90  END
```

When the program is RUN, the following output is produced.

```
-----------------------
      X            Y
-----------------------
      1            4
      2            7
      3            10
      4            13
      5            16
      6            19
      7            22
      8            25
      9            28
      10           31
-----------------------
```

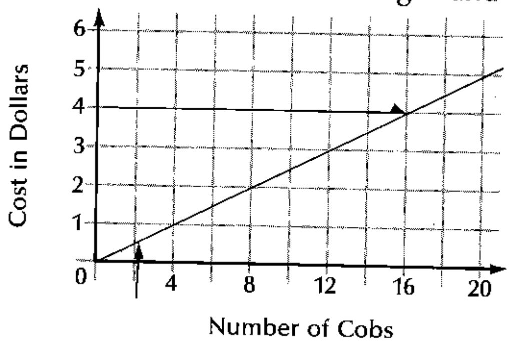
1. What is the equation for which the table of values is calculated?
2. How would you modify line 50 so that a table of values for the equation would be calculated?

   a. $y = 5x - 1$ **b.** $3x + y = 7$ **c.** $5x + 2y = -3$
3. The table of values is calculated for the x values 1 through 10. How could you modify the program to produce a table of values from -5 to 5?
4. Modify the program so that the odd values of x from 1 to 19 are used.
5. To make the program as general as possible, the equation in line 50 can be altered to read: `50 LET Y = A * X + B`

   INPUT statements can then be used to define the values of A and B when the program is RUN. Alter the program above so that the user of the program is asked to enter values for A and B at the start of the program.

Applications

A grocery store advertised corn on the cob for sale at 4 cobs for $1.00. Jackson drew a line graph to show the relationship between the number of cobs of corn bought and the total cost of the corn.

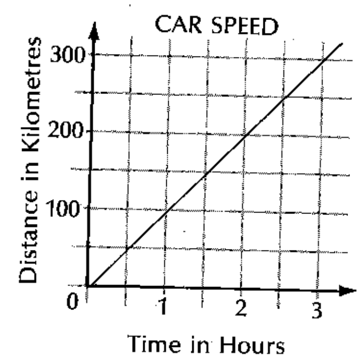


From the graph, Jackson was able to tell that 2 cobs of corn would cost $0.50 and that $4.00 would buy 16 cobs of corn.

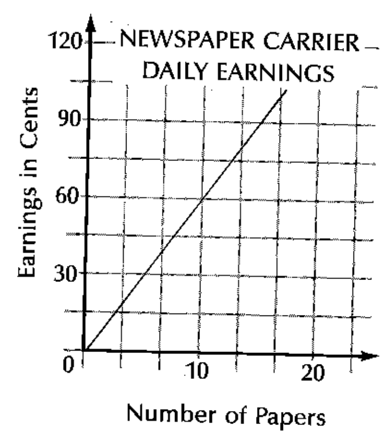
EXERCISES

Use the graphs to answer the questions.

1. How fast was the car travelling?
2. How far had the car gone after 2.5 h?
3. Did the car go the same distance each hour of travelling time?
4. How long did it take the car to travel 275 km?

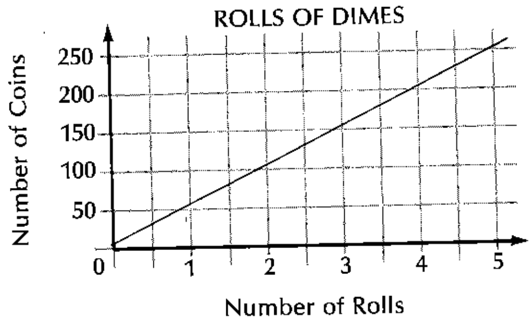


5. How much did the carrier earn for each paper delivered?
6. If a carrier delivered 35 papers each day, how much money would be earned daily? weekly? (5 d)?
7. A carrier earned $1.20/d for delivering papers. How many papers were delivered each day?
8. A jet flew 950 km/h. Draw a line graph to show the relationship between the distance flown and the time it takes to fly that distance. Write 3 questions that can be answered from the graph.

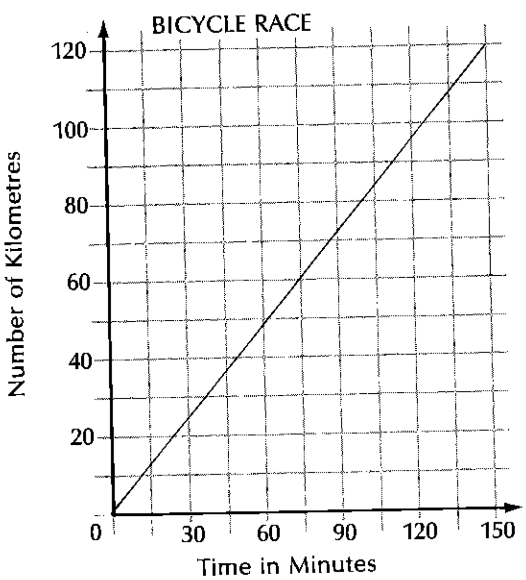


PRACTICE

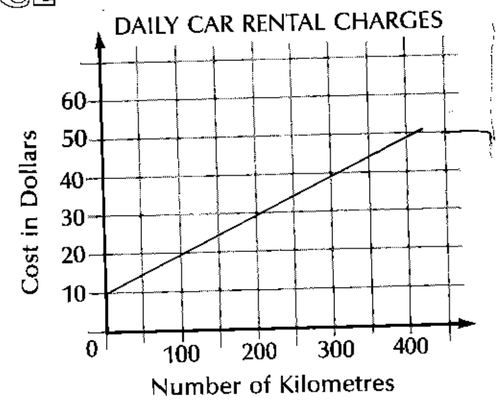
Use the graphs to answer the questions.



1. How many dimes are there in 2 rolls?
2. How many rolls of dimes are required to make $20?
3. Three rolls of dimes is how much money?

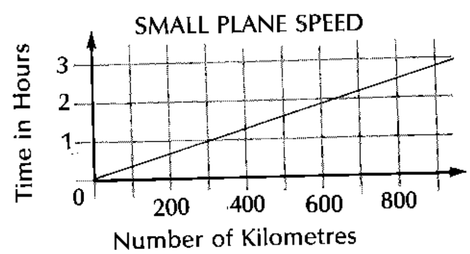


8. What was the speed of the bicycle?
9. How far did the rider go in 2 h?
10. How long did it take to bicycle 50 km?



The rental company charges a daily flat rate and an additional charge for each kilometre driven.

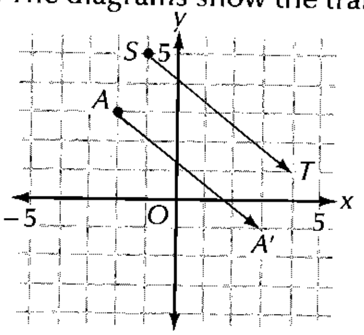
4. What is the daily flat rate?
5. What is the rate for each kilometre driven?
6. How much would a renter pay for the use of a car for one day while travelling 300 km?
7. Can you read from the graph, the cost of renting a car for 7 d while travelling 500 km?



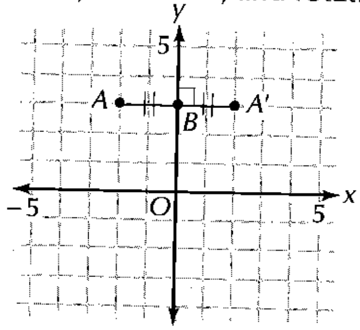
11. How fast was the plane flying?
12. How far had the plane gone in 30 min?
13. How long did it take the plane to fly 500 km?
14. Did the plane travel the same number of kilometres each hour?

Transformations on the Coordinate Plane

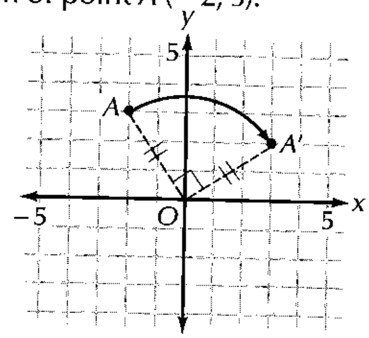
The diagrams show the translation, reflection, and rotation of point A $(-2, 3)$.



Translation ST:
5 right, down 4.
$AA' = ST$
Image: A' $(3, -1)$



Reflection in y-axis.
$AB = A'B \quad AA' \perp y$-axis
Image: A' $(2, 3)$



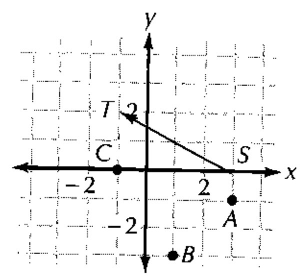
Rotation: 90° clockwise about O.
$\angle AOA' = 90° \quad OA = OA'$
Image: A' $(3, 2)$

EXERCISES

Copy and complete the table with the ordered pair locating the transformation image.

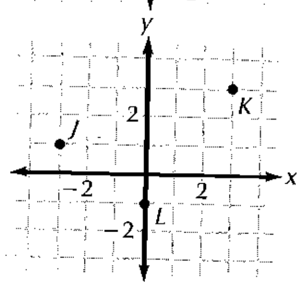
1. Locate the image of the given point after translation ST.

	Point	Image
a.	A $(3, -1)$ →	A' (■, ■)
b.	B $(1, -3)$ →	B' (■, ■)
c.	C $(-1, 0)$ →	C' (■, ■)



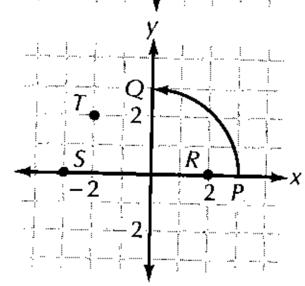
2. Locate the image of the given point after a reflection in the x-axis.

	Point	Image
a.	J $(-3, 1)$ →	J' (■, ■)
b.	K $(3, 3)$ →	K' (■, ■)
c.	L (■, ■) →	L' (■, ■)

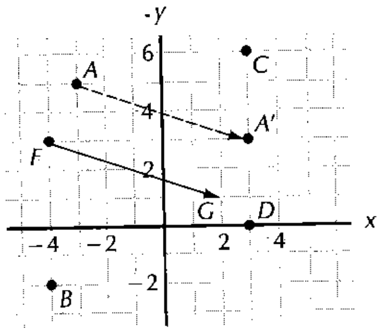


3. Locate the image of the given point after rotation PQ about $(0, 0)$.

	Point	Image
a.	R (■, ■) →	R' (■, ■)
b.	S (■, ■) →	S' (■, ■)
c.	T (■, ■) →	T' (■, ■)

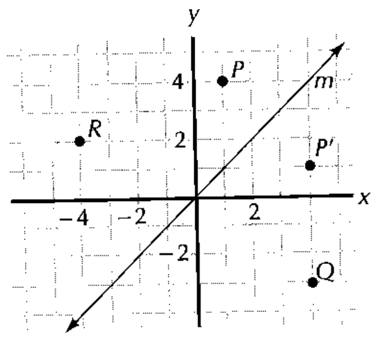


PRACTICE

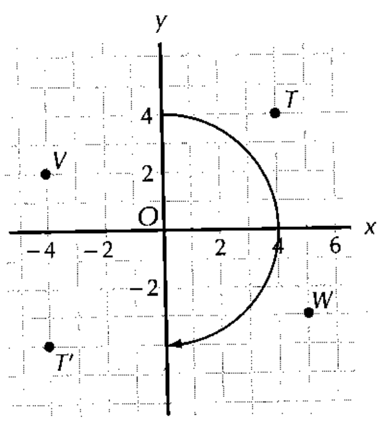


Refer to the diagram to the right.

1. Describe the translation which maps A onto A'.
2. Copy and complete.
   a. A (■, ■) → A' (■, ■)
   b. B (■, ■) → B' (■, ■)
   c. C (■, ■) → C' (■, ■)
3. Write a statement comparing AA', BB', and CC' to arrow FG.

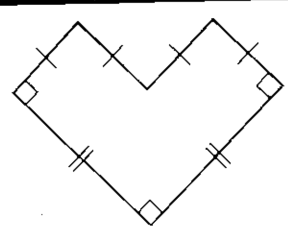


4. Describe the reflection which maps P onto P'.
5. Copy and complete.
   a. P (■, ■) → P' (■, ■)
   b. Q (■, ■) → Q' (■, ■)
   c. R (■, ■) → R' (■, ■)
6. Write a statement describing the relationship of line m to the given segment.
   a. PP' b. QQ' c. RR'



7. Describe the rotation which maps T onto T'.
8. Copy and complete.
   a. T (■, ■) → T' (■, ■)
   b. W(■, ■) → W'(■, ■)
   c. V (■, ■) → V' (■, ■)
9. What is the size of the given angle?
   a. $\angle TOT'$ b. $\angle WOW'$ c. $\angle VOV'$
10. Write a statement comparing the lengths of the given segments.
    a. TO and $T'O$ b. WO and $W'O$

Puzzler

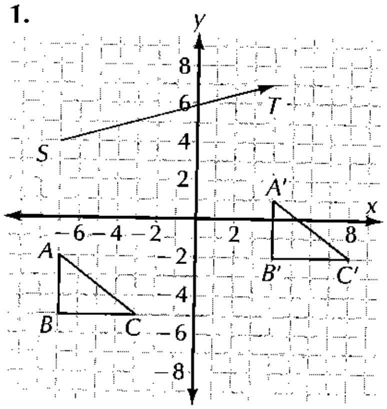


Construct a figure like the one at the right. Show how the figure can be divided into four congruent pieces.

Polygon Transformations on the Coordinate Plane

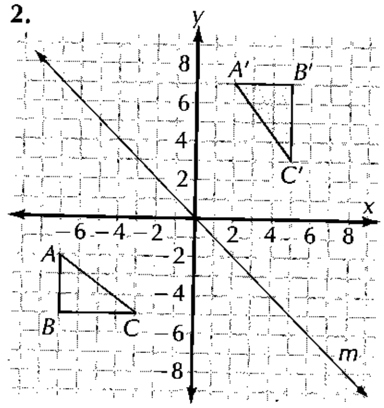
The diagrams show the translation, reflection, and rotation images of $\triangle ABC$ on the coordinate number plane.

1.



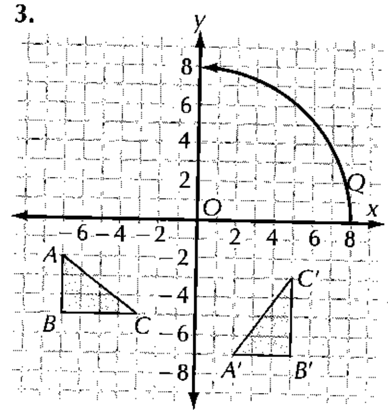
$\triangle ABC \rightarrow \triangle A'B'C'$ under the *translation ST* [R11, U3].

2.



$\triangle ABC \rightarrow \triangle A'B'C'$ under a *reflection* in line m.

3.



$\triangle ABC \rightarrow \triangle A'B'C'$ under a 90° counter-clockwise *rotation* about (0, 0).

EXERCISES

Refer to Diagram 1 above.

1. What is the translation which maps A onto A'?
2. Compare the length of each segment to the length of arrow ST.
   a. $\overline{AA'}$ b. $\overline{BB'}$ c. $\overline{CC'}$

Refer to diagram 2 above.

3. Describe the relation of line m to the given segment.
   a. $\overline{AA'}$ b. $\overline{BB'}$ c. $\overline{CC'}$

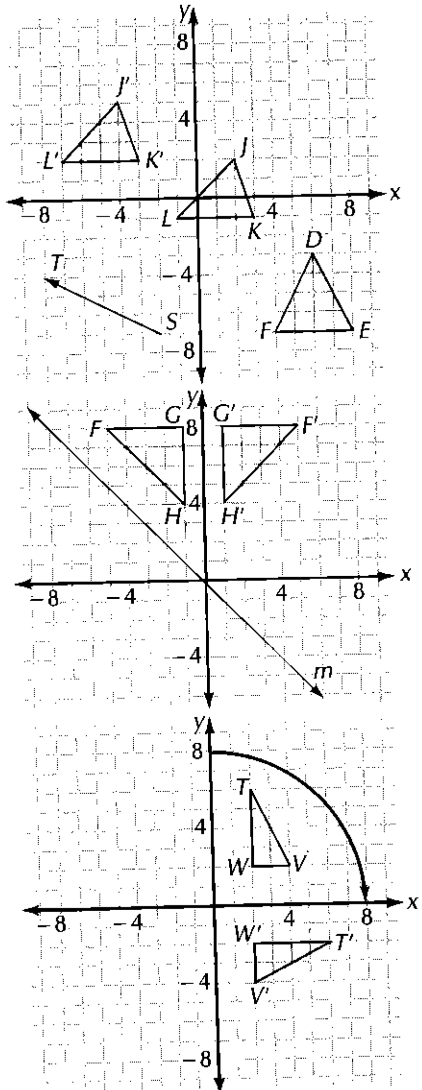
Refer to diagram 3 above.

4. Copy and complete.
   a. A (■, ■) → A' (■, ■) b. B (■, ■) → B' (■, ■) c. C (■, ■) → C' (■, ■)
5. What is the size of the given angle?
   a. $\angle AOA'$ b. $\angle BOB'$ c. $\angle COC'$
6. Write a statement comparing the lengths of AO and $A'O$.

PRACTICE

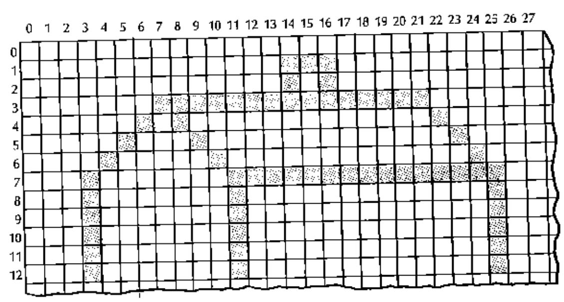
Refer to the diagrams at the right.

1. Define the translation for which $\triangle JKL \rightarrow \triangle J'K'L'$.
2. Copy and complete.
   a. J (■, ■) → J' (■, ■)
   b. K (■, ■) → K' (■, ■)
   c. L (■, ■) → L' (■, ■)
3. Locate $\triangle D'E'F'$ under the translation shown by arrow ST.
4. Describe the reflection for which $\triangle FGH \rightarrow \triangle F'G'H'$.
5. Locate the images of F, G, and H after a reflection in the x-axis.
6. Locate the images of F, G, and H for a reflection in line m.
7. Locate the images of F, G, and H for the translation [L2, D2] and a reflection in the y-axis.
8. Describe the rotation for which $\triangle TVW \rightarrow \triangle T'V'W'$.
9. Copy and complete.
   a. T (■, ■) → T' (■, ■)
   b. V (■, ■) → V' (■, ■)
   c. W (■, ■) → W' (■, ■)
10. Locate the images of T, V, and W after a 180° clockwise rotation about (0, 0).
11. Locate the images of T, V, and W after a 270° counter-clockwise rotation about (0, 0).



Computer Graphics

Make a design on grid paper. Then write a BASIC program using `PRINT TAB ( );` statements and `FOR/NEXT` loops to produce the design.



The Earth's Degree-Coordinate System

Every point on the Earth's surface can be located by an ordered pair of *longitude* and *latitude* coordinates.

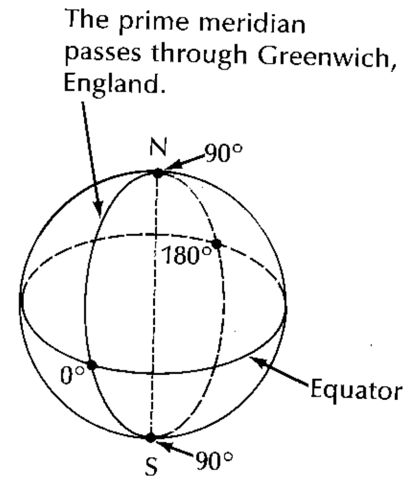
(longitude, latitude)

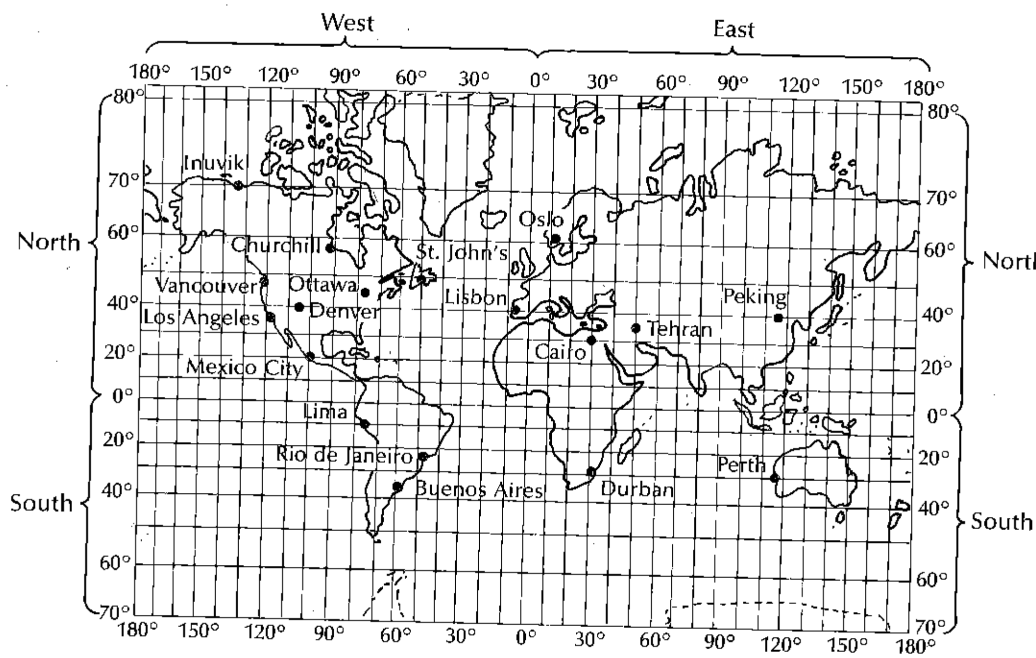
The first coordinate, longitude, is the *horizontal* distance between 0° and 180° east or west of the prime meridian.

The second coordinate, latitude, is the vertical distance between 0° and 90° north or south of the equator.

For example, Oslo, Norway is located near the point that is 10°E of the prime meridian and 60°N of the equator.

(10°E, 60°N) Oslo, Norway





Find the cities near the following locations.

1. 115°E, 40°N
2. 45°W, 25°S
3. 135°W, 70°N
4. 30°E, 30°S
5. 75°W, 10°S
6. 30°E, 30°N
7. 100°W, 20°N
8. 120°W, 35°N

Write the ordered pair locating the city to the nearest 5°.

9. St. John's
10. Vancouver
11. Peking
12. Buenos Aires

REVIEW

Using the replacement set $\{-3, -2, -1, 0, 1, 2, 3\}$, list the solution set.

1. $x > -2$ **2.** $x + 2 = -1$ **3.** $-3x = 1$

4. $2x + (-1) = -9$ **5.** $x > 3$ **6.** $-4x + (-x) = -5$

Write two more ordered pairs for each relation.

7. {(cubic centimetre, cm^3), (cubic decimetre, dm^3), (cubic metre, m^3)}

8. {(XV, 15), (CXXVIII, 128), (DCXL, 640)}

9. {(length, *l*), (height, *h*), (volume, *V*)}

Copy and complete the tables.

10.

Rule: $y = 7x$	
x	y
-1	■
0	■
1	■
2	■

11.

Rule: $y = 3x + 4$	
x	y
-2	■
-1	■
0	■
1	■

12.

Rule: $y = -2x + 1$	
x	y
-2	■
-1	■
0	■
1	■

Identify the quadrant or axis in which the ordered pair is located.

13. $(-5, 2)$ **14.** $(7, 11)$ **15.** $(-9, -1)$ **16.** $(0, 4)$ **17.** $(-3, 0)$

Make a table to find some solutions to the equations.
Use $\{-2, -1, 0, 1, 2\}$ as the possible values for x.

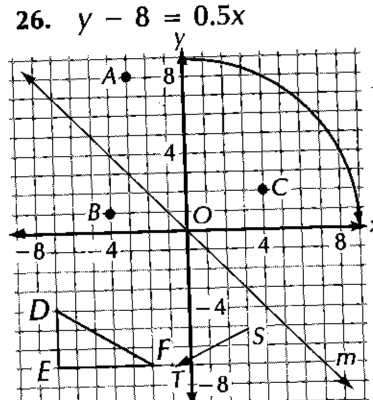
18. $y = 6x + 2$ **19.** $y = -4x + (-3)$ **20.** $y = -x + 5$

Find three solutions to each equation and then draw its graph.

21. $y = -4x + 1$ **22.** $y = 2x + (-2)$ **23.** $y = -x + 3$

For each equation, write a similar equation for y in terms of x.
Then write one ordered pair that satisfies the equation.

24. $4x + y = 14$ **25.** $x - y = 10$ **26.** $y - 8 = 0.5x$



Refer to the diagram at the right.
Copy and complete.

27. A (■, ■) → A' (■, ■) under translation ST.

28. B (■, ■) → B' (■, ■) under reflection in line m.

29. C (■, ■) → C' (■, ■) under 90° clockwise rotation about O.

30. Name the coordinates of the image of $\triangle DEF$ under each transformation.

TEST

UNIT 14

Using the replacement set $\{-3, -2, -1, 0, 1, 2, 3\}$, list the solution set.

1. $x > -2$ **2.** $x + (-2) = -4$ **3.** $-x + 3 = 0$

4. $5x + 2 = 18$ **5.** $-2x + 5 = 3$ **6.** $x > 3$

Describe the relation rule.
Then write two more ordered pairs satisfying the relation.

7. {(Nova Scotia, N.S.), (Ontario, Ont.), (British Columbia, B.C.)}

8. $\{(101_{(2)}, 5_{(10)}), (1100_{(2)}, 12_{(10)}), (11010111_{(2)}, 215_{(10)})\}$

9. $\left\{\left(0.75, \frac{3}{4}\right), \left(0.1\overline{6}, \frac{1}{6}\right), \left(0.375, \frac{3}{8}\right)\right\}$

Copy and complete the table.

10.

Rule: $y = 2x$	
x	y
-2	
-1	
0	
1	

11.

Rule: $y = -5x + 1$	
x	y
1	
0	
-1	
-2	

12.

Rule: $y = -x + 2$	
x	y
2	
1	
0	
-1	

Identify the quadrant or axis in which the ordered pair is located.

13. $(-5, -6)$ **14.** $(0, -2)$ **15.** $(2, -3)$ **16.** $(-5, -9)$ **17.** $(7, 0)$

Make a table to find some solutions to the equation.
Use $\{-2, -1, 0, 1, 2\}$ as the possible values for x.

18. $y = -4x$ **19.** $y = 3x + 5$ **20.** $y = -x + 2$

Find three solutions for the given equation and then draw its graph.

21. $y = x + (-1)$ **22.** $y = -6x - 7$ **23.** $y = -x + 1$

For each equation, write a similar equation for y in terms of x.
Then write one ordered pair that satisfies the equation.

24. $y - 5x = 30$ **25.** $2x + y = -11$ **26.** $y - 17 = 3x$

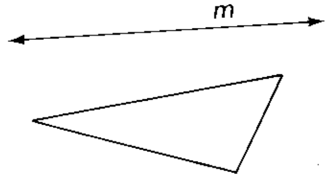
27. Draw a square with vertices $(5, -5)$, $(8 -5)$ $(8, -2)$ and $(5, -2)$ on grid paper. Locate each transformation image of the square.

a. translation [L7, U9]
b. reflection in the x-axis
c. 90° counter-clockwise rotation about $(0, 0)$

Transformations

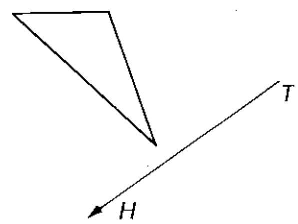
1. Copy the diagrams on grid paper. Draw the image for the given transformation.

a.



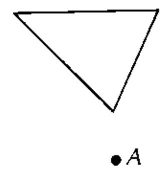
reflection in line *m*

b.



translation defined by arrow *TH*

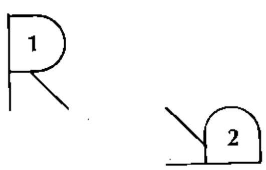
c.



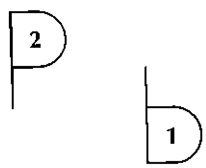
90° clockwise rotation, centre *A*

2. Describe a combination of two transformations that would map Figure 1 onto Figure 2.

a.



b.



c.

3. Copy and complete the chart.

Number of sides	Name of regular polygon	Order of rotational symmetry	Order of symmetry
3	[redacted]	[redacted]	[redacted]
4	[redacted]	[redacted]	[redacted]
5	[redacted]	[redacted]	[redacted]
6	[redacted]	[redacted]	[redacted]

4. For each image of the object, find the scale ratio and the scale factor.