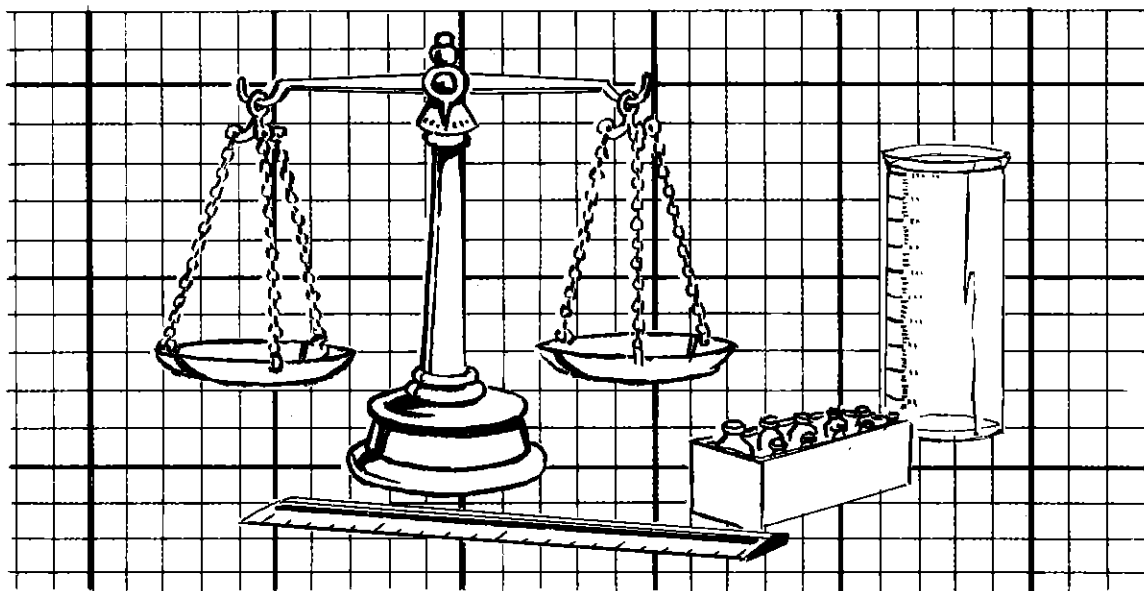


UNIT 12

Area and Volume



The Metric System

The prefixes and symbols of the metric system indicate powers of ten.

10^3	10^2	10^1	Power of Ten	$\frac{1}{10}$ or 10^{-1}	$\frac{1}{100}$ or 10^{-2}	$\frac{1}{1000}$ or 10^{-3}
kilo	hecto	deca	Metric Prefix	deci	centi	milli
k	h	da	Metric Symbol	d	c	m

The metric prefixes and symbols are combined with:

metre (m) for measuring length,
litre (L) for measuring capacity, and
gram (g) for measuring mass.

Examples of metric equivalents:

Length:
 10 m = 1 dam (decametre)
 100 m = 1 hm (hectometre)
 1000 m = 1 km (kilometre)

0.1 m = 1 dm (decimetre)
 0.01 m = 1 cm (centimetre)
 0.001 m = 1 mm (millimetre)

Capacity:
 1000 L = 1 kL (kilolitre)
 0.001 L = 1 mL (millilitre)

Mass:
 1000 g = 1 kg (kilogram)
 0.001 g = 1 mg (milligram)
 1000 kg = 1 t (tonne)

EXERCISES

- Write the metric prefix for each power of ten.
a. 10^3 b. 10^{-2} c. 10^1 d. 10^{-3} e. 10^2 f. 10^{-1}
- Which is the appropriate unit for the measurement of each?
a. height of a door b. capacity of an orange juice can
c. mass of 15 apples d. capacity of a tube of toothpaste
e. width of a table f. mass of a locomotive train
g. distance across Hudson Bay h. mass of a mouse
i. thickness of a sheet of paper j. capacity of a large swimming pool
- Copy and complete.
a. $400 \text{ cm} = \blacksquare \text{ m}$ b. $5000 \text{ mg} = \blacksquare \text{ g}$ c. $685 \text{ L} = \blacksquare \text{ kL}$
d. $0.9 \text{ m} = \blacksquare \text{ mm}$ e. $0.82 \text{ kg} = \blacksquare \text{ g}$ f. $0.4 \text{ L} = \blacksquare \text{ mL}$

PRACTICE

- Copy and complete.
a. $3000 \text{ m} = \blacksquare \text{ km}$ b. $5764 \text{ L} = \blacksquare \text{ kL}$ c. $764 \text{ g} = \blacksquare \text{ mg}$
d. $3.7 \text{ L} = \blacksquare \text{ cL}$ e. $642 \text{ mm} = \blacksquare \text{ m}$ f. $49.8 \text{ dm} = \blacksquare \text{ m}$
g. $34\,100 \text{ mg} = \blacksquare \text{ g}$ h. $1700 \text{ km} = \blacksquare \text{ m}$ i. $8726 \text{ kg} = \blacksquare \text{ t}$
j. $45 \text{ cm} = \blacksquare \text{ dm}$ k. $16 \text{ t} = \blacksquare \text{ kg}$ l. $3\,000\,000 \text{ mL} = \blacksquare \text{ kL}$
- Place a decimal point in the numeral to make the statement true.
a. The Nile River is about 66 700 km long.
b. The mass of a ping pong ball is about 245 g.
c. A cup of coffee measures about 2500 mL.
d. The mass of a cat is about 6350 kg.
e. The height of Mt. Everest is about 88 000 m.
f. The mass of an elephant is about 60 000 kg.
- Copy and complete the table of equivalent measures.

	km	hm	dam	m	dm	cm	mm
a.	?	?	?	4	40	?	?
b.	?	?	?	?	?	?	72
c.	0.05	?	?	?	?	?	?

Mega and Micro

The metric prefix **mega** (M) indicates 10^6 .
The metric prefix **micro** (μ) indicates 10^{-6} .

- a. A bacterium measures about 0.001 mm across.
About how many micrometres is that?

0.001 mm



Bacterium $10\,000\times$ larger

- b. Use a Canada highway map to find an example of two Canadian cities which are about 1 Mm apart.

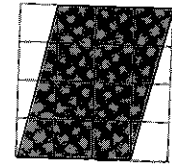
Areas of Rectangles, Squares, and Parallelograms

Area is a measure of the surface inside a closed figure.

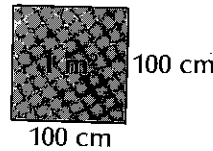
Area is expressed in various *square units*.

Commonly used metric area units are:

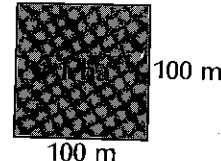
- square millimetre (mm^2).
- square centimetre (cm^2).
- square metre (m^2).
- hectare (ha).
- square kilometre (km^2).



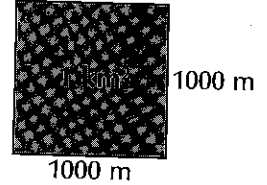
12 square units



100 cm

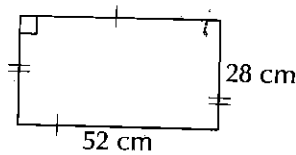


100 m
1 ha = 1 hm²



1000 m

The area (A) of a rectangle is the product of the lengths of its sides.

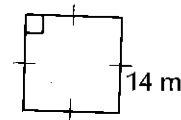


$$\text{Area} = \text{length} \times \text{width}$$

$$A = lw$$

$$A = 52 \times 28$$

$$A = 1456 \text{ (cm}^2\text{)}$$



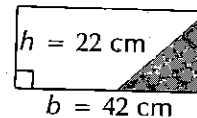
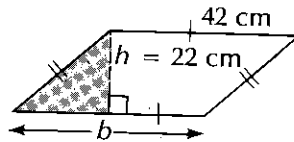
$$\text{Area} = \text{side} \times \text{side}$$

$$A = s^2$$

$$A = 14 \times 14$$

$$A = 196 \text{ (m}^2\text{)}$$

A parallelogram can be cut and rearranged as a rectangle with the same base and height. Therefore, the area of a parallelogram is the product of its base (b) and height (h).



$$\text{Area} = \text{base} \times \text{height}$$

$$A = bh$$

$$A = 22 \times 42$$

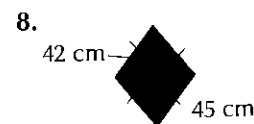
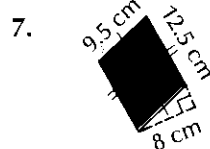
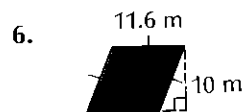
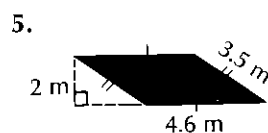
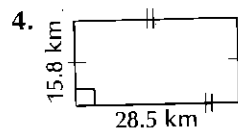
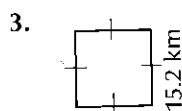
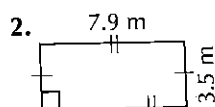
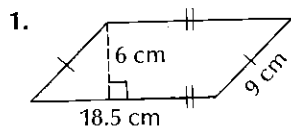
$$A = 924 \text{ (cm}^2\text{)}$$

EXERCISES

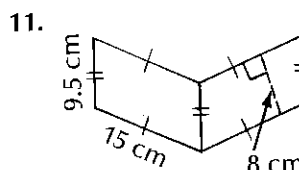
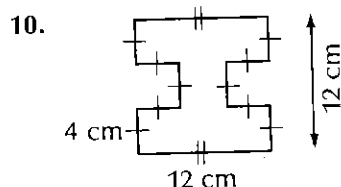
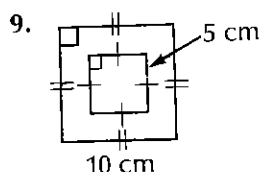
- What is the area of the rectangle?
 - $l = 6 \text{ cm}$
 $w = 13 \text{ cm}$
 - $l = 2.4 \text{ m}$
 $w = 7.3 \text{ m}$
 - $l = 9.3 \text{ km}$
 $w = 3.2 \text{ km}$
- What is the area of the square?
 - $s = 18 \text{ cm}$
 - $s = 5.2 \text{ m}$
 - $s = 8.4 \text{ km}$
- What is the area of the parallelogram?
 - $b = 7 \text{ cm}$
 $h = 12 \text{ cm}$
 - $b = 12.6 \text{ m}$
 $h = 9.7 \text{ m}$
 - $b = 11.8 \text{ km}$
 $h = 5.2 \text{ km}$

PRACTICE

Find the area.



Find the area of the shaded region.



Copy and complete.

12. $1 \text{ cm}^2 = \blacksquare \text{ mm} \times \blacksquare \text{ mm}$ $\therefore 1 \text{ cm}^2 = \blacksquare \text{ mm}^2$
13. $1 \text{ m}^2 = \blacksquare \text{ cm} \times \blacksquare \text{ cm}$ $\therefore 1 \text{ m}^2 = \blacksquare \text{ cm}^2$
14. $1 \text{ m}^2 = \blacksquare \text{ dm} \times \blacksquare \text{ dm}$ $\therefore 1 \text{ m}^2 = \blacksquare \text{ dm}^2$
15. $1 \text{ m}^2 = \blacksquare \text{ mm} \times \blacksquare \text{ mm}$ $\therefore 1 \text{ m}^2 = \blacksquare \text{ mm}^2$
16. $1 \text{ ha} = \blacksquare \text{ m} \times \blacksquare \text{ m}$ $\therefore 1 \text{ ha} = \blacksquare \text{ m}^2$
17. $1 \text{ km}^2 = \blacksquare \text{ m} \times \blacksquare \text{ m}$ $\therefore 1 \text{ km}^2 = \blacksquare \text{ m}^2$

Refer to the table.

18. Which area is equal to about 1 ha?
19. About how many ice hockey rinks would fit on a football field?
20. What is the area of each playing field?
21. What is the perimeter of each?

Sport	Playing Field Dimensions
Tennis	23.7 m by 11 m
Basketball	26 m by 14 m
Ice Hockey	61 m by 30.5 m
Football	101 m by 50.5 m
Soccer	100 m by 73 m

Areas in Scientific Notation

The given areas are written in *scientific notation*. Write each in standard form.

- a. The surface area of the planet Jupiter is $6.4 \times 10^{10} \text{ km}^2$.
- b. An influenza virus measuring 0.0001 mm will fit on a square with area $1.0 \times 10^{-8} \text{ mm}^2$.
- c. The approximate area of Canada is $9.9 \times 10^6 \text{ km}^2$.
- d. As Mercury orbits the Sun, it encloses an area of about $1.054 \times 10^{16} \text{ km}^2$.

Areas of Triangles and Trapezoids

The areas of a triangle and a trapezoid are *half of a parallelogram*.

The area of a triangle is one half the product of its base and height.

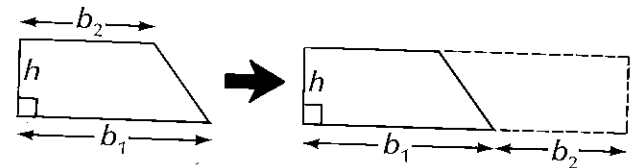
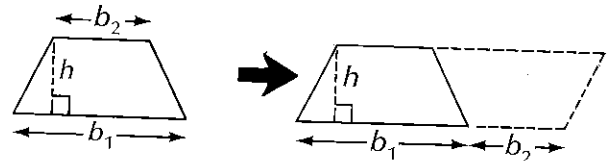
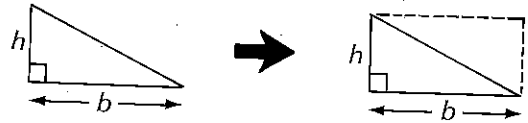
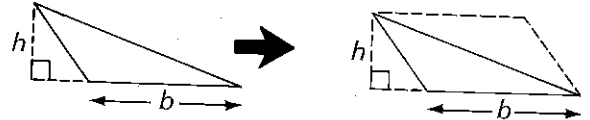
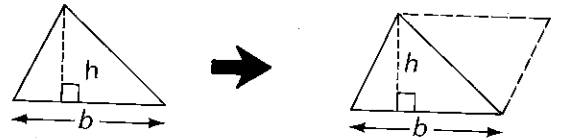
$$A = \frac{1}{2}bh$$

Any side can be the **base**.
The perpendicular distance from the base to the opposite vertex is the **height**.

The area of a trapezoid is one half the product of its height and the sum of its bases.

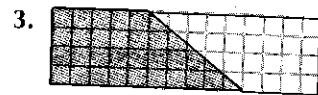
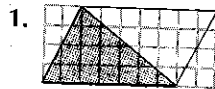
$$A = \frac{1}{2}(b_1 + b_2)h$$

The parallel sides are the **bases**.
The perpendicular distance between the bases is the **height**.

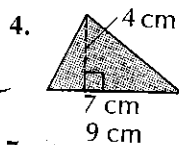


EXERCISES

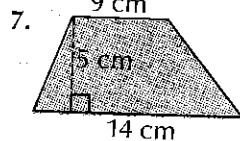
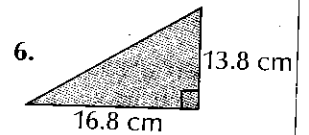
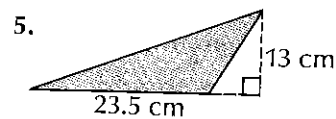
Find the shaded area in square units.



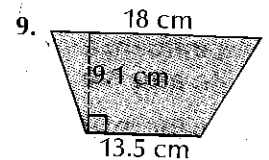
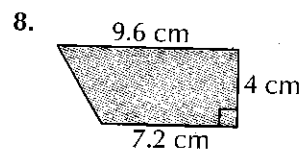
Find the area using the appropriate formula.



$b = 7$
 $h = 4$

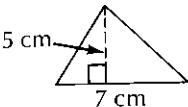
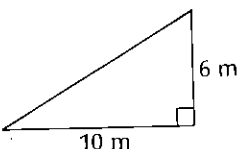
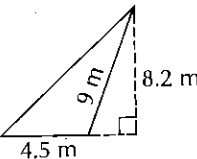
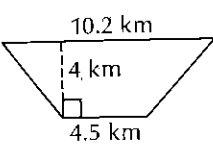
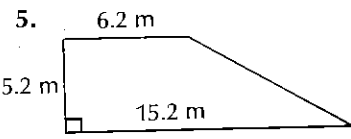
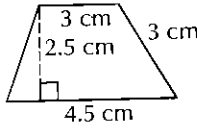
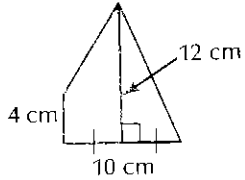
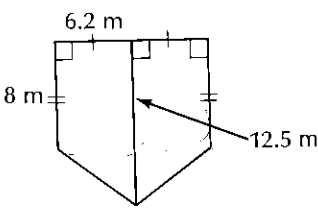
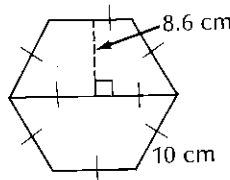


$b_1 = 14$
 $b_2 = 9$
 $h = 5$



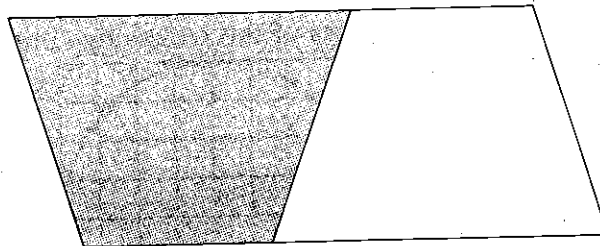
PRACTICE

Find the area.

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 

10. On grid paper, draw an acute triangle, a right triangle, and an obtuse triangle, each having a base of 7 cm and a height of 4 cm. Find the area of each.

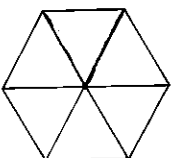
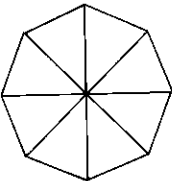
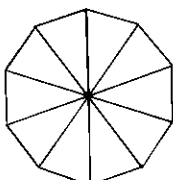
11. Measure and then calculate the area of the shaded trapezoid in square centimetres.



12. One of the equal sides of an isosceles right triangle is 6 cm. What is the area of the triangle?

Other Polygon Areas

Polygons with equal sides can be divided into congruent triangles. Write a formula for calculating the area of each polygon below. Then measure and calculate the area of each in square millimetres.

- a. 
regular hexagon
- b. 
regular octagon
- c. 
regular decagon

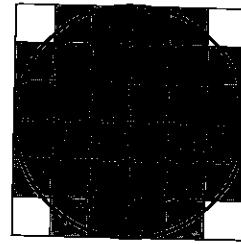
1. a regular n -sided polygon

Area of a Circle

The *approximate* area of a circle can be found by counting squares.

$$(1 \times 24) + \left(\frac{1}{2} \times 8\right) = 28$$

The area of the circle is *about* 28 square units.



Count one square unit for each red square.

Count $\frac{1}{2}$ square unit for each blue square.

The exact area of a circle is the product of pi (π) and the square of the radius.

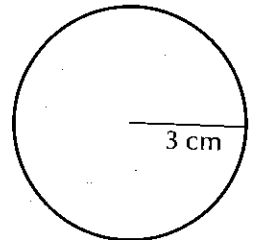
$$A = \pi r^2$$

$$\pi \approx 3.14$$

$$A \approx 3.14 \times 3^2$$

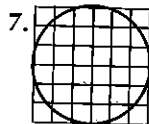
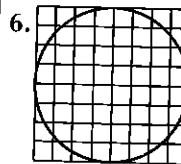
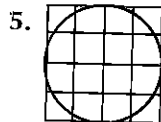
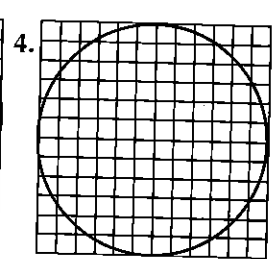
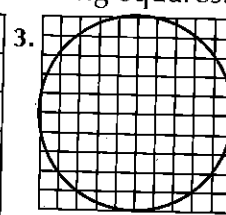
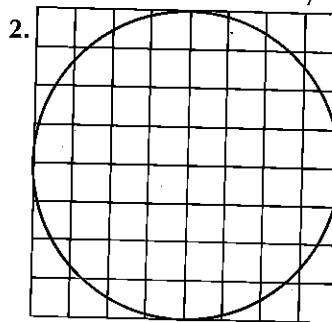
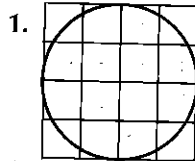
$$A \approx 3.14 \times 9$$

$$A \approx 28.26 \text{ (cm}^2\text{)}$$

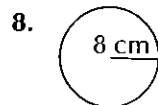


EXERCISES

Approximate the areas of the circles by counting squares.

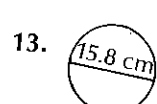
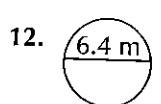
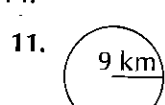
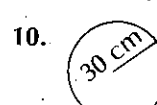
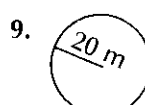


Calculate the area of the circle using the formula. Use $\pi \approx 3.14$.



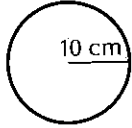
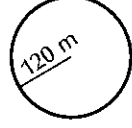
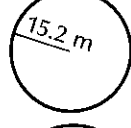
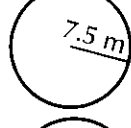
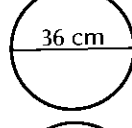



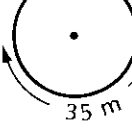
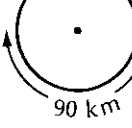



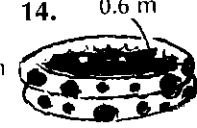


$$r = 8$$

$$A = \pi r^2$$

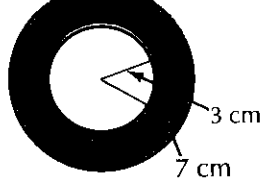

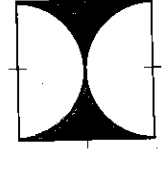
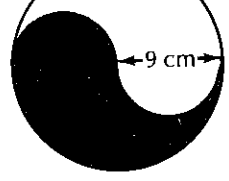


PRACTICE

Calculate the area to the nearest tenth. Use $\pi \approx 3.14$.

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10. 
11. 
12. 
13. 
14. 
15. 
16. 

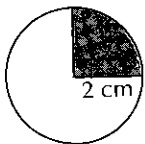
Find the area of the shaded region.

17. 
18. 
19. 
20. 

Areas of Sectors

The shaded part of the interior of the circle is called a **sector**.

The area of the sector is $\frac{90^\circ}{360^\circ}$ or $\frac{1}{4}$ of the area of the circle.



$$\text{Area of } 90^\circ \text{ sector} = \frac{90^\circ}{360^\circ} \times \text{area of circle}$$

$$A \approx \frac{1}{4} \times 3.14 \times 2^2$$

$$A \approx 3.14 \text{ (cm}^2\text{)}$$

Find the area of each sector. Use $\pi \approx 3.14$ and a radius of 2 cm.

- a. 
- b. 
- c. 
- d. 

Three-Dimensional Figures

The objects in our three-dimensional world can be thought of as *solids*, *shells*, or *skeletons*.

Three-dimensional objects with polygonal faces are called *polyhedrons*.

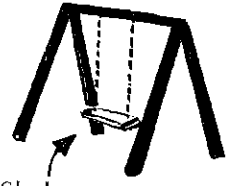
Two kinds of polyhedrons are *prisms* and *pyramids*.



Solid:

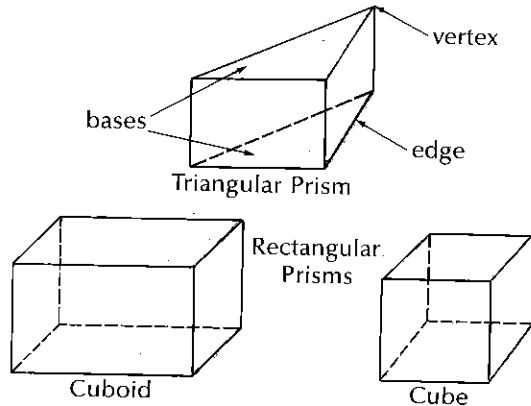


Shell:

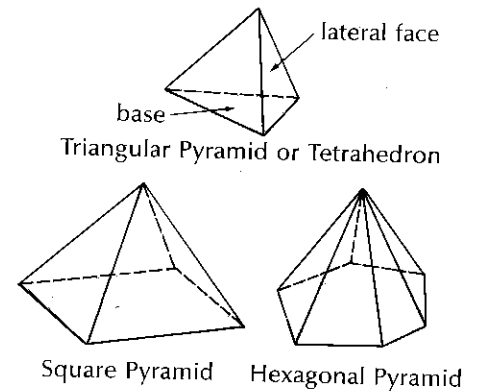


Skeleton:

Prisms have two congruent parallel bases. The lateral faces are rectangles.



Pyramids have one base. The lateral faces are triangles.



Other three-dimensional figures include the *cone*, the *cylinder*, and the *sphere*.

EXERCISES

Which three-dimensional figures have the following properties?

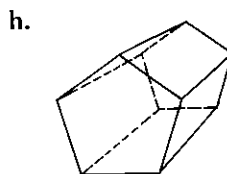
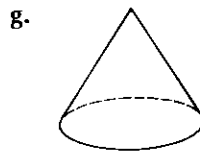
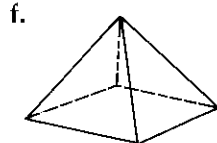
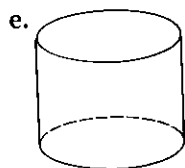
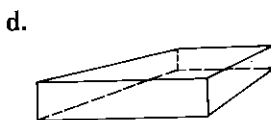
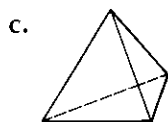
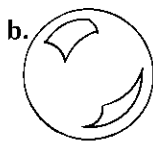
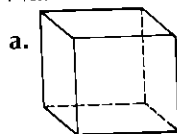
1. a square base and four triangular lateral faces
2. two triangular bases and three rectangular lateral faces
3. four triangular faces
4. six congruent square faces
5. two parallel congruent circular bases

Classify the three-dimensional figure as a solid, shell, or skeleton.

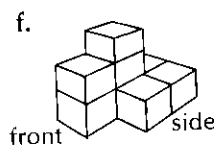
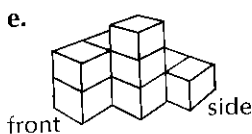
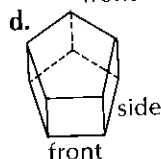
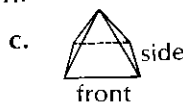
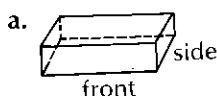
- | | | |
|----------------|-----------------|----------------------|
| 6. baseball | 7. globe | 8. coin |
| 9. house frame | 10. soccer ball | 11. clothesline tree |

PRACTICE

1. Name each three-dimensional figure.



2. Sketch a front view, a side view, and a top view of each.



Faces, Vertices, and Edges

There is an interesting relationship connecting the number of faces, edges, and vertices of a polyhedron. Leonard Euler (1707–1783) was the first to publish the relationship and it was named **Euler's Formula** after him.

Find this relationship by copying and completing the table.
Test the relationship with other polyhedrons.



Leonard Euler

Polyhedrons	Faces (F)	Vertices (V)	Edges (E)	$F + V - E$
triangular prism				
triangular pyramid				
rectangular prism				
rectangular pyramid				
pentagonal prism				
pentagonal pyramid				
hexagonal prism				
hexagonal pyramid				

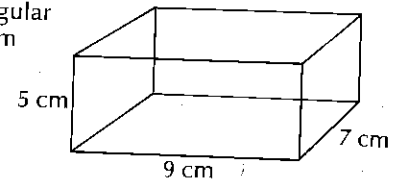
Surface Area of Prisms and Pyramids

In order to know how much paint to buy to paint a garage, you must find the **surface area** of the garage.

The surface area of a prism or pyramid is determined by finding the total area of its faces.

A **rectangular prism** has 6 faces:
4 lateral rectangular faces plus 2 rectangular bases.

Rectangular Prism



$$\begin{aligned} 2(9 \times 5) &= 90 \text{ (cm}^2\text{)} \\ 2(7 \times 5) &= 70 \text{ (cm}^2\text{)} \end{aligned}$$

$$2(7 \times 9) = + 126 \text{ (cm}^2\text{)}$$

$$286 \text{ (cm}^2\text{)}$$

Lateral area:

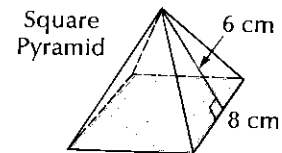
Front and back lateral faces are 9 cm by 5 cm.
Left and right lateral faces are 7 cm by 5 cm.

Area of bases:

Top and bottom faces are 7 cm by 9 cm.

$$\text{Lateral area} + \text{Area of bases} = \text{Surface area}$$

A **square pyramid** has 5 faces:
4 congruent lateral triangular faces plus one base.



$$4\left(\frac{1}{2} \times 8 \times 6\right) = 96 \text{ (cm}^2\text{)}$$

$$8 \times 8 = + 64 \text{ (cm}^2\text{)}$$

$$160 \text{ (cm}^2\text{)}$$

Lateral area:

The four lateral faces each have an 8 cm base and a 6 cm height.

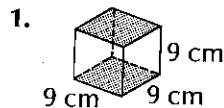
Area of base:

Base is 8 cm by 8 cm

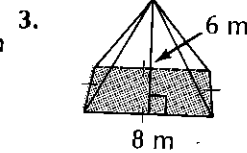
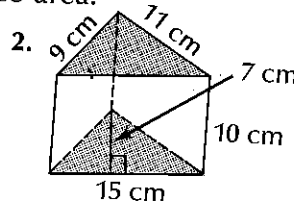
$$\text{Lateral area} + \text{Area of base} = \text{Surface area}$$

EXERCISES

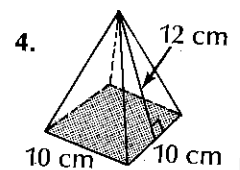
Find the total surface area.



Lateral area = ■
Area of bases = ■
Surface area = ■

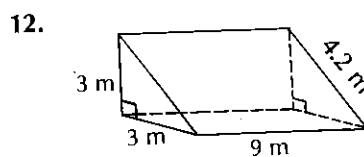
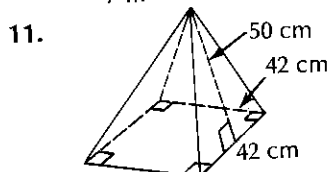
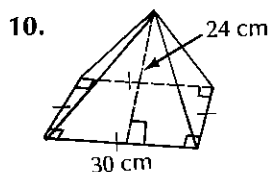
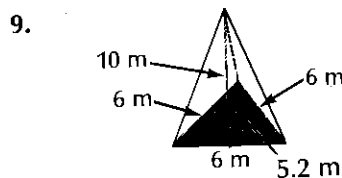
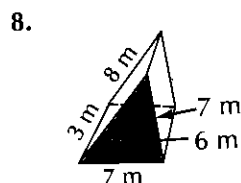
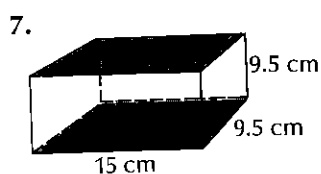
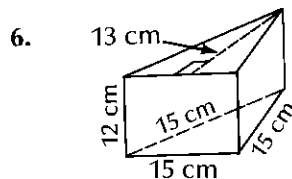
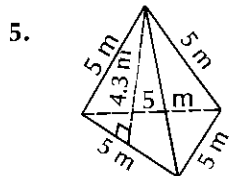
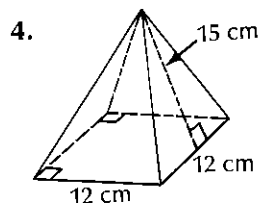
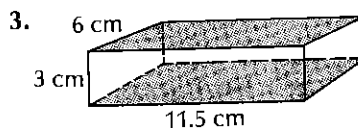
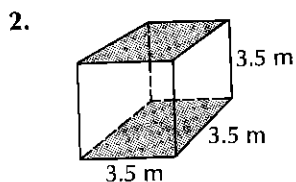
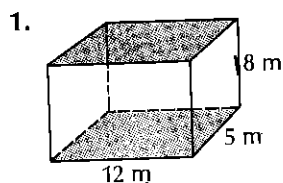


Lateral area = ■
Area of base = ■
Surface area = ■



PRACTICE

Find the total surface area.



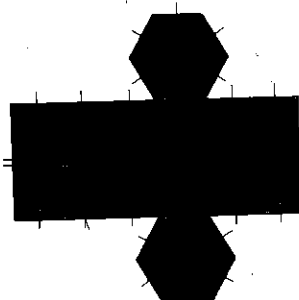
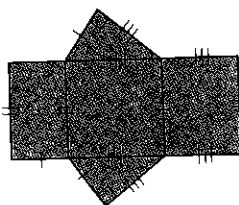
13. Suppose 1 L of paint covers 9 m^2 . If the height of each room below is 3 m, calculate the total surface area to be painted and the amount of paint needed.

- 9 m by 4 m (walls only)
- 6 m by 6 m (walls and ceiling)
- 10 m by 8 m (walls, floor, and ceiling)

Prism Patterns

Use the prism patterns shown to make your own prisms.

- What kind of prisms would the patterns make?
- Does each prism satisfy Euler's theorem?

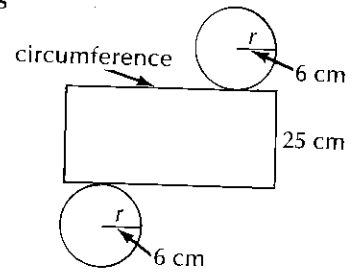


Surface Area of Cylinders and Cones

The total surface area of a cylinder is the sum of the area of its two bases and its lateral area.



The lateral surface of a cylinder is curved. It is like the label on a juice can. If this curved surface is flattened, it forms a rectangle.

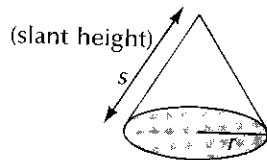


$$\begin{aligned} \text{Lateral area: length (circumference)} \times \text{width} &= (\pi d)25 \approx (3.14 \times 12)25 \approx 942 \text{ (cm}^2\text{)} \\ \text{Area of two bases: } 2 \times \text{area of base} &= 2(\pi r^2) \approx 2(3.14 \times 6^2) \approx + 226.08 \text{ (cm}^2\text{)} \end{aligned}$$

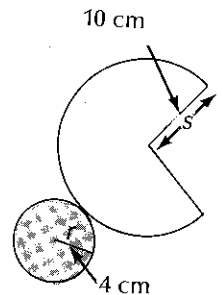
$$\text{Lateral area} + \text{Area of two bases} = \text{Surface area}$$

$$1168.08 \text{ (cm}^2\text{)}$$

The total surface area of a cone is the sum of the area of the base and its lateral area.



The lateral surface of a cone is curved. If this curved surface is flattened, it forms a sector of a circle.



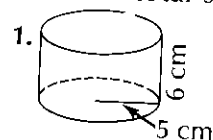
$$\begin{aligned} \text{Lateral area: } \pi rs &\approx 3.14 \times 4 \times 10 \approx 125.6 \text{ (cm}^2\text{)} \\ \text{Area of base: } \pi r^2 &\approx 3.14 \times 4^2 \approx + 50.24 \text{ (cm}^2\text{)} \end{aligned}$$

$$\text{Lateral area} + \text{Area of base} = \text{Surface area}$$

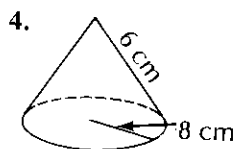
$$175.84 \text{ (cm}^2\text{)}$$

EXERCISES

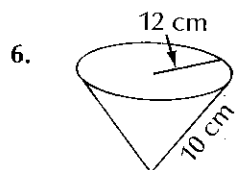
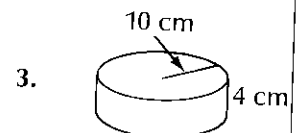
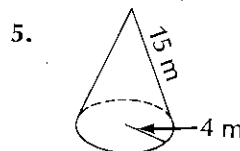
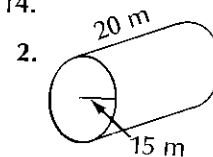
Find the total surface area. Use $\pi \approx 3.14$.



$$\begin{aligned} \text{Lateral Area} &= (\pi d)h \\ \text{Area of Bases} &= 2(\pi r^2) \\ \text{Surface Area} &\approx \blacksquare \end{aligned}$$

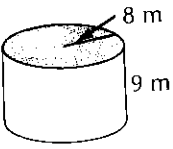
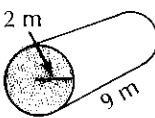
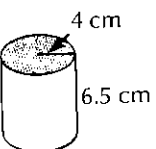
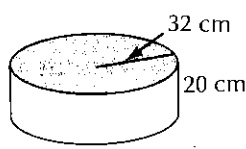
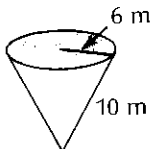
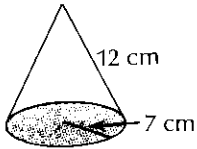
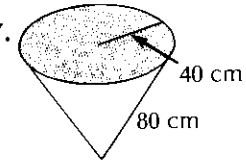
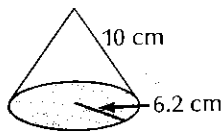


$$\begin{aligned} \text{Lateral Area} &= \pi rs \\ \text{Area of Base} &= \pi r^2 \\ \text{Surface Area} &\approx \blacksquare \end{aligned}$$



PRACTICE

Find the total surface area. Use $\pi \approx 3.14$.

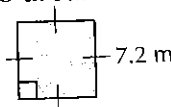
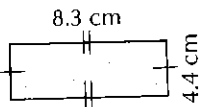
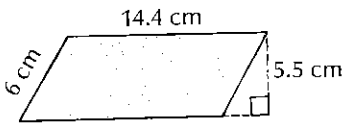
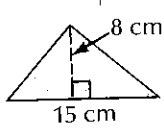
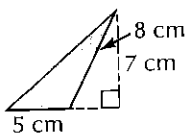
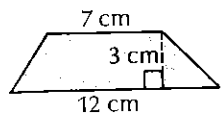
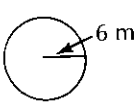
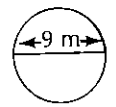
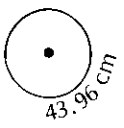
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REVIEW

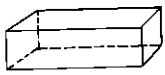
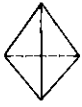
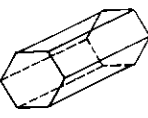

Copy and complete.

1. $6850 \text{ mm} = \blacksquare \text{ m}$
2. $4.5 \text{ kL} = \blacksquare \text{ L}$
3. $54 \text{ t} = \blacksquare \text{ kg}$

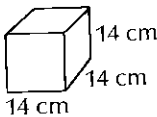
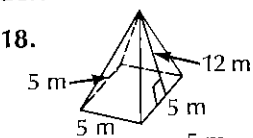
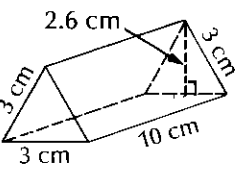
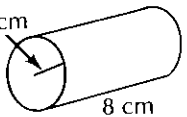
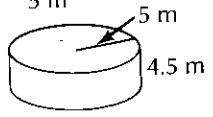
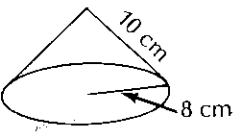
Find the area.

4. 
5. 
6. 
7. 
8. 
9. 
10. 
11. 
12. 

Name each figure and draw the top view.

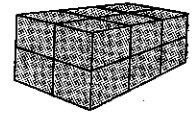
13. 
14. 
15. 
16. 

Find the total surface area of each solid.

17. 
18. 
19. 
20. 
21. 
22. 

Volume of Prisms and Pyramids

The **volume** of a solid is a measure of the space it occupies. Volume is measured in **cubic units**.



12 cubic units

Commonly used cubic units are the **cubic centimetre** (cm^3) and **cubic metre** (m^3).

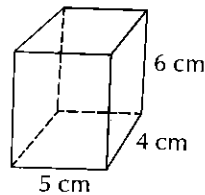
The volume (V) of a *prism* is the product of the area of the base and the height.

$$\text{Volume} = \text{area of base} \times \text{height}$$

$$V = Bh = (lw)h$$

$$V = 20 \times 6$$

$$V = 120 \text{ (cm}^3\text{)}$$



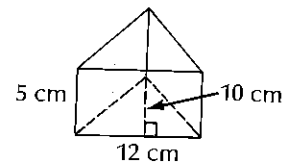
$$\text{Volume} = \text{area of base} \times \text{height}$$

$$V = Bh$$

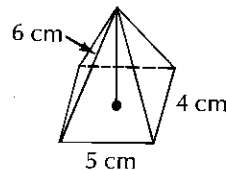
$$V = \left(\frac{1}{2} \times 12 \times 10\right) \times 5$$

$$V = \frac{600}{2}$$

$$V = 300 \text{ (cm}^3\text{)}$$



The volume of a *pyramid* is one third that of a prism with the same base and height.



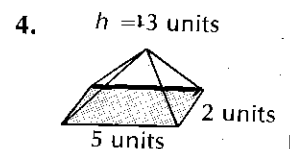
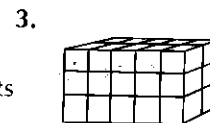
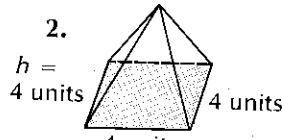
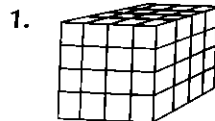
$$\text{Volume} = \frac{1}{3} \times \text{area of base} \times \text{height}$$

$$V = \frac{1}{3}Bh = \frac{1}{3}(lw)h$$

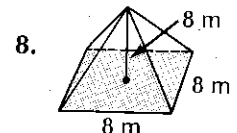
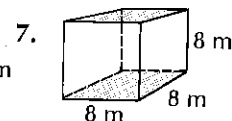
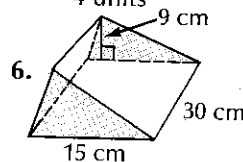
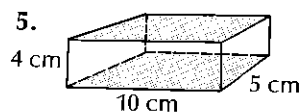
$$V = \frac{1}{3} \times (4 \times 5) \times 6 = \frac{120}{3} = 40 \text{ (cm}^3\text{)}$$

EXERCISES

What is the volume in cubic units?

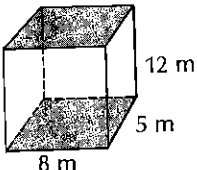
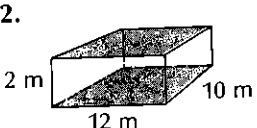
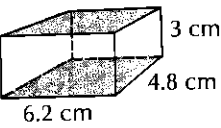
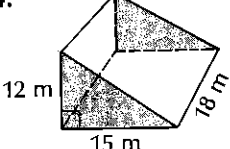
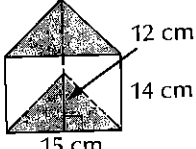
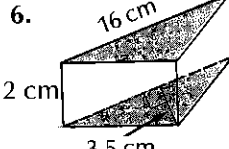
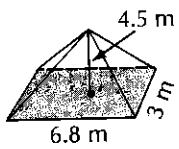
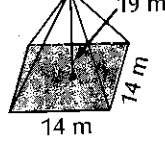
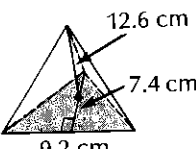
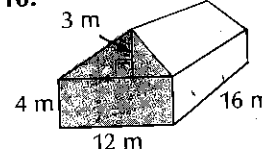
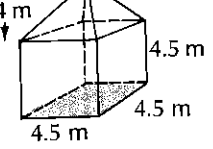
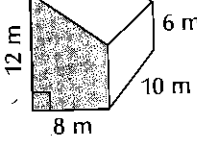


Find the volume.



PRACTICE

Find the volume.

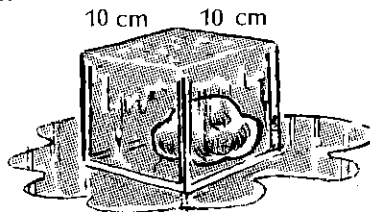
1. 
2. 
3. 
4. 
5. 
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10. 
11. 
12. 

Copy and complete.

13. 1 cm^3 is $\blacksquare \text{ mm}$ by $\blacksquare \text{ mm}$ by $\blacksquare \text{ mm}$
 $\therefore 1 \text{ cm}^3$ is $\blacksquare \text{ mm}^3$
14. 1 m^3 is $\blacksquare \text{ cm}$ by $\blacksquare \text{ cm}$ by $\blacksquare \text{ cm}$
 $\therefore 1 \text{ m}^3$ is $\blacksquare \text{ cm}^3$
15. 1 m^3 is $\blacksquare \text{ mm}$ by $\blacksquare \text{ mm}$ by $\blacksquare \text{ mm}$
 $\therefore 1 \text{ m}^3$ is $\blacksquare \text{ mm}^3$
16. 1 km^3 is $\blacksquare \text{ m}$ by $\blacksquare \text{ m}$ by $\blacksquare \text{ m}$
 $\therefore 1 \text{ km}^3$ is $\blacksquare \text{ m}^3$

Solve.

17. How many cubic metres of concrete are needed to make a sidewalk 45 m long, 1.5 m wide, and 18 cm thick?
18. When an object is submerged in water, it displaces a volume of water equal to its own volume.
 What is the volume of the rock if the water level in the tank rose 1.8 cm when the rock was submerged?



Sugar Cubes

A sugar cube measures about 1 cm on all sides. Its volume is about 1 cm^3 .

For each volume at the right, find an equivalent volume in sugar cube units.

2-story, 4 bedroom house: 1000 m^3
 Pacific Ocean: $700\,000\,000 \text{ km}^3$
 Moon: $21\,900\,000\,000 \text{ km}^3$
 Earth: $1\,083\,207\,300\,000 \text{ km}^3$

Volume of Cylinders and Cones

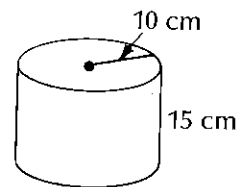
The volume (V) of a cylinder is the product of the area of the circular base and the height.

$$\text{Volume} = \text{area of base} \times \text{height}$$

$$V = Bh = (\pi r^2)h$$

$$V \approx (3.14 \times 10^2) \times 15$$

$$V \approx 4710 \text{ (cm}^3\text{)}$$



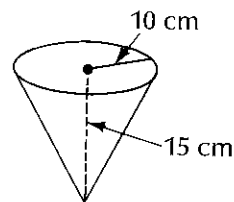
The volume of a cone is one third that of a cylinder with the same dimensions.

$$\text{Volume} = \frac{1}{3} \times \text{area of base} \times \text{height}$$

$$V = \frac{1}{3}Bh = \frac{1}{3}(\pi r^2)h$$

$$V \approx \frac{1}{3} \times (3.14 \times 10^2) \times 15$$

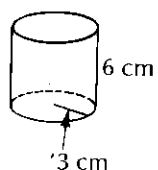
$$V \approx 1570 \text{ (cm}^3\text{)}$$



EXERCISES

Find the volume to the nearest hundredth. Use $\pi \approx 3.14$.

1.

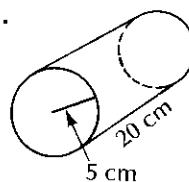


$$r = 3$$

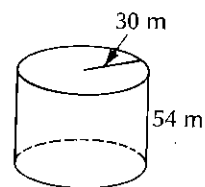
$$h = 6$$

$$V = (\pi r^2)h$$

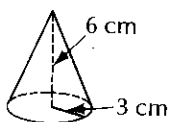
2.



3.



4.

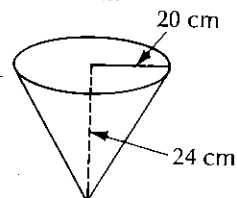


$$r = 3$$

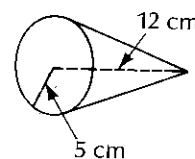
$$h = 6$$

$$V = \frac{1}{3}(\pi r^2)h$$

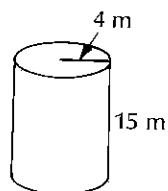
5.



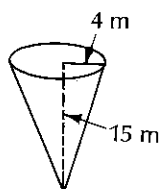
6.



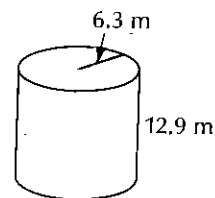
7.



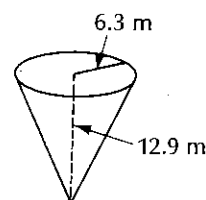
8.



9.

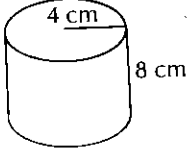
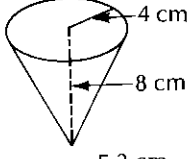
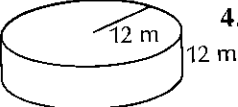
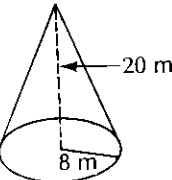
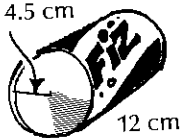

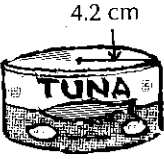
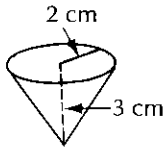
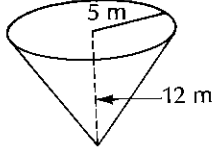

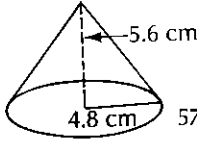
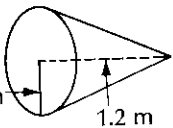


10.



PRACTICE

Find the volume to the nearest hundredth. Use $\pi \approx 3.14$.

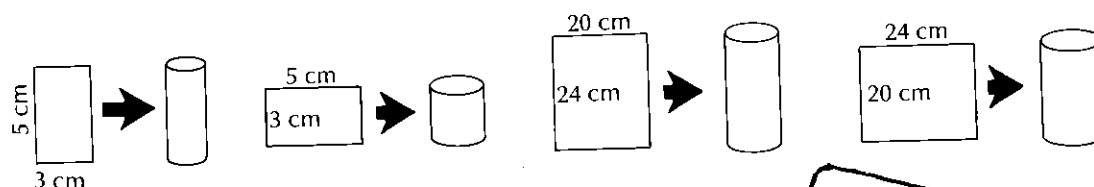
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12. 

Solve.

13. About how many cubic metres of salt are in a conical salt pile 15 m in diameter and 12 m high?
14. A cylindrical tuna fish can is to have a volume of at least 1000 cm^3 and a diameter of 10 cm. What is the minimum height of the can in centimetres.

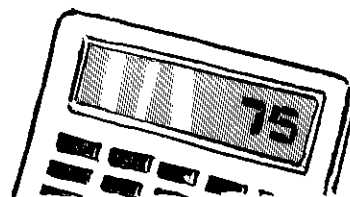
Comparing Volumes

Model pairs of cylinders with paper cut to the dimensions are shown below. Note that each pair of cylinders has the *same lateral area*.



Use a calculator to compute the pairs of volumes for each set of diagrams above. Round to two decimal places.

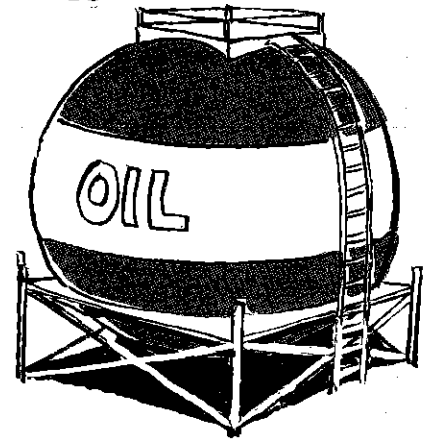
When the calculations are complete, make a conclusion about the volumes of cylinders with the same lateral area but different-sized bases.



Surface Area and Volume of Spheres

The surface of a **sphere** is the set of points that are a fixed distance from a given point, the centre.

For the spherical oil tank, all points are 10 m from its centre. Therefore the *radius* of the tank is 10 m.



Surface Area of a Sphere

$$A = 4\pi r^2$$

$$A \approx 4 \times 3.14 \times 10^2$$

$$A \approx 4 \times 3.14 \times 100$$

$$A \approx 1256 \text{ (m}^2\text{)}$$

Volume of a Sphere

$$V = \frac{4}{3}\pi r^3$$

$$V \approx \frac{4}{3} \times 3.14 \times 10^3$$

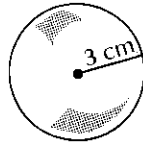
$$V \approx \frac{12\,560}{3}$$

$$V = 4\,186.666... \approx 4187 \text{ (m}^3\text{)}$$

EXERCISES

Find the surface area to the nearest tenth. Use $\pi \approx 3.14$.

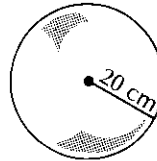
1.



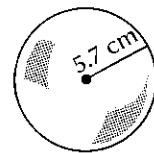
$$r = 3$$

$$A = 4\pi r^2$$

2.

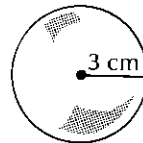


3.



Find the volume to the nearest tenth. Use $\pi \approx 3.14$.

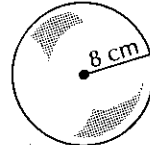
4.



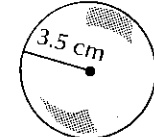
$$r = 3$$

$$V = \frac{4}{3}\pi r^3$$

5.

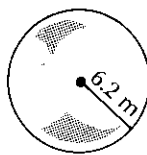


6.

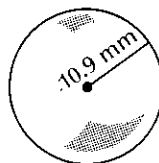


Find the volume and surface area to the nearest hundredth. Use $\pi \approx 3.14$.

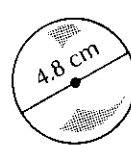
7.



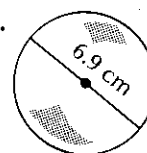
8.



9.



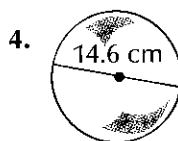
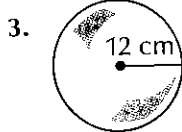
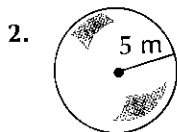
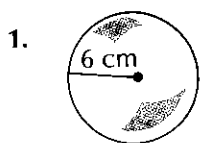
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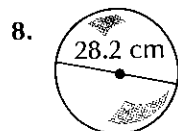
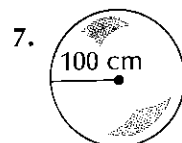
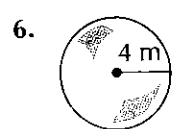
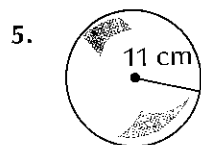
11. In a sphere of radius r , what is the ratio of the surface area to the volume?

PRACTICE

Find the surface area to the nearest tenth. Use $\pi \approx 3.14$.



Find the volume to the nearest tenth. Use $\pi \approx 3.14$.



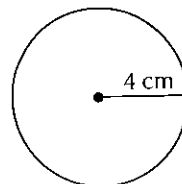
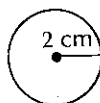
Copy and complete the table to compare the sizes of various balls used in sports.

	Sphere	Radius	Surface Area	Volume
9.	baseball	3.7 cm		
10.	golfball	2 cm		
11.	tennis ball	3.5 cm		
12.	volleyball	21 cm		
13.	basketball	24 cm		

Sphere Patterns

When the radius of a sphere is doubled, its surface area and volume increase. Copy and complete the table below to find the increase patterns for the surface area and volume. Write the surface area and volume in terms of pi.

Sphere	Radius	Surface Area	Volume
A	2 cm	16π	$\frac{32}{3}\pi$
B	4 cm	?	?
C	3 cm	?	?
D	6 cm	?	?
E	5 cm	?	?
F	10 cm	?	?



- What is the pattern for surface area?
- What is the pattern for volume?
- How would the surface area and volume of a sphere change if the radius were tripled?

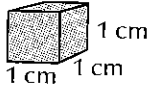
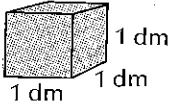
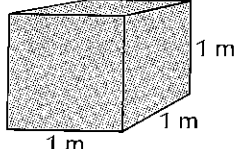
Mass and Capacity

Mass is the measure of the amount of matter in an object.

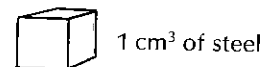
Capacity is the measure of the amount a container can hold.

Capacity may be considered the internal volume or the amount of space enclosed.

For water at 4°C:

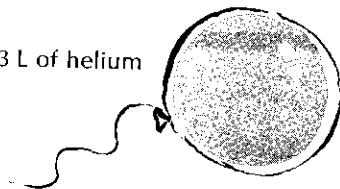
Volume:	Mass	Capacity
 1 cm ³	1 g	1 mL (millilitre)
 1 dm ³	1 kg	1 L (litre)
 1 m ³	1 t	1 kL (kilolitre)

The volume of *solid matter* is usually measured in cubic units.



The volumes of *liquids, gases, and solids* that pour are often measured in units of capacity.

3 L of helium



2 L of milk



3 L of detergent



EXERCISES

Copy and complete for water at 4°C.

- 1 cm³ → ■ g
- 1 dm³ = ■ L
- 1 m³ → ■ t
- 1 dm³ = ■ cm × ■ cm × ■ cm
∴ 1 dm³ = ■ cm³
- 1 dm³ → ■ g
2 dm³ → ■ g
- 1 dm³ = ■ mL
2 dm³ = ■ mL
- 1 m³ = ■ dm × ■ dm × ■ dm
∴ 1 m³ = ■ dm³
- 1 m³ → ■ kg
5 m³ → ■ kg
- 1 m³ = ■ L
5 m³ = ■ L
- 1 m³ = ■ cm × ■ cm × ■ cm
∴ 1 m³ = ■ cm³
- 1 m³ → ■ g
3 m³ → ■ g
- 1 m³ = ■ mL
3 m³ = ■ mL
- 500 cm³ = ■ mL
500 cm³ = ■ L
- 750 dm³ → ■ kg
750 dm³ → ■ g
- 4.5 m³ → ■ t
4.5 m³ → ■ kg
4.5 m³ → ■ g
- Why do we specify 4°C in the above questions?

PRACTICE

What unit of capacity is equivalent to each?

1. 1 cm^3 2. 870 cm^3 3. 40 dm^3 4. 2 m^3 5. 2500 m^3

What unit of volume is equivalent to each?

6. 550 mL 7. 7.8 L 8. 450 kL 9. 1.5 L 10. 10.4 kL

What is the mass of each amount of water at 4°C .

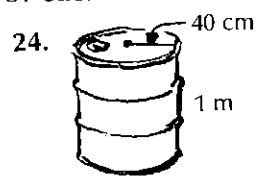
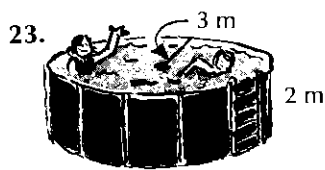
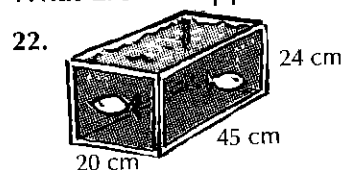
11. 5 dm^3 12. 4.2 m^3 13. 380 cm^3 14. 12.8 dm^3 15. 65 m^3

Copy and complete the table for water at 4°C .

	Volume			Capacity			Mass		
	cm^3	dm^3	m^3	mL	L	kL	g	kg	t
16.		3000	3						
17.						0.5			
18.									6
19.			10						
20.									0.1
21.						0.8			

Each container is completely filled with water at 4°C .

What are the approximate volume, capacity, and mass of each?



25. What would be the volume, capacity, and mass of each of the above containers if they were only three-fourths filled?

Density

Density is the mass of a substance per unit of volume. Density varies with temperature and atmospheric pressure. Density is generally written in terms of kilograms per cubic metre (kg/m^3).

The table shows the mass of 1 cm^3 of various substances.

Write the *density* of each substance by writing an equivalent mass in kilograms for 1 m^3 .

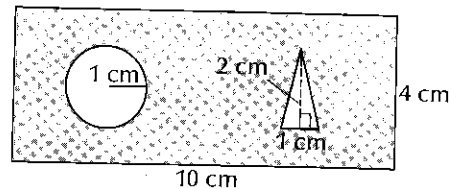
Substance	Volume	Mass	Density (kg/m^3)
Platinum	1 cm^3	21.45 g	
Gold	1 cm^3	19.29 g	
Silver	1 cm^3	10.44 g	
Water	1 cm^3	1 g	1000
Rubber	1 cm^3	0.93 g	

Area and Volume Problems

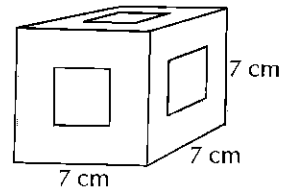
1. Sketch three rectangles with different areas having a perimeter of 24. What are the dimensions of the rectangle with the largest area?
2. Sketch four rectangles with different perimeters having an area of 64 cm^2 . What are the dimensions of the rectangle with the longest perimeter? shortest perimeter? (Use whole numbers.)

Solve to two decimal places. Use $\pi \approx 3.14$.

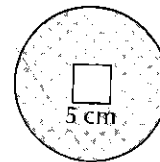
3. Find the area of the shaded part of the figure at the right.



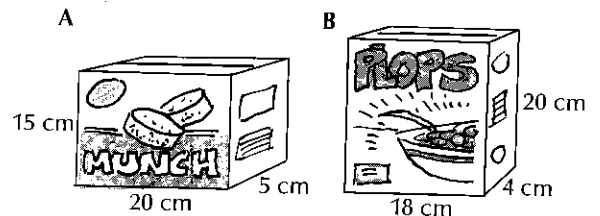
4. A kitchen floor measures 4.2 m by 3.8 m. If carpet costs $\$22/\text{m}^2$, how much would it cost to cover the kitchen floor?
5. The cube at the right has a 3 cm by 3 cm square opening on each face. What is the total surface area of the box?



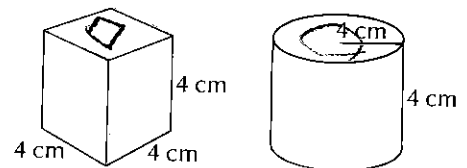
6. The circular figure at the right has a radius of 10 cm and a square hole. What is the area?



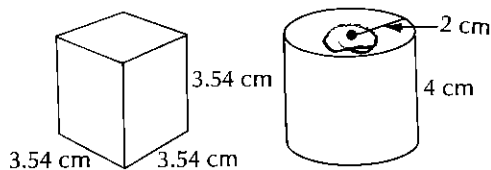
7. a. Which box at the right can hold more cereal?
b. Which box has the larger surface area?
c. Design a box that would be more economical to produce and yet hold the same amount as box A.



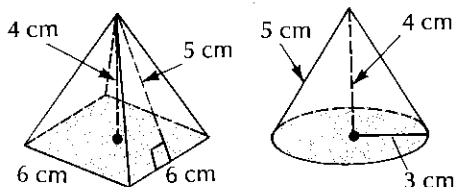
8. The cube and the cylinder at the right have similar dimensions.
 - a. Which has the larger volume?
 - b. Which has the larger surface area?



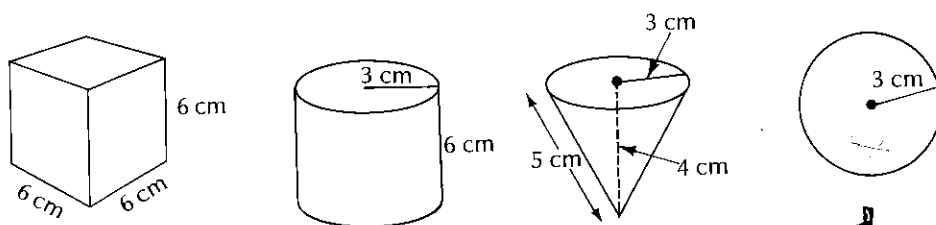
9. The cube and the cylinder at the right have approximately the same surface area. Compare their volumes.



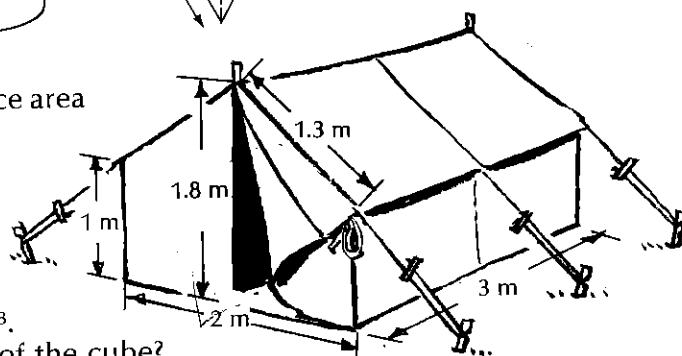
10. The cone and the square pyramid at the right have similar measures.
- Which has the larger volume?
 - Which has the larger surface area?



11. Each figure below is a container for water.
- Rank each container according to its volume.
 - What is the liquid capacity of each container?



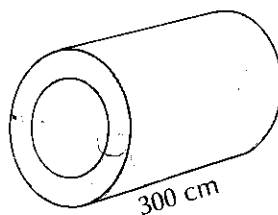
12. What are the volume and surface area of the tent?
(The tent has a canvas floor.)



13. The volume of a cube is 64 cm^3 .
What is the total surface area of the cube?
14. The total surface area of a cube is 600 cm^2 .
What is the volume of the cube?

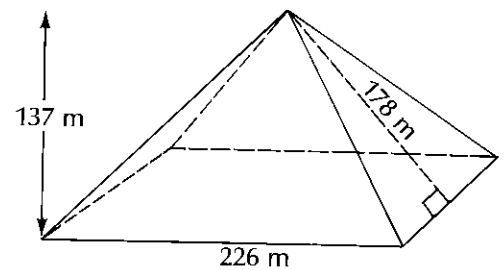
15. The pipe has an inner radius of 80 cm and an outer radius of 85 cm.

- What is the volume of metal in the pipe?
- What is the capacity of the pipe?

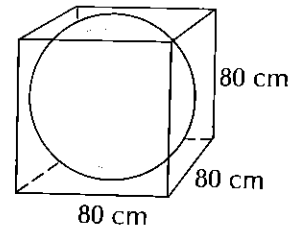


16. The Great Pyramid at Giza is one of the largest structures ever built by man. It is one of the Seven Wonders of the ancient world.

- The base of the Great Pyramid is a square. What is its area?
- What is the area of the base to the nearest hectare?
- What is the area of one lateral face of the Great Pyramid in square metres? hectares?
- Including the base, what is the total surface area of the Great Pyramid in square metres? hectares?
- About 2 300 000 blocks of stone were used to build the Great Pyramid. The mass of a stone is about 2000 kg. What is the total mass of the pyramid? Write the answer in scientific notation.
- What is the total mass of the pyramid in tonnes?
- The king's chamber inside the Great Pyramid measures 10.43 m long, 5.21 m wide, and 5.82 m high. To the nearest tenth of a cubic centimetre, what is the volume of the king's chamber?

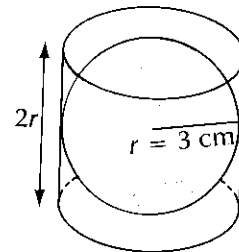


17. A sphere fits snugly inside a cube as shown at the right. What is the volume of space between the cube and the sphere in terms of π ?



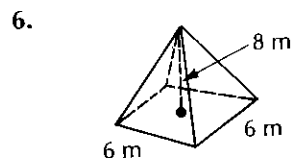
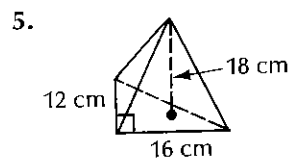
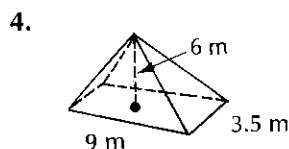
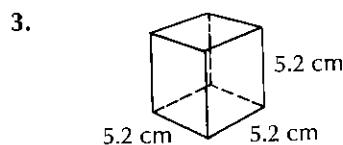
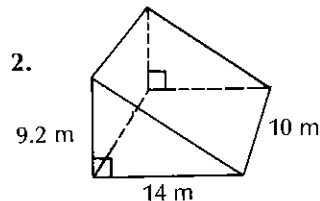
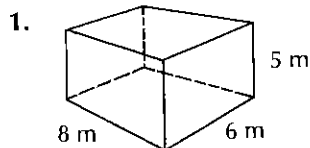
18. A sphere fits snugly inside a cylinder as shown.

- Which is greater, the lateral area of the cylinder or the surface area of the sphere?
- Find the ratio of the volume of the sphere to the volume of the cylinder in simplest terms.

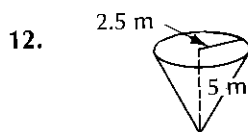
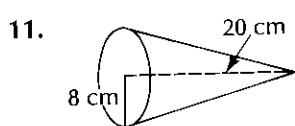
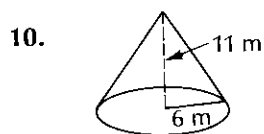
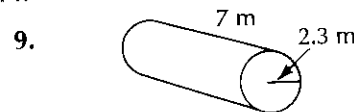
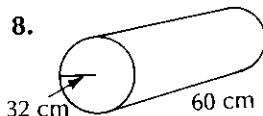


REVIEW

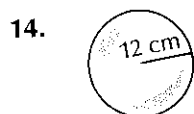
Find the volume of each.



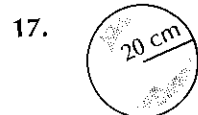
Find the volume to one decimal place. Use $\pi \approx 3.14$.



Find the surface area to one decimal place. Use $\pi \approx 3.14$.



Find the volume to one decimal place. Use $\pi \approx 3.14$.



Copy and complete for water at 4°C .

19. $20 \text{ m}^3 = \blacksquare \text{ cm}^3$

20. $700 \text{ dm}^3 = \blacksquare \text{ m}^3$

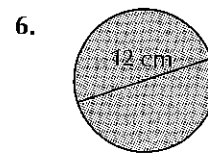
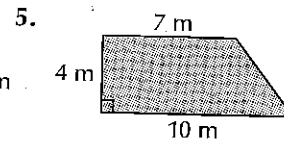
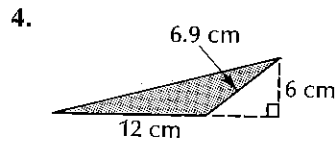
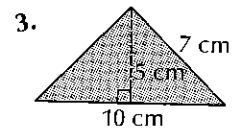
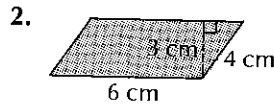
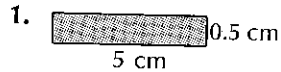
21. $9 \text{ m}^3 = \blacksquare \text{ L}$

Solve.

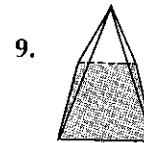
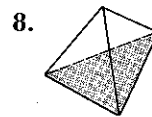
22. A sphere has a surface area of 100π . What is its radius?

23. A scoop of ice cream has the shape of a sphere with a 3 cm radius. The scoop sits on top of an ice cream cone with a 3 cm radius and a 10 cm height. Is the cone big enough to hold all the ice cream if it should melt?

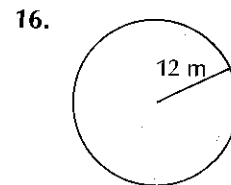
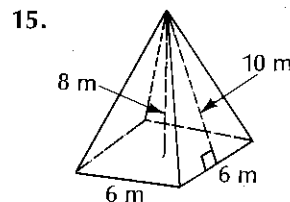
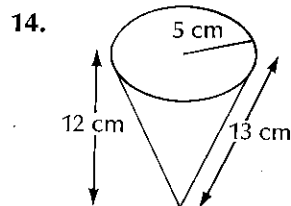
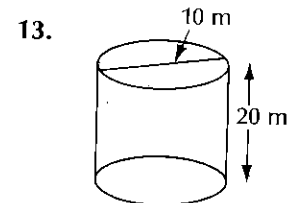
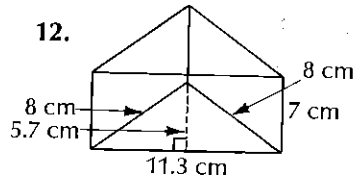
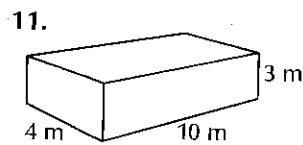
Find the area of each.



Name each three-dimensional figure.



Find the surface area and volume of each.



The container at the right has a volume of 1250 cm^3 .

17. How many litres of water could it hold?

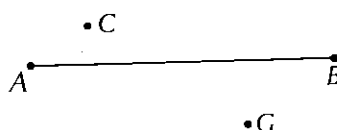
18. What is the mass of the water that the container would hold?

Solve.

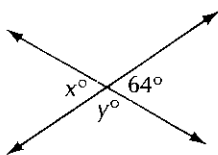
19. If you double the dimensions of a rectangular solid, how much larger will the volume be?



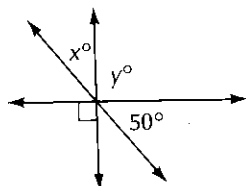
- Draw two adjacent supplementary angles, one of which is 45° .
- Draw two adjacent complementary angles, one of which is 30° .
- Draw a circle with a radius of 4 cm with a compass.
 - Calculate the circle's circumference. Use $\pi \approx 3.14$.
 - Draw chord $AB = 8$ cm.
 - Construct a perpendicular bisector to the diameter AB , dividing the circle into four congruent sectors and arcs.
 - Draw chords joining the endpoints of the diameters to form a square.
 - Bisect the four right angles around the centre of the circle.
 - Use the angle bisectors to construct an octagon.
- Draw a diagram similar to the given one.
 - Use a compass to construct $\overline{FG} \perp \overline{AB}$.
 - Use a compass to construct $\overline{CD} \parallel \overline{AB}$.
- Find each unknown angle.



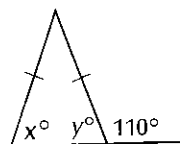
a.



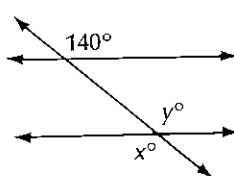
b.



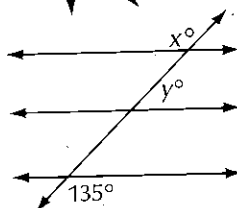
c.



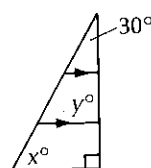
d.



e.



f.



- Make a sketch of each. Indicate congruent angles and sides and lines of symmetry.
 - trapezoid
 - rhombus
 - parallelogram
 - rectangle
 - square
 - kite
- Draw an acute triangle. Then use a compass and straightedge to construct a congruent copy of the triangle.