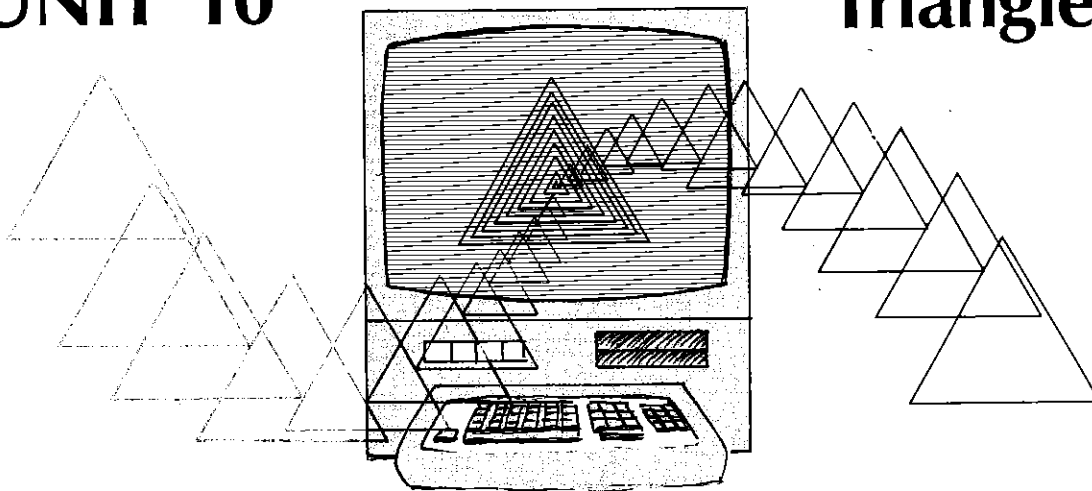


UNIT 10

Triangles



From Points to Polygons

A **point** has an exact position. It is shown as a *dot* or an *intersection* of 2 lines, segments, or curves. A point is named by a capital letter: A or P .

A **line** (straight line) is a set of points extending endlessly in opposite directions. A line is named by any two points on it: line MN or \overleftrightarrow{MN} .

A **segment** (line segment) is a part of a line bounded by two **endpoints**. A segment is named by its endpoints: segment AB or \overline{AB} .

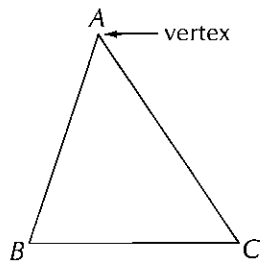
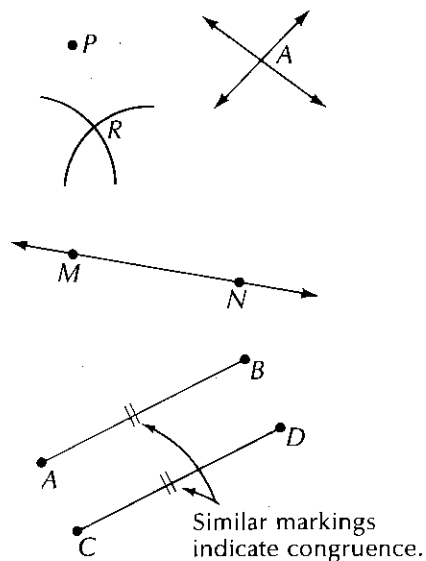
The *length* of \overline{AB} is denoted AB .

$AB = CD$ means the segments have *equal length*.

$\overline{AB} \cong \overline{CD}$ means the segments are **congruent**.

A **polygon** is a closed figure whose sides are line segments. A triangle is a polygon with 3 sides. A quadrilateral is a polygon with 4 sides.

Point of intersection of two sides is called a **vertex**. A polygon is named by its vertices; for example, triangle ABC or $\triangle ABC$.



- Audrey read some books one summer. Let r represent the number of books she read. Write an expression for each of the following in terms of r .
 - Sylvia read 4 more books than Audrey.
 - Alice read twice as many books as Audrey.
 - Bob read three less than twice as many books as Audrey.
 - Sid read four more than half as many books as Audrey.
- Evaluate for $n = 3$.
 - $2n^2 + 4n^2$
 - $8n - 5n + 6 - 4$
 - $12(9 - n) + 8$
- Solve the equation and check the solution.
 - $14a = 196$
 - $\frac{a}{-21} = 15$
 - $a - 18 = -11$
 - $3p - 17 = 50$
 - $2(y - 4) - 6 = 14$
 - $3x + 2x - 27 = 23$
- If it costs \$4.25 for the first three minutes of a telephone call, and if each additional minute costs \$1.25, which of the following represents the cost for m minutes if $m \geq 3$?
 - $4.25m + 1.25m$
 - $4.25m + 1.25(m - 3)$
 - $4.25 + 1.25(m - 3)$
- When a load is hung on a spring, the spring stretches. The amount it stretches is called the *extension*.

The table shows the extension for various masses on a spring.

Mass (kg)	0	1.2	2.0	3.2	4.8
Extension (cm)	0	0.9	1.5	2.4	3.6

- Write a formula that fits the relationship of extension and mass.
 - Find the mass attached to the spring if the extension is 2.7 cm.
- Write a variable expression for each unknown quantity. Then write an equation to solve the problem.

In a district track meet, Thurston scored 17 more points than Hedley. Dinsmore scored twice as many as Hedley. Altogether, the three teams scores 101 points. Which team won the track meet and by how much?



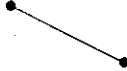
EXERCISES

Identify the type of geometric figure.

1.



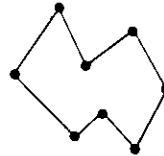
2.



3.

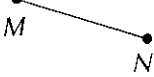


4.



Name the figure.

5.



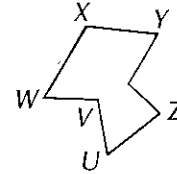
6.



7.



8.

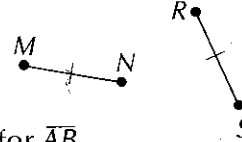


PRACTICE

Draw and label a figure for each.

1. a point H 2. a line KL 3. a segment PQ 4. a polygon MNO

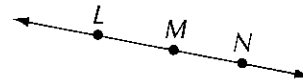
5. What relationship holds for segments MN and RS ?



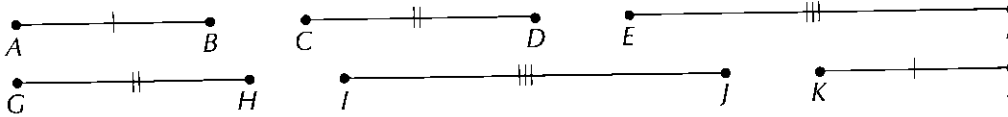
6. How is the length of \overline{AB} denoted?

7. Given $\overline{AB} \cong \overline{CD}$ and $\overline{EF} \cong \overline{CD}$, what relationship holds for \overline{AB} and \overline{EF} ?

8. Points L , M , and N are on the same line. Name the line in three ways.



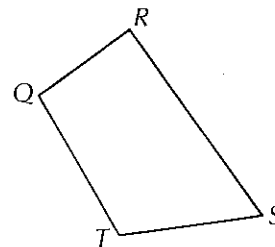
9. Use the symbol \cong to relate the congruent segments.



10. What special name is given to points Q , R , S , and T in the quadrilateral?

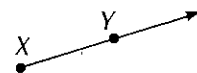
11. How many vertices does polygon $QRST$ have?

12. The quadrilateral $QRST$ may also be named $QTSR$. What six other combinations of Q , R , S , and T can be used to name the polygon?
(Note: The letters in the name must be written in order, clockwise or counter-clockwise, around the polygon.)

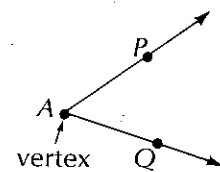


Angles

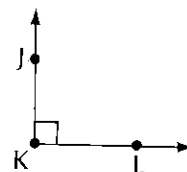
A **ray** is part of a line with one endpoint. A ray is named by its endpoint and another point: ray XY or \overrightarrow{XY} .



An **angle** is formed by 2 rays with a common endpoint. The angle formed by ray AP and ray AQ is called angle PAQ , $\angle PAQ$, or $\angle A$.



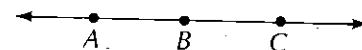
The size of an angle is measured in *degrees*. The size of a **right angle** is 90 degrees or 90° . The notation \perp indicates a right angle.



$$\angle K = 90^\circ$$

The size of a **straight angle** is 180° .

$$\angle ABC = 180^\circ$$

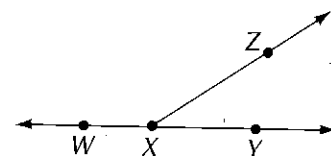


An **acute angle** is greater than 0° and less than 90° .

An **obtuse angle** is greater than 90° and less than 180° .

$\angle ZXY$ is acute.

$\angle WXZ$ is obtuse.



A **protractor** is used to measure the size of an angle.

Examples

Using the outer scale:

$$\angle EFH = 50^\circ$$

$$\angle EFI = 120^\circ$$

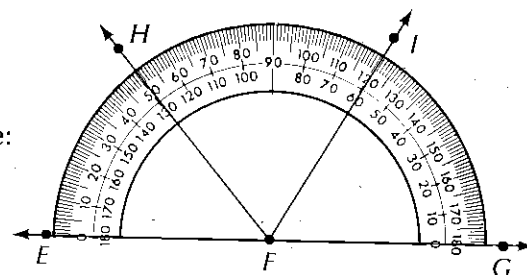
$$\angle EFG = 180^\circ$$

Using the inner scale:

$$\angle GFI = 60^\circ$$

$$\angle GFH = 130^\circ$$

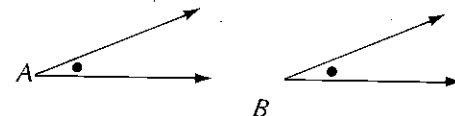
$$\angle GFE = 180^\circ$$



Congruent angles are the same size.

$$\angle A \cong \angle B$$

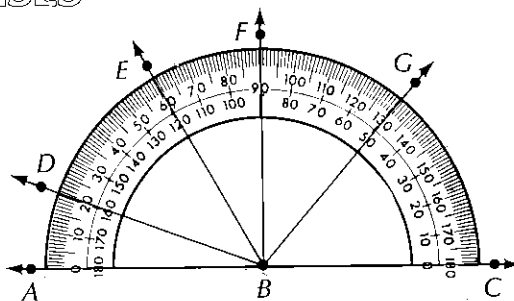
Marking angles in similar ways indicates congruence.



EXERCISES

What is the size of angle?

- | | | |
|------------------|------------------|------------------|
| 1. $\angle CBF$ | 2. $\angle CBG$ | 3. $\angle CBD$ |
| 4. $\angle CBA$ | 5. $\angle ABE$ | 6. $\angle ABG$ |
| 7. $\angle GBE$ | 8. $\angle DBF$ | 9. $\angle EBC$ |
| 10. $\angle FBE$ | 11. $\angle DBG$ | 12. $\angle DBC$ |

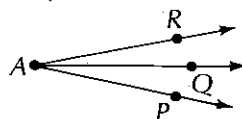


PRACTICE

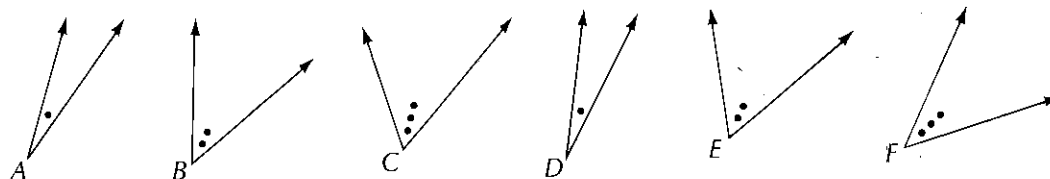
1. Draw and label a figure.

- | | | |
|-----------------------------|----------------------------|------------------------------|
| a. ray MN | b. $\angle ABC$ | c. \overleftrightarrow{MN} |
| d. $\angle F$ | e. a right angle | f. an acute angle |
| g. an obtuse angle | h. a straight angle | i. a 90° angle |
| j. $\angle ABC = 180^\circ$ | k. $\angle DEF = 75^\circ$ | l. $\angle GHI = 10^\circ$ |
| m. $\angle K = 130^\circ$ | n. $\angle L = 45^\circ$ | o. $\angle M = 135^\circ$ |
| p. $\angle G = 5^\circ$ | q. $\angle V = 170^\circ$ | r. $\angle T = 15^\circ$ |

2. For the figure at the right, why would it be incorrect to rename $\angle PAQ$ as $\angle A$.

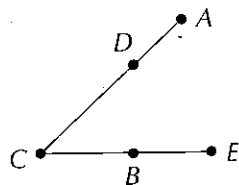


3. What notation is used to indicate a right angle in a diagram?
4. Use the congruence symbol \cong to relate the angles.



5. Suppose $\angle ABC \cong \angle DEF$ and $\angle DEF \cong \angle GHI$. What relationship holds for $\angle ABC$ and $\angle GHI$?

6. The angle at the right may be named $\angle ACE$. List 4 other ways to identify the angle.



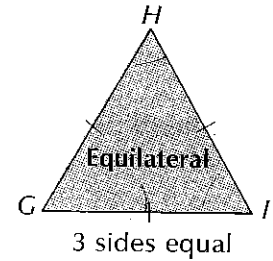
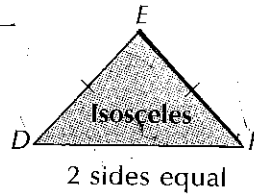
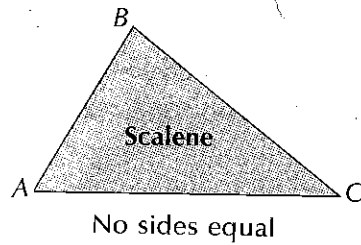
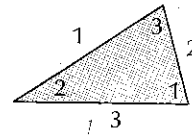
7. Name the vertex of each angle.

- | | | | | | |
|-----------------|-----------------|---------------|---------------|-----------------|---------------|
| a. $\angle RST$ | b. $\angle STU$ | c. $\angle G$ | d. $\angle B$ | e. $\angle XYZ$ | f. $\angle P$ |
|-----------------|-----------------|---------------|---------------|-----------------|---------------|

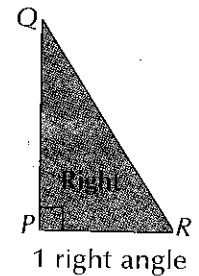
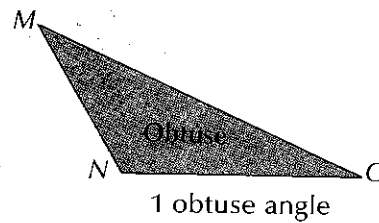
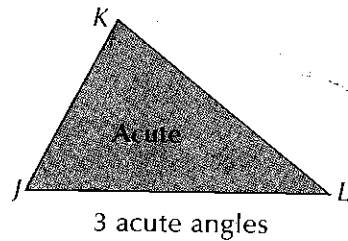
Triangles

A triangle is a polygon consisting of 3 segments and 3 angles.

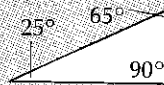
Triangles are classified as *scalene*, *isosceles*, or *equilateral* according to their number of equal sides.



Triangles are classified as *acute*, *right*, or *obtuse* according to the types of their angles.



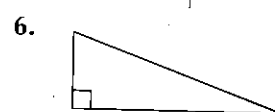
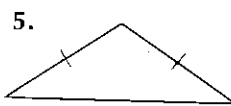
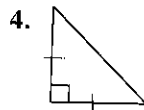
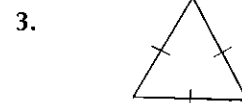
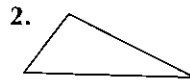
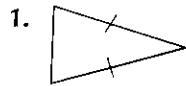
The sum of the angles of any triangle is 180° .



$$25^\circ + 65^\circ + 90^\circ = 180^\circ$$

EXERCISES

Classify each triangle in two ways: by sides and by angles.



Draw each kind of triangle on grid paper.

7. scalene

8. right

9. isosceles

10. scalene right

11. isosceles right

12. acute

13. obtuse

14. obtuse isosceles

15. equilateral

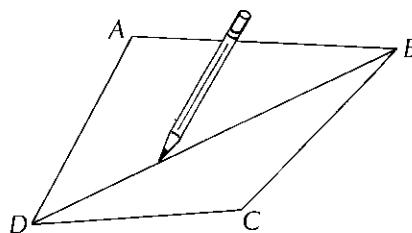
PRACTICE

- Measure the angles of the triangles at the top of the previous page.
 - $\triangle ABC$: $\angle A$, $\angle B$, $\angle C$
 - $\triangle DEF$: $\angle D$, $\angle E$, $\angle F$
 - $\triangle GHI$: $\angle G$, $\angle H$, $\angle I$
 - $\triangle JKL$: $\angle J$, $\angle K$, $\angle L$
 - $\triangle MNO$: $\angle M$, $\angle N$, $\angle O$
 - $\triangle PQR$: $\angle P$, $\angle Q$, $\angle R$
- Show that the sum of the angles for each triangle above is 180° .
- In $\triangle GHI$ above, what relationship holds between $\angle G$, $\angle H$, and $\angle I$?
 - Guess and test*: Is this relationship true for all equilateral triangles?
- In $\triangle DEF$ above, what relationship holds for $\angle D$ and $\angle F$?
 - Guess and test*: Is this relationship true for all isosceles triangles?
- In $\triangle ABC$ above, what relationship holds for $\angle A$, $\angle B$, and $\angle C$?
 - Guess and test*: Is this relationship true for all scalene triangles?
- In $\triangle PQR$ above, what is the sum of angles Q and R ?
 - Guess and test*: Is this sum the same for all right triangles?
- What kind of triangle has these angle sizes?
 - 38° , 104° , 38°
 - 80° , 65° , 35°
 - 25° , 90° , 65°
 - 110° , 35° , \blacksquare
 - 16° , 74° , \blacksquare
 - 60° , 60° , \blacksquare
 - 90° , 20° , \blacksquare
 - 30° , 30° , \blacksquare
- Can a triangle have a straight angle? Explain.
- Make the statement true. Write *All*, *Some*, or *No*.
 - \blacksquare equilateral triangles are isosceles.
 - \blacksquare isosceles triangles are right.
 - \blacksquare equilateral triangles are right.
 - \blacksquare obtuse triangles are right.
 - \blacksquare right triangles are equilateral.
 - \blacksquare right triangles are scalene.

More and More Triangles

Frank drew diagonal DB in the polygon at the right to show that the sum of the angles in a quadrilateral is 360° .

Find a pattern with the table below to help answer the questions.

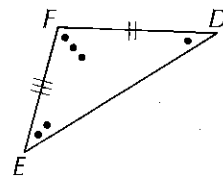
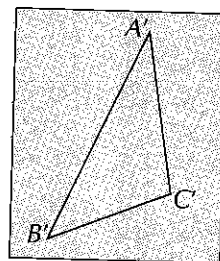
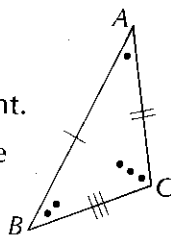


What is the sum of the angles of a nonagon (9-sided polygon)?
 a 12-sided polygon?
 a 100-sided polygon?

Polygon	Number of Sides	Number of Angles	Sum of Angles
Triangle	?	?	?
Quadrilateral	?	?	?
Pentagon	?	?	?

Congruent Triangles

Triangles ABC and DEF are congruent.
This means they have the same shape and the same size.



In other words, a tracing $A'B'C'$ of triangle ABC will fit exactly onto triangle DEF .

For two congruent triangles:

1. corresponding sides are congruent.
2. corresponding angles are congruent.

$$\triangle ABC \cong \triangle DEF$$

$$AB \cong DE$$

$$AC \cong DF$$

$$BC \cong EF$$

$$\angle A \cong \angle D$$

$$\angle B \cong \angle E$$

$$\angle C \cong \angle F$$

Side-Side-Side Rule (SSS):

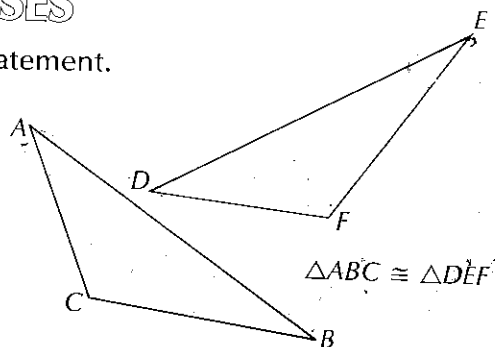
If the three sides of one triangle are congruent to the three sides of another triangle, the triangles are congruent.

EXERCISES

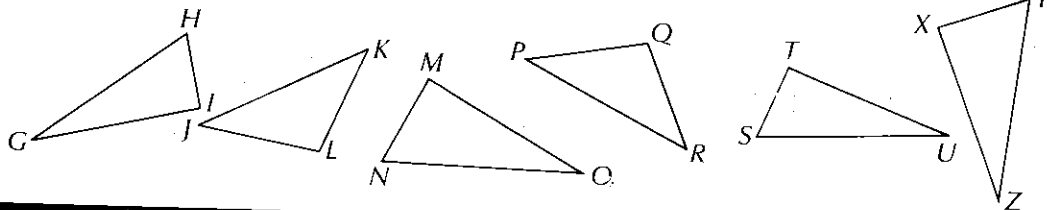
1. Copy and complete the congruence statement.

- | | |
|---------------------------------------|---------------------------------------|
| a. $\angle A \cong \blacksquare$ | b. $\angle C \cong \blacksquare$ |
| c. $\angle B \cong \blacksquare$ | d. $\overline{AC} \cong \blacksquare$ |
| e. $\overline{CB} \cong \blacksquare$ | f. $\overline{AB} \cong \blacksquare$ |

2. Trace triangles ABC and DEF .
Mark the corresponding congruent parts on the two triangles.

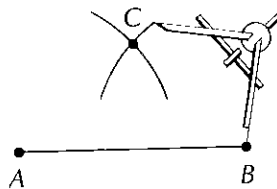


3. Use the congruence symbol \cong to relate the triangles.
Trace each triangle to check for congruency.



PRACTICE

- Given: $\triangle MNO \cong \triangle RST$.
Copy and complete each congruence statement.
 - $\angle N \cong \blacksquare$
 - $\angle O \cong \blacksquare$
 - $\overline{NO} \cong \blacksquare$
 - $\overline{OM} \cong \blacksquare$
- Given: $\triangle FGH$ and $\triangle IJK$ for which $\overline{FG} \cong \overline{IJ}$, $\overline{HG} \cong \overline{KJ}$, and $\overline{HF} \cong \overline{KI}$.
Copy and complete each congruence statement.
 - $\triangle IJK \cong \blacksquare$
 - $\angle J \cong \blacksquare$
 - $\angle F \cong \blacksquare$
 - $\angle I \cong \blacksquare$
- Given: $\triangle ABC$ and $\triangle DEF$ for which $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$.
Draw an example to show that $\triangle ABC$ is not necessarily congruent to $\triangle DEF$.
- Given: $\triangle ABC \cong \triangle DEF$ and $\triangle GHI \cong \triangle DEF$.
How are $\triangle ABC$ and $\triangle GHI$ related?
- Use ruler and compass to complete the construction.
Given: $AB = 3$ cm, $AC = 2$ cm, $BC = 2$ cm.
Construct isosceles triangle ABC following these steps.
 - Draw \overline{AB} 3 cm in length with a ruler.
 - Adjust the compass radius to 2 cm.
Draw two arcs centred at A and B intersecting at C .
 - Draw \overline{AC} and \overline{BC} .
- Use a compass and ruler to construct a triangle for each set of sides. Some constructions are impossible. Explain why.
 - $AB = 10$ cm, $BC = 8$ cm, $AC = 6$ cm
 - $AB = 9$ cm, $BC = 9$ cm, $AC = 9$ cm
 - $AB = 12$ cm, $BC = 4$ cm, $AC = 10$ cm
 - $AB = 8$ cm, $BC = 5$ cm, $AC = 13$ cm
 - $AB = 6$ cm, $BC = 12$ cm, $AC = 4$ cm

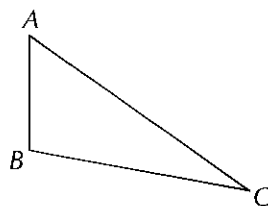


Investigating Triangles

Given: $\triangle ABC$, in which $AB < BC < AC$.

How are $\angle A$, $\angle B$, and $\angle C$ related?

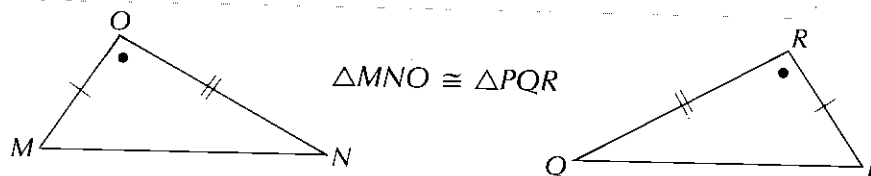
Test your conclusion for several other triangles.



Congruent Triangles

Side-Angle-Side Rule (SAS):

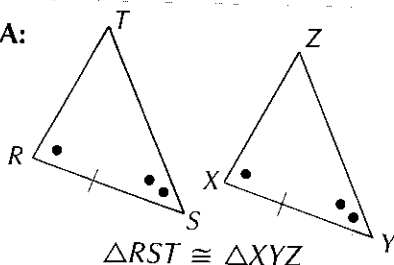
If two sides and the contained angle of one triangle are respectively congruent to two sides and the contained angle of another triangle, the triangles are congruent.



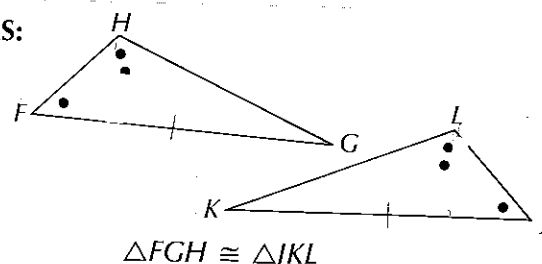
Angle-Side-Angle Rule (ASA) or (AAS):

If two angles and a side of a triangle are respectively congruent to two angles and the corresponding side of another triangle, the triangles are congruent.

ASA:



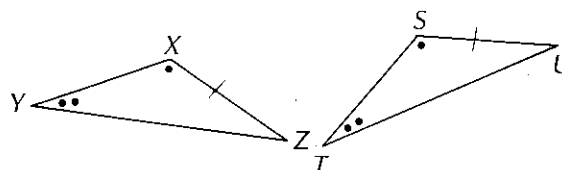
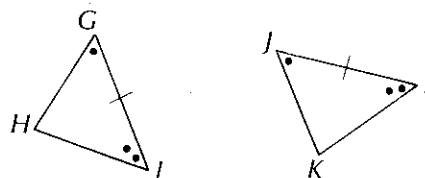
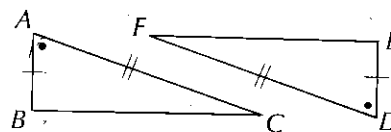
AAS:



EXERCISES

Copy and complete each statement.
Refer to the diagrams at the right.

1. a. $\overline{AB} \cong \blacksquare$ b. $\angle A \cong \blacksquare$
 c. $\overline{AC} \cong \blacksquare$ d. $\triangle ABC \cong \blacksquare$
 e. $\overline{BC} \cong \blacksquare$ f. $\angle C \cong \blacksquare$
2. a. $\angle I \cong \blacksquare$ b. $\angle G \cong \blacksquare$
 c. $\overline{GI} \cong \blacksquare$ d. $\triangle GHI \cong \blacksquare$
 e. $\angle H \cong \blacksquare$ f. $\overline{GH} \cong \blacksquare$
3. a. $\angle X \cong \blacksquare$ b. $\angle Y \cong \blacksquare$
 c. $\overline{XZ} \cong \blacksquare$ d. $\triangle XYZ \cong \blacksquare$
 e. $\overline{YZ} \cong \blacksquare$ f. $\angle Z \cong \blacksquare$



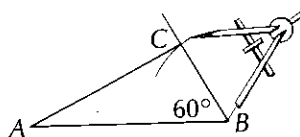
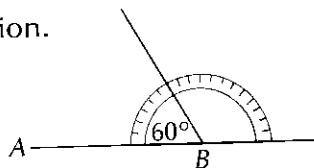
PRACTICE

1. Use ruler and protractor to complete the construction.

Given: $AB = 3$ cm, $BC = 2$ cm, $\angle B = 60^\circ$

Construct $\triangle ABC$ following these steps

- Draw \overline{AB} 3 cm in length with a ruler.
- Place the protractor vertex at B and draw a 60° angle.
- Draw \overline{BC} 2 cm in length with a ruler.
- Draw \overline{AC} .



2. Use a compass, ruler, and protractor to construct each triangle.

- $\triangle ABC$ for which $AB = 4$ cm, $BC = 5$ cm, $\angle B = 35^\circ$.
- $\triangle DEF$ for which $DE = 7$ cm, $\angle D = 45^\circ$, $\angle E = 45^\circ$.
- $\triangle GHI$ for which $GI = 9$ cm, $GH = 2$ cm, $\angle G = 75^\circ$.
- $\triangle JKL$ for which $JK = 4$ cm, $KL = 4$ cm, $\angle K = 50^\circ$.

3. Given: $\triangle MNO$ and $\triangle PQR$ for which $MN = 8$ cm, $NO = 7$ cm, $\angle N = 90^\circ$, and $QR = 7$ cm, $RP = 8$ cm, $\angle R = 90^\circ$.

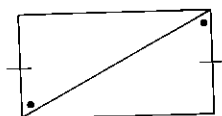
- Construct the two triangles.
- Are the two triangles congruent? Explain.

4. Given: $\triangle STU$ and $\triangle VWX$ for which $ST = 5$ cm, $TU = 9$ cm, $\angle S = 105^\circ$, and $VW = 5$ cm, $WX = 9$ cm, $\angle W = 105^\circ$.

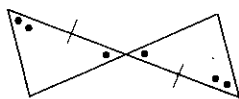
- Construct the two triangles.
- Are the two triangles congruent? Explain.

5. Are the two triangles congruent? If yes, state the rule which establishes the congruence: SSS, SAS, ASA, or AAS.

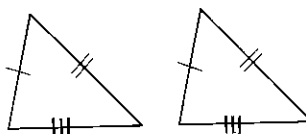
a.



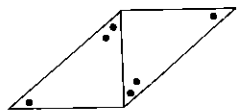
b.



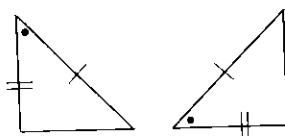
c.



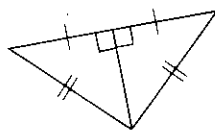
d.



e.



f.



Construction Challenge

Given: Two triangles ABC for which $BC = 5$ cm, $AB = 3$ cm, $\angle C = 35^\circ$.

Construct the two triangles so they are *not* congruent.

Similar Triangles

$\triangle ABC$ and $\triangle DEF$ are called **similar triangles**.
Their corresponding angles are congruent.
Similar triangles have the same shape but may have different sizes.

Properties of Similar Triangles:

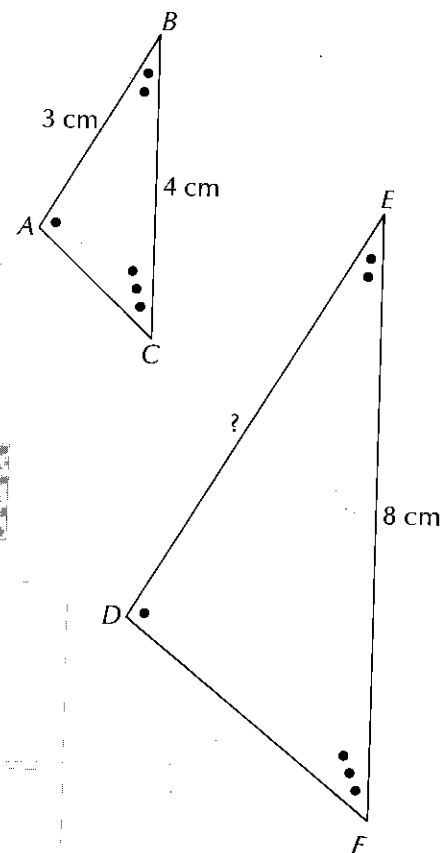
1. Corresponding angles are equal.
2. Corresponding sides are proportional.

The length of \overline{DE} can be found using a proportion.

$$\frac{AB}{DE} = \frac{BC}{EF}$$

$$\frac{3}{DE} = \frac{4}{8}$$

$$\begin{aligned} DE &= 3 \times 2 \\ DE &= 6 \end{aligned}$$



Angle-Angle-Angle Rule (AAA):

If the angles of one triangle are respectively congruent to the angles of another triangle, the triangles are similar.

Proportional Sides Rule

If the lengths of corresponding sides of two triangles are proportional, the triangles are similar.

Given: $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$.

$\therefore \triangle ABC$ and $\triangle DEF$ are similar.

Given: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{1}{2}$

$\therefore \triangle ABC$ and $\triangle DEF$ are similar.

EXERCISES

Copy and complete proportion.

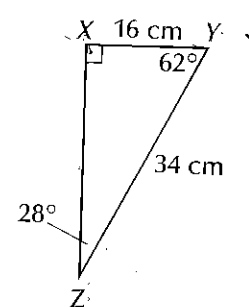
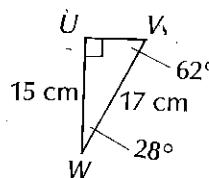
1. $\frac{UV}{XY} = \frac{VW}{\blacksquare}$ 2. $\frac{UV}{XY} = \frac{\blacksquare}{XZ}$

3. $\frac{VW}{YZ} = \frac{UW}{\blacksquare}$ 4. $\frac{XZ}{UW} = \frac{XY}{\blacksquare}$

Substitute numerical values. Then solve.

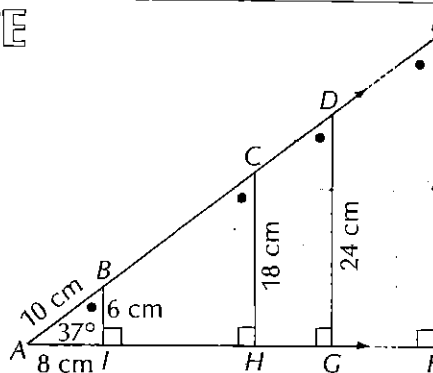
5. Solve for UV . 6. Solve for XZ .

$$\frac{UV}{XY} = \frac{WV}{ZY} \quad \frac{XZ}{UW} = \frac{ZY}{WV}$$



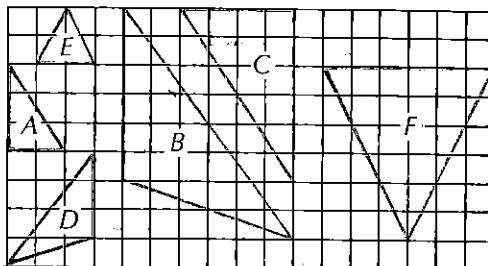
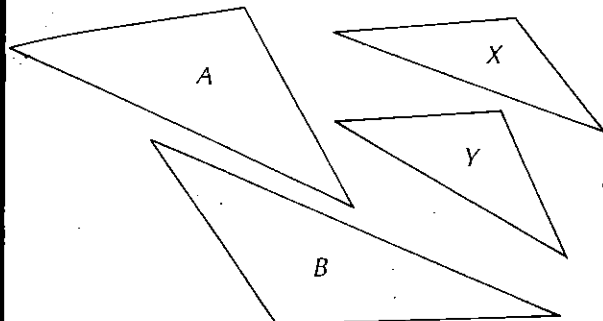
PRACTICE

- Use a proportion to find each length.
 - AH
 - AG
 - AC
 - AD
 - If $EF = 30$ cm, $AE = \blacksquare$ and $AF = \blacksquare$.
 - If $EF = 40$ cm, $AE = \blacksquare$ and $AF = \blacksquare$.
 - If $EF = 80$ cm, $AE = \blacksquare$ and $AF = \blacksquare$.
 - If $EF = 600$ cm, $AE = \blacksquare$ and $AF = \blacksquare$.



- Match the similar triangles by measuring the angles.

- Match the similar triangles.



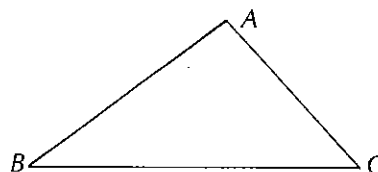
REVIEW

- Draw and label each figure.
 - a ray
 - a segment
 - a polygon
 - a 60° angle
- Complete each statement.
 - $\angle A = \blacksquare^\circ$
 - $\angle B = \blacksquare^\circ$
 - $\angle C = \blacksquare^\circ$
 - $\angle A + \angle B + \angle C = \blacksquare^\circ$
- Classify each triangle.
 -
 -
 -
 -
- Is the triangle congruent to the given triangle? Explain.

Given:

a.

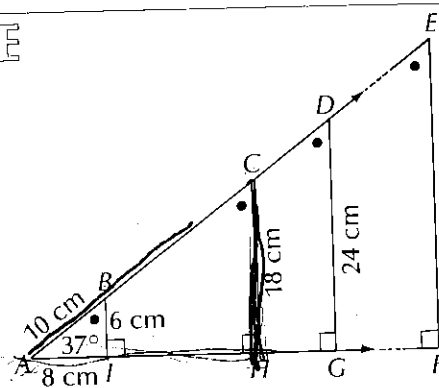
b.



PRACTICE

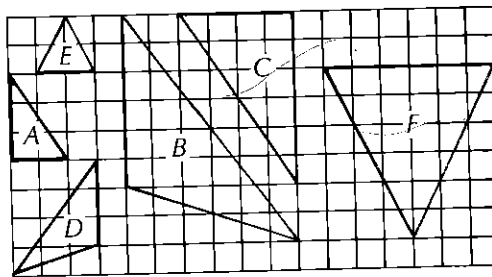
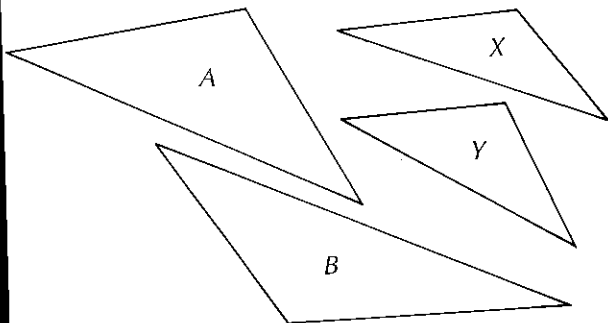
1. Use a proportion to find each length.

- a. AH b. AG c. AC d. AD
 e. If $EF = 30$ cm, $AE = \blacksquare$ and $AF = \blacksquare$.
 f. If $EF = 40$ cm, $AE = \blacksquare$ and $AF = \blacksquare$.
 g. If $EF = 80$ cm, $AE = \blacksquare$ and $AF = \blacksquare$.
 h. If $EF = 600$ cm, $AE = \blacksquare$ and $AF = \blacksquare$.



2. Match the similar triangles by measuring the angles.

3. Match the similar triangles.



REVIEW

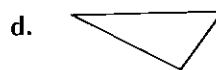
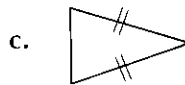
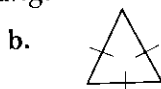
1. Draw and label each figure.

- a. a ray b. a segment c. a polygon d. a 60° angle

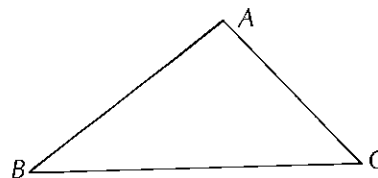
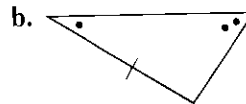
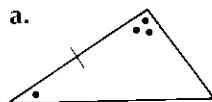
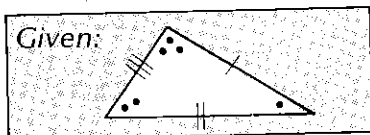
2. Complete each statement.

- a. $\angle A = \blacksquare^\circ$ b. $\angle B = \blacksquare^\circ$
 c. $\angle C = \blacksquare^\circ$ d. $\angle A + \angle B + \angle C = \blacksquare^\circ$

3. Classify each triangle.



4. Is the triangle congruent to the given triangle? Explain.

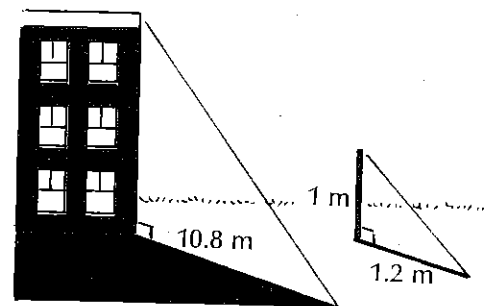


Applications of Similar Triangles

The height of the building can be approximated using similar triangles. The triangles formed by the building and its shadow and the stick and its shadow are similar.

Both the building and the stick are *perpendicular* to the ground.

The lengths of the shadows cast by the stick and the building are *proportional* to their heights.



$$\frac{\text{height of building (metres)} \rightarrow x}{\text{height of stick (metres)} \rightarrow 1} = \frac{10.8 \leftarrow \text{shadow of building (metres)}}{1.2 \leftarrow \text{shadow of stick (metres)}}$$

$$1.2x = 10.8$$

$$x = \frac{10.8}{1.2}$$

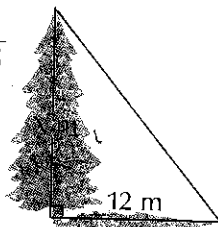
$$x = 9$$

The height of the building is approximately 9 m.

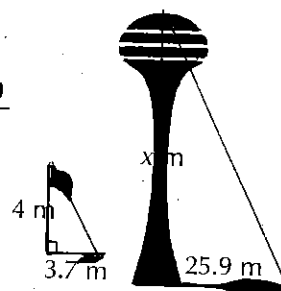
EXERCISES

Solve the proportion to approximate the missing height.

1. $\frac{x}{5.7} = \frac{12}{4.5}$

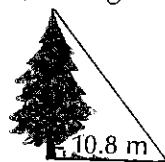
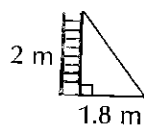


2. $\frac{x}{4} = \frac{25.9}{3.7}$

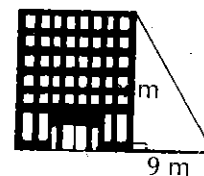
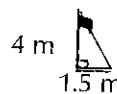


What is the approximate height of the taller object?

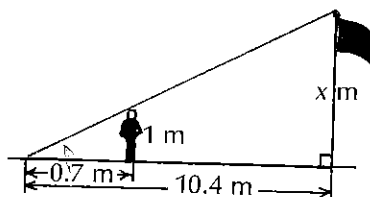
3.



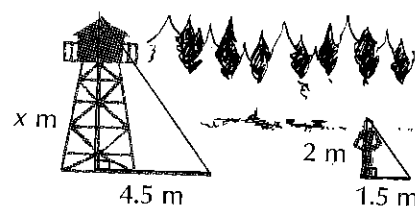
4.



5.



6.

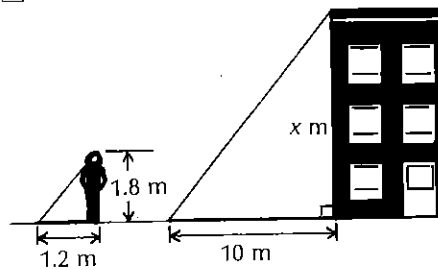


PRACTICE

Solve.

1. A woman 1.8 m tall casts a shadow 1.2 m long at the same time that a building casts a shadow 10 m long.

What is the approximate height of the building?

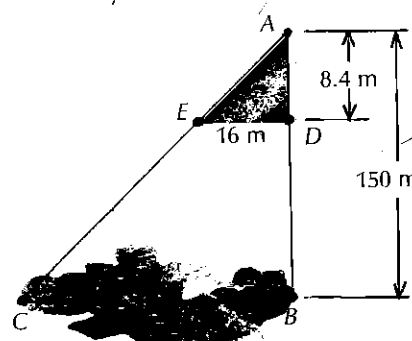


2. A 2 m high wall casts a shadow 2.5 m long. The shadow of a boy standing on the wall is 1.75 m long. Approximately how tall is the boy?
3. A 12 m high flagpole casts a 5 m shadow. At the same time, a teacher walking by casts a 0.8 m shadow. Approximately how tall is the teacher in centimetres?

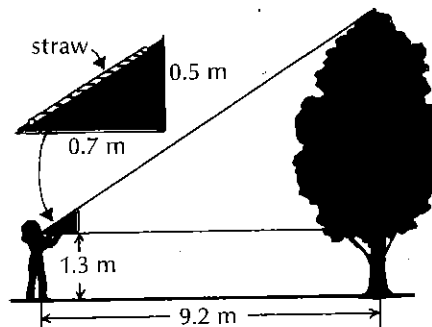
4. To measure the length of Loon Lake from points B to C , Charlie stood at point A , 150 m from point B . He then sighted point C . Charlie next marked off a smaller triangle ADE using his sightings of points C and B from point A . He made AD 8.4 m and DE 16 m.

Assuming $\angle ADE$ and $\angle ABC$ are right angles, explain why $\angle AED$ is congruent to $\angle ACB$?

What is the approximate length of Loon Lake?



5. Judy attaches a straw to the longest side of a cardboard right triangle to measure a tree's height. If she stands 9.2 m away from the tree, she can sight its top through the straw. Judy's eye level is 1.3 m above the ground. What is the approximate height of the tree?

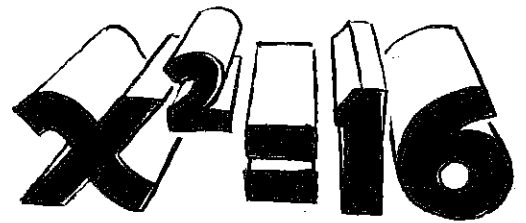


Outdoor Math

Try the methods used above to approximate:

- a. the height of a flagpole.
- b. the height of a tree.
- c. the height of a building.
- d. the height of a telephone pole.
- e. the width of a road.
- f. the length of a building.

Square Roots



Which integers squared equal 16?

$$4^2 = 16 \quad \text{and} \quad (-4)^2 = 16$$

Both 4 and -4 are called **square roots** of 16.

The symbol $\sqrt{\quad}$ is used for the **positive square root** of a number.

positive square root:

$$\sqrt{16} = 4$$

negative square root:

$$-\sqrt{16} = -4$$

The square root of a number can be checked by squaring.

Examples

$$\sqrt{25} = 5 \text{ since } 5^2 = 25$$

$$\sqrt{0.09} = 0.3 \text{ since } (0.3)^2 = 0.09$$

$$-\sqrt{25} = -5 \text{ since } (-5)^2 = 25$$

$$\sqrt{\frac{4}{81}} = \frac{2}{9} \text{ since } \left(\frac{2}{9}\right)^2 = \frac{4}{81}$$

The square root of a number can sometimes be found by factoring.

$$\begin{aligned} \sqrt{100} &= \sqrt{2 \times 2 \times 5 \times 5} \\ &= \sqrt{2 \times 2} \times \sqrt{5 \times 5} \\ &= 2 \times 5 = 10 \end{aligned}$$

The square root of a rational number can often be simplified by finding the square root of the numerator and the square root of the denominator.

Example: $\sqrt{\frac{9}{49}} = \frac{\sqrt{9}}{\sqrt{49}} = \frac{3}{7}$

EXERCISES

Simplify. Check by squaring.

1. $\sqrt{16}$

2. $\sqrt{81}$

3. $\sqrt{144}$

4. $\sqrt{36}$

5. $\sqrt{196}$

6. $\sqrt{400}$

7. $\sqrt{225}$

8. $\sqrt{1}$

9. $\sqrt{0.81}$

10. $\sqrt{0.49}$

11. $\sqrt{0.0001}$

12. $\sqrt{0.0004}$

13. $\sqrt{\frac{9}{16}}$

14. $\sqrt{\frac{4}{81}}$

15. $\sqrt{\frac{1}{144}}$

16. $\sqrt{\frac{25}{36}}$

17. $\sqrt{\frac{49}{121}}$

18. $\sqrt{\frac{64}{256}}$

19. $\sqrt{\frac{100}{144}}$

20. $\sqrt{\frac{1}{196}}$

Write the positive and the negative square roots for each.

21. 16

22. 9

23. 121

24. 81

25. 0.16

26. 0.09

27. 1.21

28. 0.81

PRACTICE

Simplify. Check by squaring.

- | | | | |
|---------------------------|-----------------------------|-----------------------------|----------------------------|
| 1. $\sqrt{4}$ | 2. $\sqrt{49}$ | 3. $\sqrt{169}$ | 4. $\sqrt{64}$ |
| 5. $\sqrt{1.21}$ | 6. $\sqrt{0.04}$ | 7. $\sqrt{0.64}$ | 8. $\sqrt{0.01}$ |
| 9. $\sqrt{\frac{36}{81}}$ | 10. $\sqrt{\frac{16}{900}}$ | 11. $\sqrt{\frac{25}{121}}$ | 12. $\sqrt{\frac{169}{4}}$ |

Use prime factorization to simplify the expression.

- | | | | |
|---|---|---|---|
| 13. $\sqrt{4 \times 9}$ | 14. $\sqrt{16} \times \sqrt{25}$ | 15. $\sqrt{81 \times 100}$ | 16. $\sqrt{4} \times \sqrt{49}$ |
| 17. $\sqrt{2 \times 2 \times 3 \times 3}$ | 18. $\sqrt{5 \times 5 \times 7 \times 7}$ | 19. $\sqrt{2 \times 2 \times 7 \times 7}$ | 20. $\sqrt{3 \times 3 \times 7 \times 7}$ |
| 21. $\sqrt{324}$ | 22. $\sqrt{625}$ | 23. $\sqrt{576}$ | 24. $\sqrt{256}$ |
| 25. $\sqrt{196}$ | 26. $\sqrt{1225}$ | 27. $\sqrt{484}$ | 28. $\sqrt{441}$ |
| 29. $\frac{\sqrt{121}}{\sqrt{196}}$ | 30. $\frac{\sqrt{400}}{\sqrt{256}}$ | 31. $\frac{\sqrt{169}}{\sqrt{625}}$ | 32. $\frac{\sqrt{3600}}{\sqrt{10\,000}}$ |
| 33. $\sqrt{\frac{441}{324}}$ | 34. $\sqrt{\frac{1296}{1225}}$ | 35. $\sqrt{\frac{6400}{289}}$ | 36. $\sqrt{\frac{225}{2025}}$ |

37. A square has an area of 12.25 cm^2 . What is the length of each side?
38. A rectangle's length is twice its width. Its area is 128 cm^2 . What is the length of each side?

Using the Square Root Key

The square root of 64 is 8 because $8 \times 8 = 64$.

To find $\sqrt{64}$ press 6 4 $\sqrt{}$

Use a calculator to find the square root.

- | | | |
|------------------|---------------------|---------------------|
| a. $\sqrt{100}$ | b. $\sqrt{169}$ | c. $\sqrt{256}$ |
| d. $\sqrt{400}$ | e. $\sqrt{529}$ | f. $\sqrt{784}$ |
| g. $\sqrt{1089}$ | h. $\sqrt{1681}$ | i. $\sqrt{2209}$ |
| j. $\sqrt{2500}$ | k. $\sqrt{4225}$ | l. $\sqrt{7396}$ |
| m. $\sqrt{8100}$ | n. $\sqrt{11\,449}$ | o. $\sqrt{16\,641}$ |

Approximating Square Roots

What number squared is equal to 2?
Or what is the square root of 2?

$$1^2 = 1 < 2 \quad \text{and} \quad 2^2 = 4 > 2$$

Therefore: $1 < \sqrt{2} < 2$

Try a number between 1 and 2.

$$(1.5)^2 = 2.25 > 2$$

Therefore: $1 < \sqrt{2} < 1.5$

Try a number between 1 and 1.5.

$$(1.3)^2 = 1.69 < 2$$

Therefore: $1.3 < \sqrt{2} < 1.5$

As we close in, we see that $\sqrt{2}$ is *about* 1.4.

$$(1.4)^2 = 1.96 \approx 2$$

\approx means
approximately equal to.

The square root of 2 is a *nonterminating, nonrepeating* decimal.
Such a decimal is called an **irrational number**.

$$\sqrt{2} = 1.414\,213\,56 \dots$$

The square roots of many numbers are irrational and can only be approximated as decimals.

EXERCISES

Use the given information to complete the statement.
Check by squaring.

1. $(2.3)^2 = 5.29$
 $(2.1)^2 = 4.41$
 $\sqrt{5} \approx \blacksquare$

2. $(3.8)^2 = 14.4$
 $(3.5)^2 = 12.25$
 $\sqrt{13} \approx \blacksquare$

3. $(4.7)^2 = 22.09$
 $(4.3)^2 = 18.49$
 $\sqrt{20} \approx \blacksquare$

4. $(1.65)^2 = 2.7225$
 $(1.85)^2 = 3.4225$
 $\sqrt{3} \approx \blacksquare$

5. $(2.52)^2 = 6.3504$
 $(2.72)^2 = 7.3984$
 $\sqrt{7} \approx \blacksquare$

6. $(3.22)^2 = 10.3684$
 $(3.42)^2 = 11.6964$
 $\sqrt{11} \approx \blacksquare$

Approximate the square root to the nearest tenth.

- | | | | |
|-----------------|-------------------|------------------|-------------------|
| 7. $\sqrt{31}$ | 8. $\sqrt{3100}$ | 9. $\sqrt{90}$ | 10. $\sqrt{9000}$ |
| 11. $\sqrt{46}$ | 12. $\sqrt{4600}$ | 13. $\sqrt{151}$ | 14. $\sqrt{1.51}$ |



PRACTICE

- Copy and complete the table.
Approximate each square root to the nearest hundredth.

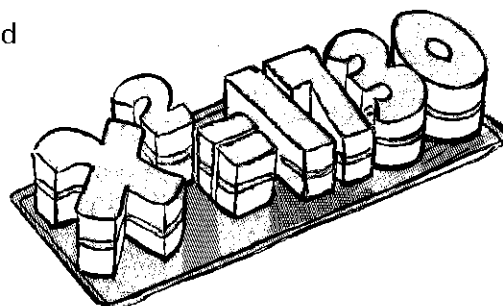
n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
\sqrt{n}	1	?	?	2	?	?	?	?	3	3.16	?	?	?	?	?	4

- Use the above table to approximate each square root to the nearest tenth. (Hint: $\sqrt{30} = \sqrt{3} \times \sqrt{10}$).
 - $\sqrt{30}$
 - $\sqrt{50}$
 - $\sqrt{60}$
 - $\sqrt{70}$
 - $\sqrt{110}$
 - $\sqrt{120}$
 - $\sqrt{130}$
 - $\sqrt{150}$
- A square has an area of 65 m². What is the length of each side (to the nearest tenth of a metre)?
- Name the dimensions of a square that has the same area as a 5 by 11 rectangle.

A Divide-and-Average Method

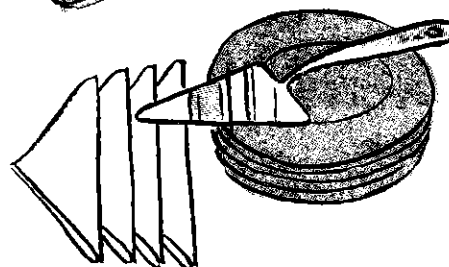
The square root of a number can be approximated using the divide-and-average method.

To find the square root of 1730, first make an estimate, say 40. Then divide 1730 by your first estimate. Round the quotient to the nearest tenth.



Step 1: Divide: $1730 \div 40 \approx 43.3$.
Now average the quotient and your first estimate (to the nearest tenth).

$$\text{Average: } \frac{40 + 43.3}{2} \approx 41.7$$



Step 2: Divide: $1730 \div 41.7 \approx 41.5$

$$\text{Average: } \frac{41.7 + 41.5}{2} \approx 41.6$$

Step 3: Divide: $1730 \div 41.6 \approx 41.6$

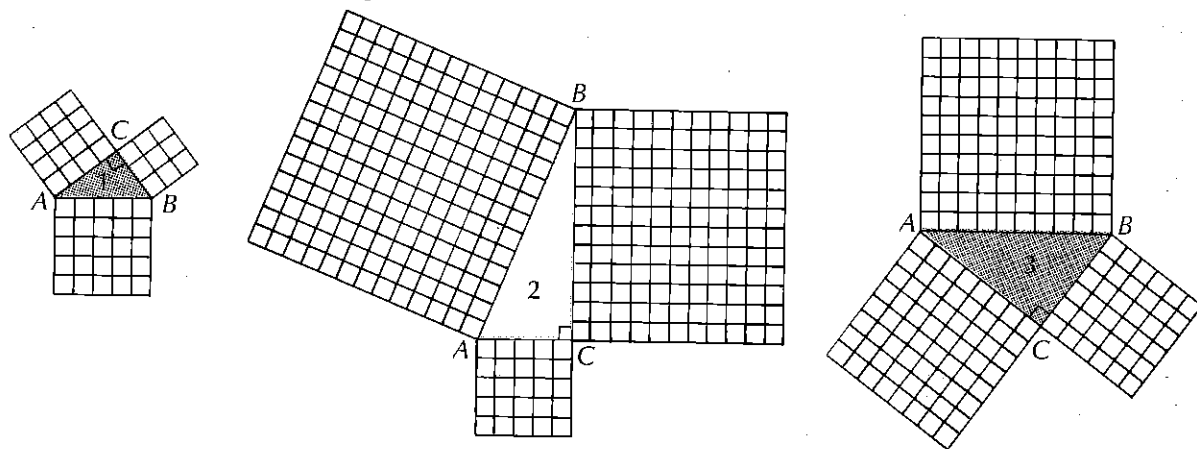
\therefore The square root of 1730 is about 41.6.

Use the divide-and-average method to approximate the square root to the nearest tenth.

- $\sqrt{105}$
- $\sqrt{340}$
- $\sqrt{85}$
- $\sqrt{450}$
- $\sqrt{180}$
- $\sqrt{260}$

The Pythagorean Theorem

In the diagrams below, squares have been constructed on the three sides of three right triangles.



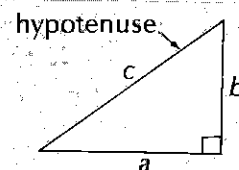
The table below summarizes the areas of the constructed squares above.

	Area of Square on \overline{AB}	Area of Square on \overline{AC}	Area of Square on \overline{BC}	Sum of Areas of Squares on \overline{AC} and \overline{BC}
right $\triangle 1$	25	16	9	25
right $\triangle 2$	169	25	144	169
right $\triangle 3$	100	64	36	100

The data in the table points out an important property of all *right triangles* known as the **Pythagorean Theorem**.

In any right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

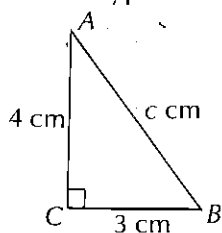
$$a^2 + b^2 = c^2$$



Examples:

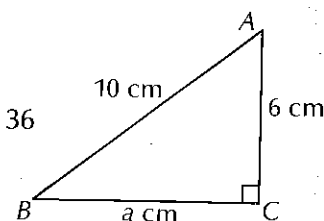
What is the length of the hypotenuse?

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 4^2 + 3^2 \\ c^2 &= 16 + 9 \\ c^2 &= 25 \\ c &= \sqrt{25} \\ c &= 5 \end{aligned}$$



What is a ?

$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 6^2 &= 10^2 \\ a^2 + 36 &= 100 \\ a^2 &= 100 - 36 \\ a^2 &= 64 \\ a &= \sqrt{64} \\ a &= 8 \end{aligned}$$



The length of the hypotenuse is 5 cm.

The length of the side is 8 cm.

EXERCISES

Simplify each expression.

1. $5^2 + 7^2$

2. $2^2 + 6^2$

3. $1.2^2 + 1.8^2$

Solve for c .

4. $c^2 = 49$

5. $c^2 = 1.21$

6. $c^2 = 900$

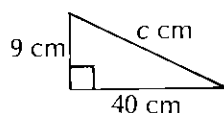
7. $c^2 = 8^2 + 15^2$

8. $c^2 = 15^2 + 20^2$

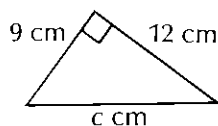
9. $c^2 = 1.4^2 + 4.8^2$

The length of the hypotenuse is c cm. Solve for c .

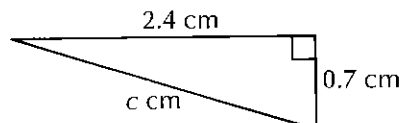
10. $c^2 = a^2 + b^2$
 $c^2 = 9^2 + 40^2$



11.

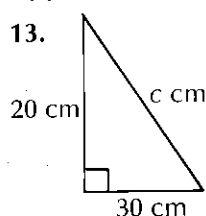


12.

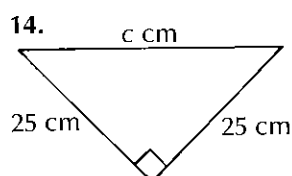


Approximate the length of the hypotenuse to the nearest tenth.

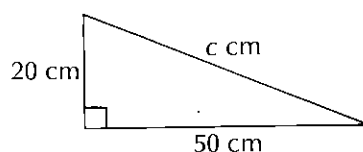
13.



14.



15.



Solve for b .

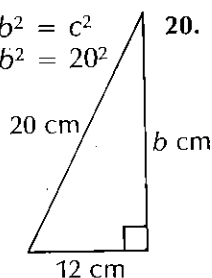
16. $10^2 = 8^2 + b^2$

17. $13^2 = 5^2 + b^2$

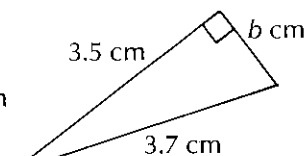
18. $2.6^2 = 2.4^2 + b^2$

The length of the side is b cm. Solve for b .

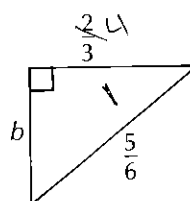
19. $a^2 + b^2 = c^2$
 $12^2 + b^2 = 20^2$



20.

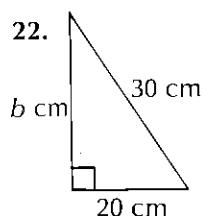


21.

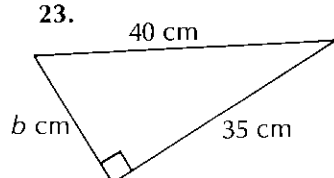


Approximate the length of the side to the nearest tenth.

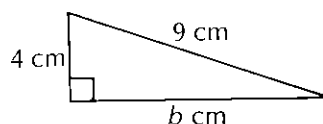
22.



23.

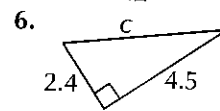
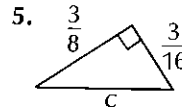
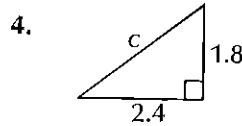
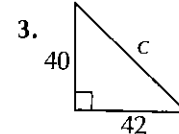
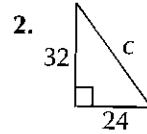
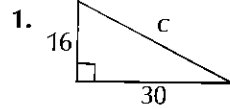


24.

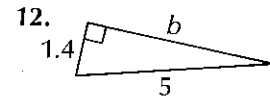
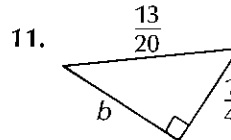
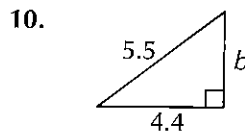
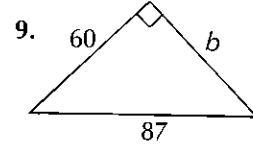
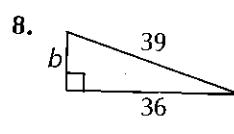
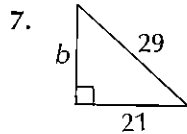


PRACTICE

The length of the hypotenuse is c units. Solve for c .



The length of the leg is b units. Solve for b .

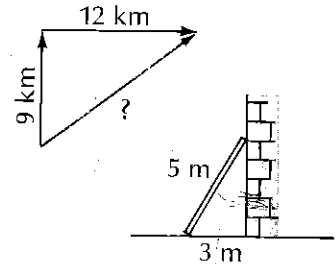


Approximate the missing length to the nearest tenth for each right triangle.

13. side = 5 cm; side = 2 cm; hypotenuse \approx ■ cm
14. hypotenuse = 4 m; two congruent sides \approx ■ cm each
15. two sides = 100 cm each; hypotenuse \approx ■ cm
16. hypotenuse = 45 m; side = 30 cm, side \approx ■ cm

Solve.

17. Marilyn hiked 9 km north and 12 km east. How far was she from her starting point?
18. The foot of a 5 m ladder is placed 3 m from a wall. How high up the wall does the ladder reach?
19. A rectangular field measures 320 m by 240 m. What is the length of its diagonal?
20. If the square of the longest side of a triangle is equal to the sum of the squares of the other two sides, the triangle is right-angled. Draw a triangle for each to see if it is right-angled.
 - a. 5 cm, 12 cm, 13 cm
 - b. 4.2 cm, 5.6 cm, 6 cm
 - c. 4.8 cm, 1.4 cm, 5 cm
 - d. 1.6 cm, 1.2 cm, 2 cm



The Right Triangle

M+

MR

$\sqrt{\quad}$

A calculator can be useful in solving a right triangle.

To find $\sqrt{5^2 + 12^2}$, press $\boxed{1} \boxed{2} \boxed{\times} \boxed{M+} \boxed{5} \boxed{\times} \boxed{=} \boxed{+} \boxed{MR} \boxed{=} \boxed{\sqrt{\quad}}$.

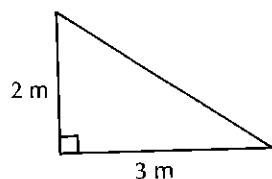
To find $\sqrt{25^2 - 7^2}$, press $\boxed{7} \boxed{\times} \boxed{M+} \boxed{2} \boxed{5} \boxed{\times} \boxed{=} \boxed{-} \boxed{MR} \boxed{=} \boxed{\sqrt{\quad}}$.

Using a calculator, find the value of each expression.

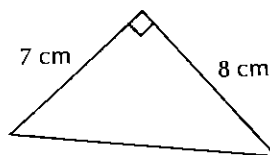
1. $\sqrt{20^2 - 12^2}$ 2. $\sqrt{15^2 + 8^2}$ 3. $\sqrt{3.2^2 + 2.4^2}$ 4. $\sqrt{3.7^2 - 3.5^2}$

Find the length of the hypotenuse to the nearest tenth.

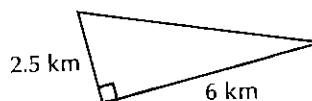
5.



6.

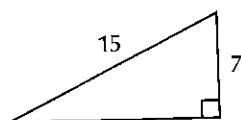


7.

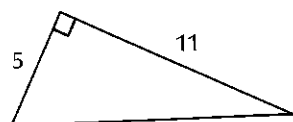


Find the missing length to the nearest hundredth.

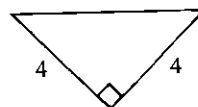
8.



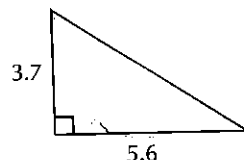
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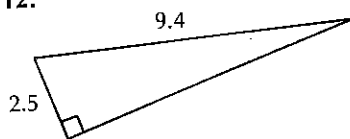
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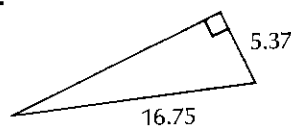
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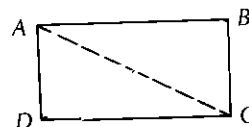
12.



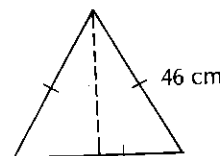
13.



14. A rectangular field is 67 m long and 32 m wide. How much shorter (to the nearest tenth of a metre) is the distance from A to C than the distance from A to B to C?



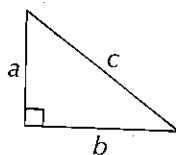
15. An equilateral triangle has sides of 46 cm in length. How long is each altitude of the triangle (to the nearest tenth of a centimetre)?



Pythagorean Triples

Any set of three *counting* numbers (a, b, c) that satisfy the equation $a^2 + b^2 = c^2$ is called a **Pythagorean Triple**.

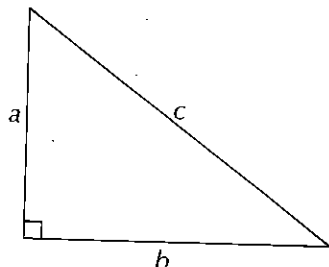
Example:



$$\begin{aligned} a &= 3 \\ b &= 4 \\ c &= 5 \end{aligned}$$

3, 4, and 5 are a Pythagorean triple because $3^2 + 4^2 = 5^2$.

Note: A new Pythagorean triple is made when each number in the above triple is *doubled*, *tripled*, and so on.



$$\begin{aligned} a &= 6 \\ b &= 8 \\ c &= 10 \end{aligned}$$

6, 8, and 10 are a Pythagorean triple because $6^2 + 8^2 = 10^2$.

$$\begin{aligned} a &= 9 \\ b &= 12 \\ c &= 15 \end{aligned}$$

9, 12, and 15 are a Pythagorean triple because $9^2 + 12^2 = 15^2$.

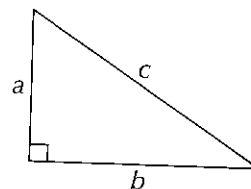
If a, b , and c form a Pythagorean triple and n is a counting number, then an, bn , and cn also form a Pythagorean triple.

EXERCISES

Find the missing number in each Pythagorean triple.

1. $a = 5$
 $b = 12$
 $c = \blacksquare$

2. $a = 9$
 $b = 12$
 $c = \blacksquare$



3. $a = 10$
 $b = \blacksquare$
 $c = 26$

4. $a = 18$
 $b = \blacksquare$
 $c = 30$

5. $a = 36$
 $b = 48$
 $c = \blacksquare$

6. $a = 7$
 $b = \blacksquare$
 $c = 25$

7. $a = \blacksquare$
 $b = 48$
 $c = 50$

8. $a = \blacksquare$
 $b = 96$
 $c = 100$

PRACTICE

Find the missing number in each Pythagorean triple.

- | | | |
|----------------|-----------------|-------------------|
| 1. 15, 20, ■ | 2. 15, 36, ■ | 3. 21, 20, ■ |
| 4. 30, 16, ■ | 5. 36, 48, ■ | 6. 24, 45, ■ |
| 7. 21, ■, 35 | 8. 45, ■, 53 | 9. ■, 35, 37 |
| 10. 45, 200, ■ | 11. ■, 84, 85 | 12. 11, ■, 61 |
| 13. 72, ■, 97 | 14. 120, ■, 169 | 15. ■, 161, 289 |
| 16. 90, 56, ■ | 17. 600, ■, 769 | 18. 4800, 4601, ■ |

Solve.

19. Find five Pythagorean triples whose numbers are multiples of the numbers in the triple 3, 4, and 5.
20. Find all Pythagorean triples for which the hypotenuse is not greater than 20.

Computer Power

The BASIC program will quickly output Pythagorean triples.

```

10 REM
20 REM
30 REM
40 FOR A = 1 TO 50
50 FOR B = 1 TO 50
60 LET M = A * A + B * B
70 LET C = INT ( SQR ( M ) )
80 IF M = C * C THEN PRINT A,B,C
90 NEXT B
100 NEXT A
110 END
    
```

Use the program above for these questions.

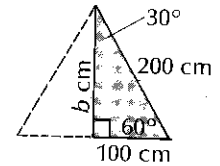
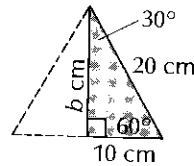
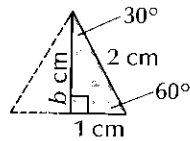
1. In which line does the program check whether $A^2 + B^2 = C^2$?
2. In line 80, what happens if M does not equal $C \cdot C$?
3. These sets of integers are not printed. Explain why.
 - a. 7, 7, 10 b. 39, 52, 65 c. 18, 7, 25
4. Marianne suggests that line 50 be changed to:

50 FOR B = A TO 50

What effect would this have on the output?

30°–60° Right Triangles

A 30°–60° right triangle has angles of 30°, 60°, and 90°. It can be thought of as half of an equilateral triangle. The hypotenuse is always twice as long as the shorter leg.



The length of the longer leg of each triangle is an *irrational* number that can be approximated as a decimal.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 1^2 + b^2 &= 2^2 \\ 1 + b^2 &= 4 \\ b^2 &= 3 \\ b &= \sqrt{3} \\ &= 1.732\ 05\ \dots \\ &\approx 1.73 \end{aligned}$$

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 10^2 + b^2 &= 20^2 \\ 100 + b^2 &= 400 \\ b^2 &= 300 \\ b &= \sqrt{300} \\ &= \sqrt{100} \times \sqrt{3} \\ &= 17.3205\ \dots \\ &\approx 17.3 \end{aligned}$$

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 100^2 + b^2 &= 200^2 \\ 10\ 000 + b^2 &= 40\ 000 \\ b^2 &= 30\ 000 \\ b &= \sqrt{30\ 000} \\ &= \sqrt{10\ 000} \times \sqrt{3} \\ &= 173.205\ 08\ \dots \\ &\approx 173.2 \end{aligned}$$

Note the pattern for 30°–60° right triangles.

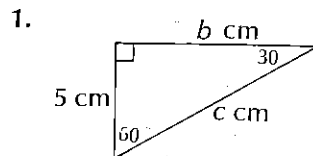
short side → 1
long side → $\sqrt{3}$

short side → 10
long side → $10 \times \sqrt{3}$

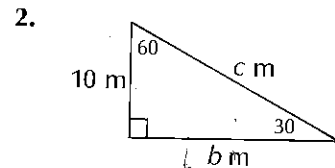
short side → 100
long side → $100 \times \sqrt{3}$

If the shorter leg of a 30°–60° right triangle has length a , the hypotenuse has length $2a$ and the other side has length $a \times \sqrt{3}$.

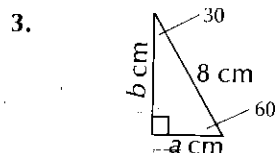
Find the missing lengths for the 30°–60° right triangles. Use $\sqrt{3} \approx 1.7$.



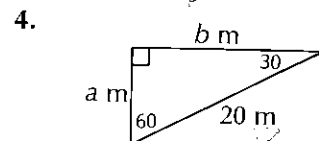
$$\begin{aligned} b &= \blacksquare \\ c &= \blacksquare \end{aligned}$$



$$\begin{aligned} b &= \blacksquare \\ c &= \blacksquare \end{aligned}$$



$$\begin{aligned} a &= \blacksquare \\ b &= \blacksquare \end{aligned}$$

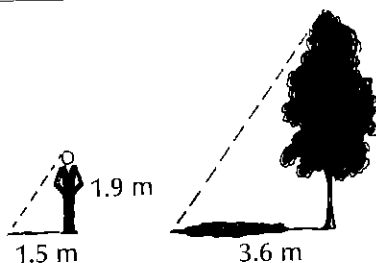


$$\begin{aligned} a &= \blacksquare \\ b &= \blacksquare \end{aligned}$$

5. An equilateral triangle measures 100 cm on all sides. What is the length of its altitude?

REVIEW

1. A man 1.9 m tall casts a shadow 1.5 m long while a nearby tree casts a shadow 3.6 m long. How tall is the tree?



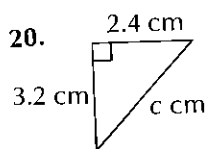
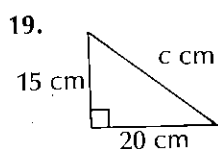
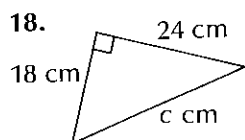
Find each square root.

2. $\sqrt{16}$ 3. $\sqrt{625}$ 4. $\sqrt{1024}$ 5. $\sqrt{1296}$
 6. $\sqrt{\frac{4}{25}}$ 7. $\sqrt{0.64}$ 8. $\sqrt{\frac{25}{144}}$ 9. $\sqrt{0.09}$

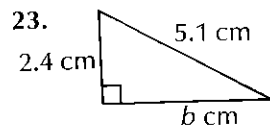
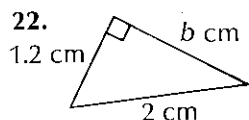
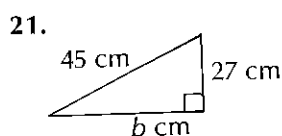
Approximate each square root to the nearest tenth.

10. $\sqrt{2}$ 11. $\sqrt{3}$ 12. $\sqrt{5}$ 13. $\sqrt{6}$
 14. $\sqrt{10}$ 15. $\sqrt{15}$ 16. $\sqrt{1500}$ 17. $\sqrt{30}$

The length of the hypotenuse is c cm. Solve for c .



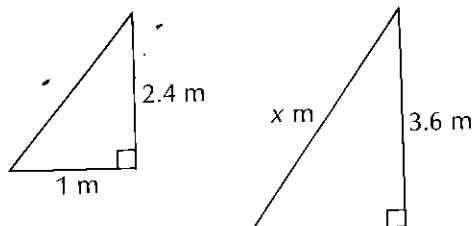
The length of the leg is b cm. Solve for b .



Find the missing number in each Pythagorean triple.

24. 33, ■, 55 25. 6, ■, 10 26. ■, 12, 13
 27. 24, ■, 25 28. 48, 36, ■ 29. 40, 42, ■

30. These two right triangles are similar. Find the length of the hypotenuse of the larger triangle.



1. Draw and label a figure for each.
 - a. line RS
 - b. polygon $ABCD$
 - c. point H
 - d. segment DE
 - e. ray GH
 - f. right angle XYZ
 - g. acute angle FGH
 - h. obtuse angle MNO
 - i. straight angle BCD

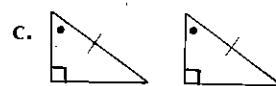
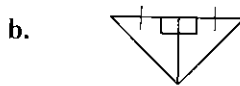
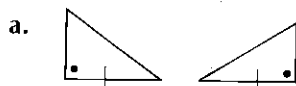
2. What kind of triangle has these angle sizes?

- a. $95^\circ, 25^\circ, 60^\circ$
- b. $35^\circ, 75^\circ$
- c. $30^\circ, 60^\circ$

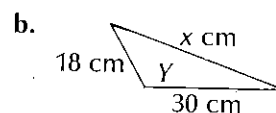
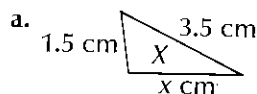
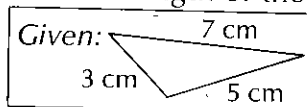
3. Copy and complete the congruence statement.

Given: $\triangle ABC \cong \triangle DEF$ a. $\angle C \cong \blacksquare$ b. $\overline{AB} \cong \blacksquare$

4. Are the triangles congruent? If yes, state the rule which establishes the congruence: **SSS**, **SAS**, **ASA**, or **AAS**.



5. Triangles X and Y below are similar to the given triangle. Find the length of the third side of each triangle.



6. A 3 m pole casts a 4.2 m shadow at the same time a building casts a 50.4 m shadow. How tall is the building?

7. Find the square root.

a. $\sqrt{900}$

b. $\sqrt{1.69}$

c. $\sqrt{\frac{36}{49}}$

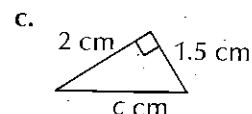
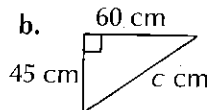
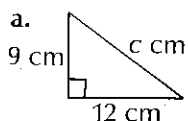
8. Approximate the square root to the nearest tenth.

a. $\sqrt{8}$

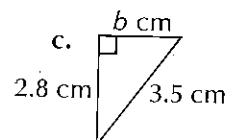
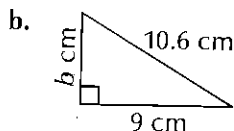
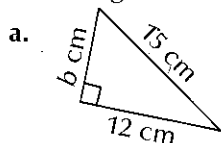
b. $\sqrt{20}$

c. $\sqrt{12}$

9. The length of the hypotenuse is c cm. Solve for c .



10. The length of the side is b cm. Solve for b .



1. Consider the set of counting numbers less than 25.
Write the ratio for each.
 - a. even numbers to odd.
 - b. prime numbers to composite.
 - c. numbers divisible by 3 to numbers divisible by 4.
2. Solve for the missing term.
 - a. $15:24 = x:56$
 - b. $\frac{x}{16} = \frac{45}{40}$
 - c. $12:15 = 60:x$
3. Solve.
 - a. It cost Mr. Santana \$450 to pave 3 m^2 of patio with flagstones.
How much would it cost him to pave 10 m^2 ?
 - b. How much should a dozen donuts cost if they are sold at
2 for 75 cents?
 - c. About how much should 2 cans of peas cost if they are on
sale at 3 cans for 79 cents?
 - d. How long would it take the Lee family to travel 700 km
if they travel 450 km in 9 h?
4. Write as a percent.
 - a. $\frac{7}{50}$
 - b. $1\frac{1}{2}$
 - c. $\frac{6}{1000}$
5. Write as a fraction.
 - a. 72%
 - b. 275%
 - c. 4.5%
6. Find the percent.
 - a. 45 out of 50
 - b. 2 out of 80
 - c. 16 out of 20
7. Find the part.
 - a. 35% of 80
 - b. $9\frac{1}{2}\%$ of 1000
 - c. 180% of 60
8. Find the whole.
 - a. 60% of what is 30?
 - b. 300% of what is 1500?
 - c. $10\frac{1}{2}\%$ of what is 21?
9. Solve.
 - a. There are 250 Grade 8 students in Willowbrook School. One
hundred twenty of the students are boys. What percent of
the students are girls?
 - b. Trent figures that he spends 12.5% more this year on his
model cars than he did last year. If he spent \$124.50 last
year, how much will he be spending this year?