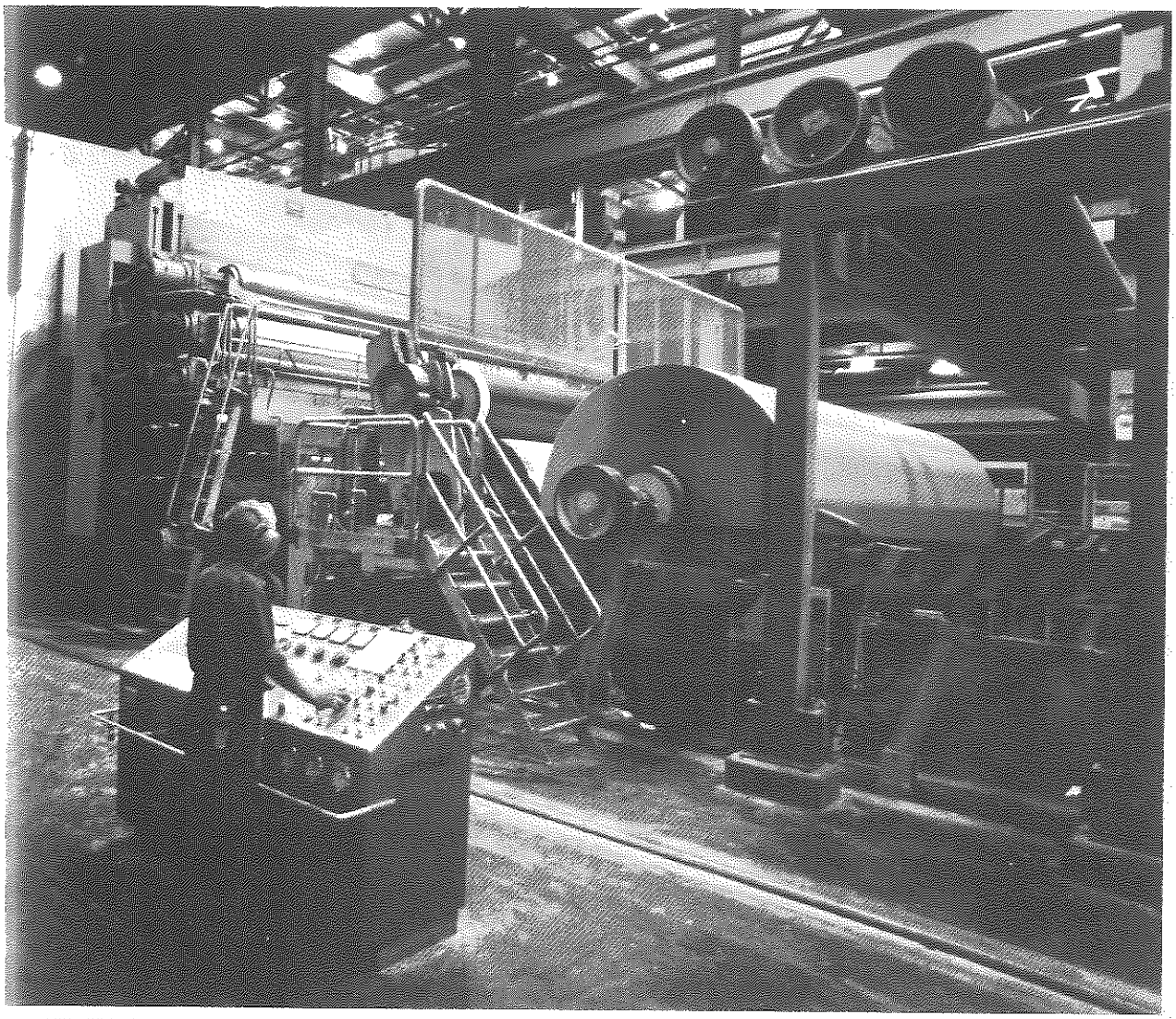
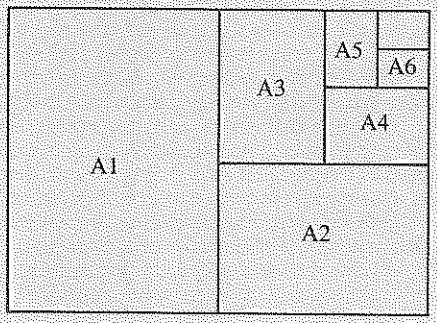
6 Ratio, Proportion, and Rate





In a move to conserve the Earth's resources, an international standard of paper sizes was created. The largest, A0, has an area of 1 m^2. The ratio of the lengths of its sides are such that cutting it in half results in the next largest size, A1, with side lengths in the same ratio. The next size, A2, is obtained by cutting an A1 sheet in half, and so on. The length-to-width ratio is the same for all sizes.

There is only one length-to-width ratio that yields similar rectangles in this way. Can you find what it is? (See Section 6-3, *Example 3*.)

6-1 RATIOS

The newspaper headline indicates that 80 out of 100 Canadians buy lottery tickets. Therefore, 20 out of 100 do not buy tickets.

80 out of every 100 Canadians buy lottery tickets

The comparison of the numbers of people who do not buy lottery tickets and those who do, can be written as a *ratio*, 20 : 80. This is read, "twenty to eighty".

A ratio is a comparison of quantities measured in the same units. The numbers 20 and 80 are the *terms* of the ratio. The order of the terms is important; they may not be interchanged.

When each term of a ratio is multiplied or divided by the same non-zero number, an equivalent ratio is produced. For example, $20 : 80 = 200 : 800 = 2 : 8 = 1 : 4$

The ratio 1 : 4 is in *lowest terms* because the only factor that the terms have in common is 1.

A ratio can be written in fractional form; that is, 20 : 80 can be written as $\frac{20}{80}$.

Although a ratio can be written in fractional form, a ratio is *not* a fraction. A fraction is a comparison of part(s) of an amount (the numerator) to the whole amount (the denominator). A ratio is a comparison of two amounts, the second of which may not be the whole amount.

For example, the ratio described previously, $\frac{20}{80}$, can be written as $\frac{1}{4}$. But this does not mean that $\frac{1}{4}$ of the people do not buy lottery tickets. In fact, $\frac{1}{4}$ in this context has no meaning as a fraction because the denominator does not represent the whole group of people considered.

Example 1. Write each ratio in lowest terms.

a) 9 : 6 b) 16 : 12 c) 25 : 20 d) $\frac{60}{15}$

Solution.

a) 9 : 6
Divide by 3.
$9 : 6 = 3 : 2$

b) 16 : 12
Divide by 4.
$16 : 12 = 4 : 3$

c) 25 : 20
Divide by 5.
$25 : 20 = 5 : 4$

d) $\frac{60}{15}$
Divide by 15.
$\frac{60}{15} = \frac{4}{1}$

Example 2. The table shows the knockout records of four boxers in the Winnipeg Boxing Club. Which two boxers have the same ratio of knockouts to fights?

Name	Fights	Knockouts
Doug James	28	7
Mike Seiling	24	8
Gary Burgess	20	5
Dan Stoffer	24	4

Solution The knockouts-to-fights ratios are
James $7 : 28 = 1 : 4$, Seiling $8 : 24 = 1 : 3$, Burgess $5 : 20 = 1 : 4$,
Stoffer $4 : 24 = 1 : 6$
Doug James and Gary Burgess have the same knockouts-to-fights ratio.

Example 3. A parking lot contains domestic and foreign cars in the ratio 7 : 4. If there are 77 cars in the lot, how many of them are foreign?

Solution. The ratio of domestic cars to foreign cars is 7 : 4.
Therefore, 7 out of 11 cars are domestic and 4 out of 11 cars are foreign.
The number of foreign cars is $\frac{4}{11}(77) = 28$
There are 28 foreign cars in the lot.

Example 4. Fuel X is composed of ingredients A and B in the ratio 3 : 5. Fuel Y is composed of ingredients A and B in the ratio 4 : 7. Which fuel is richer in ingredient A?

Solution. Ingredient A is $\frac{3}{8}$ of fuel X. Ingredient A is $\frac{4}{11}$ of fuel Y.
Raise these fractions to a common denominator.
$\frac{3}{8} = \frac{33}{88}$ and $\frac{4}{11} = \frac{32}{88}$
Since $\frac{33}{88} > \frac{32}{88}$, then $\frac{3}{8} > \frac{4}{11}$
Fuel X is richer in ingredient A.

EXERCISES 6-1

(A)

1. Explain each statement without using the word "ratio".
   a) Mrs. Adams and Mr. Singh divided the profits in the ratio 3 : 2.
   b) The ratio of girls to boys in the class is 7 : 5.
   c) Mrs. Arbor's chain saw runs on a 25 : 1 mixture of gasoline and oil.
   d) The scale of a map is 1 : 250 000.
   e) Brass is an alloy of copper and zinc in the ratio 3 : 2.

2. Write each ratio in lowest terms.
   a) 40 : 12 b) 5 : 65 c) 28 : 8 d) 32 : 52
   e) 12 : 72 f) 50 : 250 g) $\frac{60}{12}$ h) $\frac{144}{9}$

3. State which is the greater ratio.
   a) $\frac{5}{8}$ or $\frac{3}{5}$ b) 6 : 7 or 7 : 8 c) 6 : 5 or 12 : 11 d) 8 : 3 or 13 : 5

B

4. "Gran's" cookies have raisins and chocolate chips in the ratio 3 : 7. "Mum's" cookies have raisins and chocolate chips in the ratio 5 : 11. Which brand has the greater ratio of raisins to chocolate chips?

5. Air consists of oxygen and nitrogen in the approximate ratio 1 : 4, and negligible amounts of other gases.
   a) What fraction of air is oxygen? b) What fraction of air is nitrogen?

6. Sterling silver is an alloy of silver and copper in the ratio 37 : 3.
   a) What fraction of a sterling silver fork is silver?
   b) If the mass of a sterling silver ingot is 500 g, how much silver does it contain?

7. Write an equivalent ratio with a second term of 1.
   a) 5 : 2 b) 2 : 0.5 c) 3 : 10 d) 4 : 0.8

8. Write an equivalent ratio with a second term of 24.
   a) 5 : 6 b) 8 : 48 c) 27 : 36 d) 5 : 0.6

9. At a school dance, there are 15 teachers, 275 girls, and 225 boys. Express the following ratios in lowest terms.
   a) girls to boys b) teachers to girls c) students to teachers

10. A newspaper costs 25¢ each day from Monday to Friday and 75¢ on Saturday. What is the ratio of:
    a) the cost on Saturday to the cost for one week
    b) the cost on Monday to the cost for one week?

11. If the ratio of domestic cars to foreign cars in Metropolitan Toronto is 9 : 5, how many domestic cars might you expect to find in a lot containing 247 cars?

12. The length and the width of a rectangle are in the ratio 9 : 7. If the perimeter is 256 cm, what are the dimensions of the rectangle?

C

13. The front gear wheels of a ten-speed bicycle have 40 and 52 teeth. The back gear wheels have 14, 17, 20, 24, and 28 teeth.
    a) Write the ten different gear ratios (front:back).
    b) Arrange the ten gear ratios in order from lowest to highest.

14. The ratio of the mass of a hydrogen atom to the average mass of a person (70 kg) is about the same as the ratio of the average mass of a person to the mass of the sun. The mass of a hydrogen atom is about 1.7×10^{-29} kg. What is the approximate mass of the sun?

6-2 APPLICATIONS OF RATIOS

The amount of gold in jewellery and coins is measured in karats (K) with 24 K representing pure gold.

The mark 14 K on a ring means the ratio of the mass of gold in the ring to the mass of the ring is 14 : 24.

Since the second term of the ratio describing the purity of gold is always 24, it is omitted when describing the gold content.

In many ratios, only the first term is stated. The second term is omitted since it is always the same in each type of application of the ratio. Some further examples of ratios in which only the first terms are stated are Consumer Price Index, Mach numbers, and Intelligence Quotients.

Example 1. A gold bracelet is marked 18 K.

a) Express the gold content of the bracelet as a fraction in lowest terms.

b) The mass of the bracelet is 52 g and the value of pure gold is \$25.50/g. Find the value of the gold in the bracelet.

Solution.

a) Mass of gold : mass of bracelet $= 18 : 24$
$= 3 : 4$

b) Mass of gold in the bracelet $= \frac{3}{4}(52\text{ g})$
$= 39\text{ g}$

Value of gold in the bracelet $= 39(\$25.50)$
$= \$994.50$

The gold in the bracelet is worth \$994.50.

The *Consumer Price Index (CPI)* is a measure of the change in the cost of living. The ratio compares the price of 300 selected items at any time, to the price of the same items in 1981. The CPI is a ratio with its second term 100.

$$\frac{\text{CPI in year A}}{100} = \frac{\text{Cost of 300 items in year A}}{\text{Cost of 300 items in 1981}}$$

What is the Consumer Price Index today?

Example 2. In 1981 a family spent \$14 000. If the CPI in 1990 were 187.4, how much would the family spend, to the nearest \$100, in that year for the same items?

Solution.

$$\frac{\text{CPI in 1990}}{100} = \frac{\text{Cost of items in 1990}}{\text{Cost of items in 1981}}$$

Substitute the given information.

$$\frac{187.4}{100} = \frac{\text{Cost of items in 1990}}{\$14\,000}$$

$$\text{Cost of items in 1990} = \$14\,000\left(\frac{187.4}{100}\right)$$

$$= \$26\,236$$

In 1990, the family would spend about \$26 200.

To compare more than two quantities, ratios with more than two terms are used. For example, the gravities of the Earth, Jupiter, and Mars are in the ratio 5 : 13 : 2. This means that a person on Jupiter would weigh $\frac{13}{5}$ of her or his weight on Earth, and $\frac{13}{2}$ of her or his weight on Mars.

Example 3. The profits in a business are to be shared by the three partners in the ratio 2 : 3 : 5. The profit for the year was \$176 500. Calculate each partner's share.

Solution. Since the profit is shared in the ratio 2 : 3 : 5, we can think of the profit as consisting of a total of $(2+3+5)$ or 10 shares.

The 1st partner's share is $\frac{2}{10}(\$176\,500)$ or \$35 300.

The 2nd partner's share is $\frac{3}{10}$ (\$176 500) or \$52 950.

The 3rd partner's share is $\frac{5}{10}$ (\$176 500) or \$88 250.

EXERCISES 6-2

(A)

1. Express the gold content as a ratio in lowest terms.
   a) a 22 K gold coin
   b) a 16 K gold pin
   c) a charm marked 9 K, the legal minimum for an article to be called gold

2. A 14 K gold ring has a mass of 24.7 g.
   a) Find the mass of the gold in the ring.
   b) At \$25.50/g, find the value of the gold in the ring.

(B)

3. In 1986, the Consumer Price Index rose to 131.1. How much did it cost a family in 1986 for goods that cost them \$8700 in 1981?

4. Three people contributed to buy a lottery ticket, in the ratio 2 : 5 : 3. If the ticket wins \$25 000, how should the prize be divided?

5. Four partners in a business agreed to share the profits in the ratio 4 : 2 : 3 : 6. The first year's profits were \$84 000. Calculate each partner's share.

6. a) A 1-cent coin minted before 1860 had a mass of 4.50 g and contained copper, tin, and zinc in the ratio 95 : 4 : 1. What mass of tin did each coin contain?
   b) From 1876 to 1920, the mass of each 1-cent coin minted was 5.67 g, and the ratio of copper to tin to zinc was 95.5 : 3 : 1.5. What mass of copper did each coin contain?
   c) After 1942, each 1-cent coin minted had a mass of 3.24 g and contained copper, tin, and zinc in the ratio 98 : 0.5 : 1.5. What mass of zinc did each coin contain?

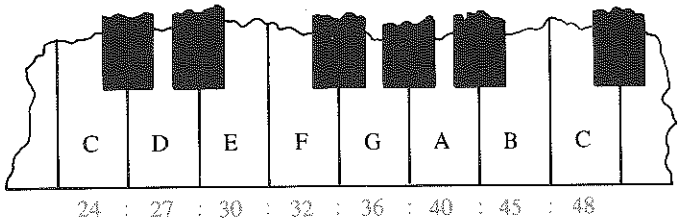
7. The *Intelligence Quotient (IQ)* is the first term of a ratio with a second term of 100. The IQ is always stated to the nearest whole number. It is calculated from this formula. The formula applies to physical ages up to 20 years.

$$\frac{\text{IQ}}{100} = \frac{\text{mental age}}{\text{physical age}}$$

   a) Find the IQ of each child.
      i) a twelve-year old with a mental age of 12.5
      ii) a seven-year old with a mental age of 6.8
   b) Find the mental age of each child.
      i) a nine-year old with an IQ of 100
      ii) a six-year old with an IQ of 150 (genius level)

8. A *Mach number* is a ratio with second term 1. The Mach number of an airplane is the ratio of its speed to the speed of sound at the same altitude and temperature. Assume an altitude where the speed of sound is 1085 km/h.
   a) Calculate the Mach number of an airplane flying at each speed. Give the answers to one decimal place.
      i) 3255 km/h ii) 1302 km/h iii) 1000 km/h
   b) Find the speed of the North American Aviation X-15A-2 flying at Mach 6.72.

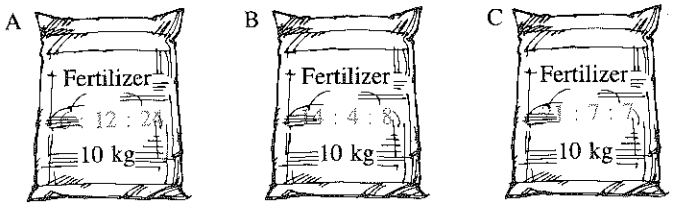
9. The frequencies of the notes in the musical scale of C major are related by an eight-term ratio.



Musical instruments are usually tuned so that A in the scale has a frequency of 440 Hz (cycles per second). Find the frequencies of the other notes in the scale.

10. Chemical fertilizers usually contain nitrogen, phosphorus, and potassium. The amount of each nutrient present is expressed as a percent of the total mass of the fertilizer in the three-term ratio, nitrogen : phosphorus : potassium.

A, B, and C are 10 kg bags of three kinds of fertilizer.



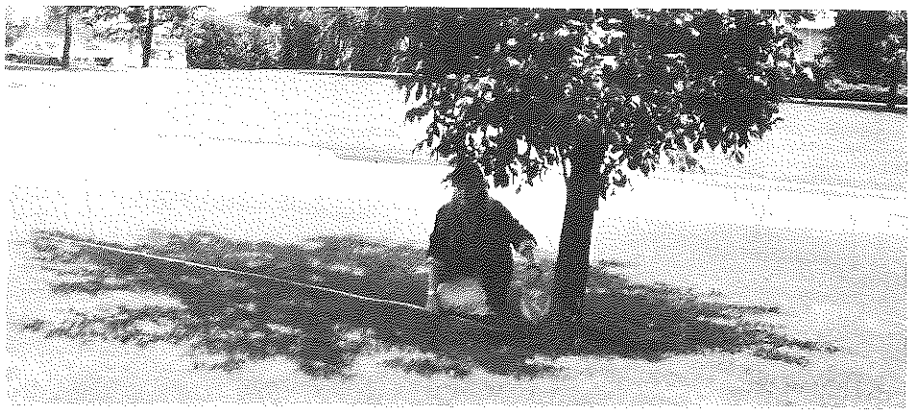
a) Which fertilizer contains the most nitrogen?
b) Which fertilizer has the greatest ratio of nitrogen to phosphorus?
c) Which fertilizer contains the most nutrients?
d) Why are the ratios not expressed in lowest terms?

11. The ratio of the approximate distances of the Earth and Uranus from the Sun is 4 : 77, and that of Mars and Uranus from the Sun is 2 : 25.
    a) What is the ratio of the approximate distances of the Earth and Mars from the Sun?
    b) How far is Mars from the Sun if the Earth's distance is 150 Gm?

12. The angles of a triangle are in the ratio 2 : 3 : 4. What are their measures?

13. The angles of a quadrilateral are in the ratio 2 : 3 : 3 : 4. What are their measures?

14. A sphere and a cone are each designed to fit snugly (at separate times) inside a cylinder of radius R and height $2R$.
    a) Write the ratio of the volume of the sphere to the volume of the cone to the volume of the cylinder.
    b) How many times is the volume of the cylinder as great as the volume of the largest sphere it will contain?



6-3 PROPORTIONS

One method of finding the height of a tall tree is to measure its shadow, and then compare that with the length of the shadow of an object whose height is known. If both shadows are measured at the same time of day, the ratio of their lengths will be equal to the ratio of the heights of the object and the tree.

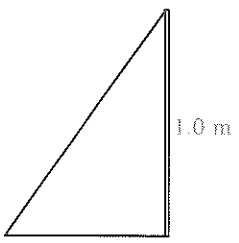
$$\frac{\text{height of tree}}{\text{height of stick}} = \frac{\text{length of tree's shadow}}{\text{length of stick's shadow}}$$

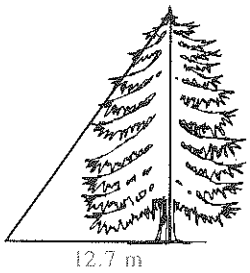
Substitute the known measures.

$$\frac{\text{height of tree}}{1.0} = \frac{12.7}{0.7}$$

$$\text{height of tree} = \frac{12.7}{0.7}$$

$$\doteq 18.1$$





The tree is about 18 m high.

This solution used the fact that two ratios were equal.

A statement that two ratios are equal is called a *proportion*. For example, the statement $a : b = c : d$, is a proportion.

Since ratios can be written in fractional form, the statement $\frac{a}{b} = \frac{c}{d}$ is also a proportion.

To simplify an expression of this type, multiply both sides of the proportion by the lowest common denominator.

$$\frac{a}{b}(bd) = \frac{c}{d}(bd)$$

$$ad = cb$$

This equation can be used to find the unknown term in a proportion when the other three terms are known.

Example 1. Find each value of x.

a) $\frac{7}{30} = \frac{14}{x}$ b) $\frac{3}{19} = \frac{x}{7}$

Solution. a) $\frac{7}{30} = \frac{14}{x}$

$7x = 14(30)$

$x = \frac{14(30)}{7}$

$= 60$

b) $\frac{3}{19} = \frac{x}{7}$

Multiply both sides by 7.

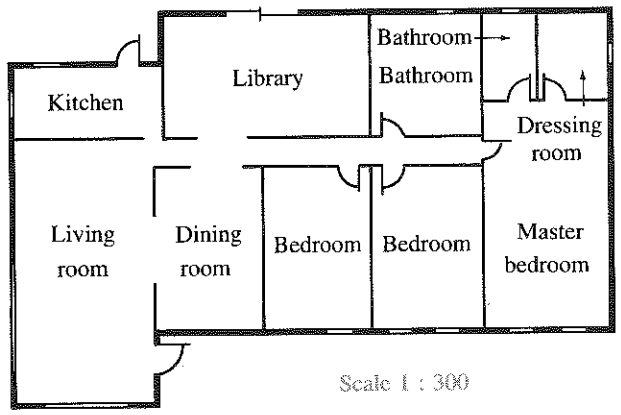
$\frac{3(7)}{19} = x$

$\frac{21}{19} = x$

Example 2. The scale, 1 : 300, of the floor plan indicates that 1 mm on the plan corresponds to 300 mm, or 0.3 m, in the actual building.

a) Find the dimensions, in metres, of the living room.

b) Find the total floor area to the nearest square metre.



Solution. a) By measurement of the floor plan, the living room is 35 mm by 20 mm.

This corresponds to 35(0.3 m) by 20(0.3 m), or 10.5 m by 6.0 m.

The living room is 10.5 m by 6.0 m.

b) The floor plan can be divided into two rectangles. One rectangle comprises the living room and the kitchen. This measures 45 mm by 20 mm, or 45(0.3 m) by 20(0.3 m).

The other rectangle comprises all the other rooms. It measures 60 mm by 42 mm, or 60(0.3 m) by 42(0.3 m).

Total area $= [45(0.3) \times 20(0.3)] + [60(0.3) \times 42(0.3)]$

$= 307.8$

The floor area is approximately 308 m^2.

6-4 RATES

When a person buys a car, one of the considerations is usually how much gasoline the car uses. The fuel consumption, in litres, is compared to the distance the car travels, in hundreds of kilometres. Such a comparison is called a *rate* because the two quantities that are compared have different units.

Which of the cars above has the better fuel consumption?

Many familiar measurements are expressed as rates.

- speed in kilometres per hour
- pulse rate in heart beats per minute
- unit pricing on supermarket shelves in cents per 100 g
- wages in dollars per hour
- sports statistics in goals per game

Example 1. A car's fuel consumption is quoted as 5.5 L/100 km.

a) Find how much fuel is needed to travel 275 km.

b) Find how far the car will travel on 48 L of gasoline.

Solution.

a) A fuel consumption of 5.5 L/100 km means that the fuel for 100 km is 5.5 L.

The fuel for 1 km would be $\left(\frac{5.5}{100}\right)$ L.

The fuel for 275 km would be $\left(\frac{5.5}{100} \times 275\right)$ L or 15.125 L.

About 15 L of fuel are needed for a trip of 275 km.

b) On 5.5 L of fuel the car travels 100 km.

On 1 L of fuel the car could travel $\left(\frac{100}{5.5}\right)$ km.

On 48 L of fuel the car could travel $\left(\frac{100}{5.5} \times 48\right)$ km or $872.\overline{72}$ km.

The car travels about 873 km on 48 L of fuel.

Example 2. In one season, Casey batted at the rate of 2 hits for every 5 official times at bat.

a) At this rate, how many hits should he get in 400 times at bat?

b) How many times should Casey have at bat to get 180 hits?

Solution. Casey's batting average is $\frac{2}{5}$ or 0.4.

a) In 400 times at bat, Casey should get 400(0.4) or 160 hits.
If the rate is constant, Casey should get 160 hits.

b) Casey bats 5 times to get 2 hits.

So he will bat $\frac{5}{2}$ times to get 1 hit.

To get 180 hits, Casey will bat $\frac{5}{2}(180)$ or 450 times.

If the rate is constant, Casey should bat about 450 times to get 180 hits.

EXERCISES 6-4

(A)

1. Dale drives 45 km on 3 L of gasoline.
   a) Find how far would she drive on:
      i) 2 L of gasoline ii) 5 L of gasoline.
   b) Find how much gasoline Dale needs to drive:
      i) 60 km ii) 270 km.

2. Bob can type at the rate of 30 words per minute.
   a) Find how long it would take him to type:
      i) 20 words ii) 100 words.
   b) Find how many words Bob can type in:
      i) 4 min ii) 6.5 min.

3. A car uses fuel at the rate of 7.2 L/100 km.
   a) Find how much fuel is needed to travel 360 km.
   b) Find how far the car will travel on a full tank of 85 L.

(B)

4. An electronic typewriter can type 540 words per minute.
   a) Find how long it will take to type:
      i) 1000 words ii) 1 000 000 words.
   b) How many words can it type in:
      i) 1 h ii) 1 week.

5. In the first 20 games of the baseball season, Reggie Jackson hit 12 home runs. If he continued at this rate, how many home runs would he hit in 160 games?

6. A 350 g box of cornflakes costs \$1.75. A 525 g box of the same cereal costs \$2.29.
   a) Find the unit price, in cents per 100 g, for each box of cereal.
   b) Which box is the better buy?

7. Adrian works in a factory making pies. He is paid $9.55 per hour for a 7.5 h shift. He is paid "time and a half" for the first 4 h of overtime in a day. Then he is paid double time for any additional hours worked that day.
   In one week, Adrian worked these hours.

Monday	Tuesday	Wednesday	Thursday	Friday
15	7.5	7.5	11.5	7.5

   What are Adrian's gross wages for this week?

8. In the first 6 games of the football season, Dave Cutler scored 83 points. If he continued scoring at this rate, how many points would he score in 14 games?

9. If 18 houses are built in 45 days, find how long at this rate it would take to build:
   a) 63 houses b) 144 houses.

10. A brand of liquid detergent is sold in 2 sizes — $2.09 for 500 mL, and $3.55 for 1 L.
   a) Find the unit price for each size of detergent. Which is the better buy?
   b) The smaller size is "on special" for one week at a price of $1.79. For this week, which is the better buy?

11. Marilyn works in a car assembly plant. She is paid $14.50 per hour for an 8 h shift. She receives an additional 72.5¢ per hour for working the night shift.
   In one month, Marilyn works two 40 h day shifts and two 40 h night shifts. Find her gross wages for the month.

12. Milk is sold in 4 L bags and 2 L and 1 L cartons. The milk is priced at $3.49, $2.31, and $1.20, respectively. By how much would the cost of each carton have to be reduced so that its unit price was equal to that of the 4 L bags?

13. Two girls, 60 km apart, start cycling toward each other at the same time. One girl cycles at 18 km/h. How fast must the other girl cycle if they are to meet in 1.5 h?

14. Car A and car B leave Halifax on the same road 1 h apart. Car A leaves first and travels at a steady 80 km/h. How fast must car B travel to overtake car A in 4 h?

15. Machine X makes 200 boxes in 3 min and machine Y makes 200 boxes in 2 min. With both machines working, how long will it take to make 200 boxes?

16. A worker is paid $8.60/h for a 40 h week and time and a half for overtime. How many hours are worked to earn $414.95 in one week?

17. A study shows that an office staff of x people will consume y cups of coffee over a period of z days. At this rate, how long would it take a staff of $3x$ people to consume $\frac{y}{12}$ cups of coffee?

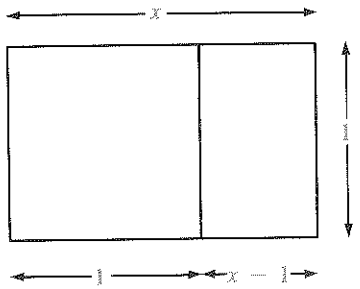
THE MATHEMATICAL MIND

The Golden Ratio

Twenty-three centuries ago Euclid posed the question, "What are the dimensions of a rectangle that has the property that when it is divided into a square and a rectangle, the smaller rectangle has the same shape as the original?"

The answer to Euclid's question can be found using this proportion.

$$\frac{\text{Length of original rectangle}}{\text{Width of original rectangle}} = \frac{\text{Length of smaller rectangle}}{\text{Width of smaller rectangle}}$$



Let the length of the original rectangle be represented by x units and let its width be 1 unit.
The side of the square will then be 1 unit, and the proportion can be written.

$$\frac{x}{1} = \frac{1}{x - 1}$$

Multiply both sides of the proportion by $x - 1$.

$$x^2 - x = 1$$
$$x^2 - x - 1 = 0$$

The positive solution of the equation $x^2 - x - 1 = 0$ is called the *golden ratio*.

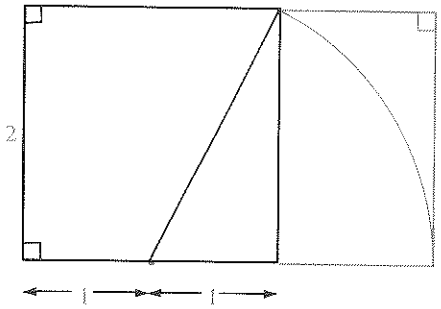
QUESTIONS

1. Use your calculator and the method of guess and check to solve the equation $x^2 - x - 1 = 0$.
2. Write down the value of the golden ratio to three decimal places.
3. Simplify this expression.

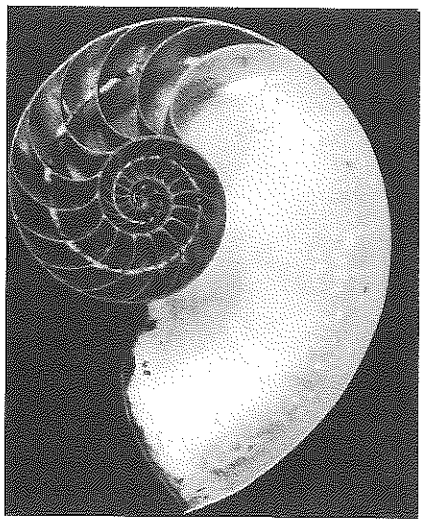
$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + 1}}}}$$

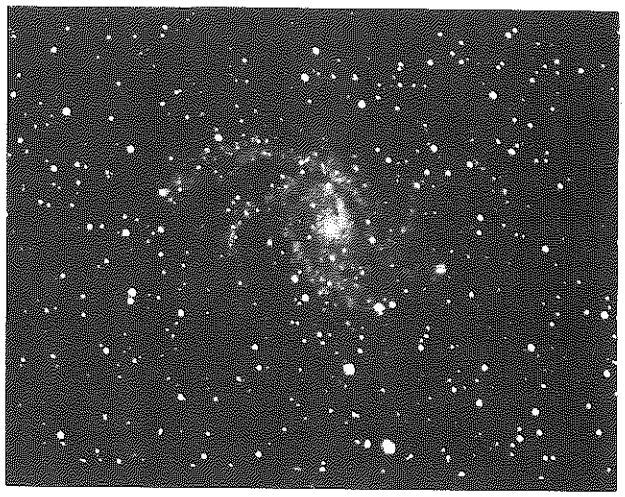
How close is this value to the value of the golden ratio?

A rectangle with a length-to-width ratio of the golden ratio is called a *golden rectangle*. Study the diagram. Then, using a pair of compasses and a straightedge, construct a golden rectangle.



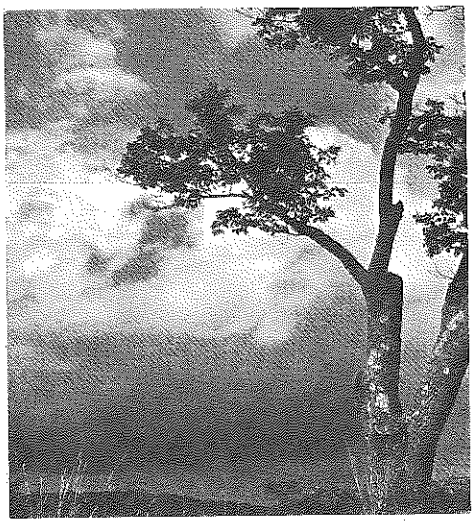
The rectangle obtained by dividing a golden rectangle into a square and a rectangle is itself a golden rectangle. Repeating this process with the small rectangle a number of times gives a set of successively smaller golden rectangles. The result of joining a set of corresponding vertices of these rectangles with a curve is a *spiral*. Such spirals can be found throughout nature, from the spiral galaxy in the heavens to the nautilus shells in the oceans.





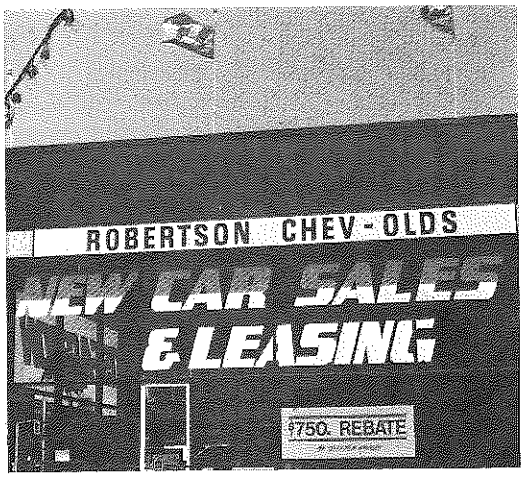
Since Euclid's time, the golden ratio has been found in mathematics, architecture, art, and nature. In fact, the golden ratio is probably second only to π in its frequency of occurrence in the mathematical sciences.

6-5 PERCENT

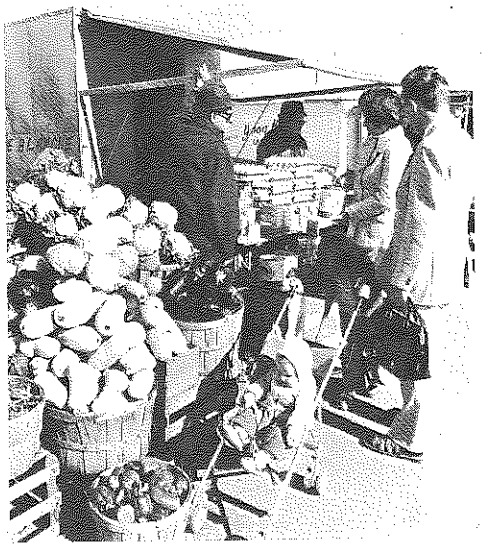


The weather forecast says that there is a 40% chance of rain today.

A store is closing down and offers everything for sale at 50% off.



A car manufacturer offers 6.9% financing to a customer who is buying a car.



A newspaper quotes a cost of living increase of 2.4% last month because of price rises in fuel and fresh produce.

A *percent* is a fraction with a denominator 100. For example,

40% is $\frac{40}{100}$ and 6.9% is $\frac{6.9}{100}$.

The word ''percent'' means ''per hundred''.

Example 1. Write each ratio as a percent.

a) $\frac{27}{100}$ b) 6:100 c) 7:25

Solution. a) $\frac{27}{100} = 27\%$

b) 6 : 100 written in fractional form is $\frac{6}{100}$, which is 6%.

c) 7 : 25 written in fractional form is $\frac{7}{25}$.

Multiply numerator and denominator by a factor that produces a denominator of 100.

$$\frac{7}{25} = \frac{7}{25} \times \frac{4}{4}$$
$$= \frac{28}{100}$$
$$= 28\%$$

Each solution could have been obtained by writing the ratio in fractional form and then multiplying by 100%.

For example, 7 : 25 can be written as $\left(\frac{7}{25}\right)100\%$ which is 28%

Here is a general rule for expressing ratios as percents.

> To change a ratio to a percent, write the ratio in fractional form and multiply by 100%.

Example 2. Write each ratio as a percent.

a) 2 : 5 b) 0.1 : 4 c) 19 : 6

Solution. Write each ratio in fractional form.

a) $\frac{2}{5} = \left(\frac{2}{5}\right)100\%$
$= 40\%$

b) $\frac{0.1}{4} = \left(\frac{0.1}{4}\right)100\%$
$= 2.5\%$

c) $\frac{19}{6} = \left(\frac{19}{6}\right)100\%$
$= \frac{950}{3}\%$
$\doteq 317\%$

Similarly, decimals can be written as percents by multiplying by 100%.

> To change a decimal to a percent, multiply the decimal by 100%.

Example 3. Express each decimal as a percent.
a) 0.64 b) 0.018 c) 0.0073 d) 2.15

Solution.
a) $0.64 = (0.64)100\%$
$= 64\%$

b) $0.018 = (0.018)100\%$
$= 1.8\%$

c) $0.0073 = (0.0073)100\%$
$= 0.73\%$

d) $2.15 = (2.15)100\%$
$= 215\%$

Conversely, to express a percent as a decimal, divide by 100%.

To change a percent to a decimal, divide the percent by 100%.

Example 4. Express each percent as a decimal.
a) 27% b) 7.5% c) 156% d) 0.9%

Solution.
a) $27\% = \dfrac{27\%}{100\%} = 0.27$

b) $7.5\% = \dfrac{7.5\%}{100\%} = 0.075$

c) $156\% = \dfrac{156\%}{100\%} = 1.56$

d) $0.9\% = \dfrac{0.9\%}{100\%} = 0.009$

Example 5. Express each percent as a fraction in lowest terms.
a) 18% b) 175% c) 12.5% d) 0.4%

Solution.
a) $18\% = \dfrac{18}{100} = \dfrac{9}{50}$

b) $175\% = \dfrac{175}{100} = \dfrac{7}{4}$

c) $12.5\% = \dfrac{12.5}{100} = \dfrac{125}{1000} = \dfrac{1}{8}$

d) $0.4\% = \dfrac{0.4}{100} = \dfrac{4}{1000} = \dfrac{1}{250}$

EXERCISES 6-5

(A)

1. Write each ratio as a percent.
   a) 7 : 100 b) 18.5 : 100 c) 57 : 100 d) 0.8 : 100
   e) 365 : 100 f) 36.5 : 100 g) 540 : 100 h) 1875 : 100

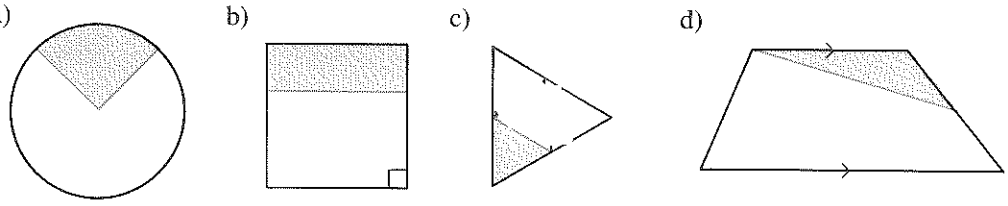
2. Express each decimal as a percent.
   a) 0.38 b) 0.57 c) 0.81 d) 0.06 e) 0.035
   f) 0.072 g) 0.091 h) 0.007 i) 0.0086 j) 0.0051
   k) 0.0007 l) 3.6 m) 3.06 n) 3.006 o) 30.6

3. Express each percent as a decimal.
   a) 24% b) 39% c) 57.4% d) 3% e) 5.8%
   f) 11.5% g) 1.6% h) 0.9% i) 137% j) 264%
   k) 375% l) 375.8% m) 0.1% n) 2.03% o) 0.25%

(B)

4. Write each ratio as a percent.
   a) 1 : 4 b) 5 : 8 c) 7 : 10 d) 11 : 20 e) 5 : 6
   f) 3 : 5 g) 8 : 5 h) 2 : 3 i) 13 : 10 j) 31 : 40
   k) 20 : 3 l) 19 : 50 m) 1 : 25 n) 7 : 40 o) 11 : 200

5. Estimate what percent of each figure is colored.
   a) b) c) d)



6. Write as a percent.
   a) one-half b) three-quarters c) seven-eighths
   d) one-hundredth e) one-thousandth f) two-thirds

7. Express each percent as a fraction in lowest terms.
   a) 26% b) 35% c) 64% d) 75%
   e) 62.5% f) 125% g) $81\frac{1}{3}\%$ h) $16\frac{2}{3}\%$
   i) 185% j) 360% k) 0.8% l) 0.125%

8. What percent of the mass of a 16 K gold ring is gold?

9. The weights of a person on Mars and on the Earth are in the ratio 2 : 5. What percent of the weight on Mars is a person's weight on Earth?

(C)

10. A fertilizer contains nitrogen, phosphorus, and potassium in the ratio 1 : 2 : 4 by mass. If 58% of the mass consists of materials other than these nutrients, what percent of the total mass of the fertilizer is phosphorus?

PROBLEM SOLVING

Construct a Table

An aircraft flies from city A to city B against the wind at an average speed of 600 km/h. On the return trip, the average speed is 1000 km/h. What is the average speed for the round trip?

Understand the problem

- What does average speed mean?
- How can it be calculated?
- Is the average speed for the return trip, the average of 600 km/h and 1000 km/h?

Think of a strategy

- Construct a table showing the time, the distance, and the average speed for each journey.

Carry out the strategy

- Write the given information in the table.

	Distance	Time	Speed
A → B			600 km/h
B → A			1000 km/h

- To complete the table, we let d represent the distance between A and B in kilometres.
- Then we use the equation: time = distance ÷ speed to calculate the times for the trips A to B and B to A. These expressions are entered in the table.

	Distance	Time	Speed
A → B	d km	$\frac{d}{600}$ h	600 km/h
B → A	d km	$\frac{d}{1000}$ h	1000 km/h

- To calculate the average speed we use the equation:
  Average speed = total distance ÷ total time

$$= \frac{2d}{\frac{d}{600} + \frac{d}{1000}}$$

Multiply numerator and denominator by 3000. Then divide numerator and denominator by d.

$$= \frac{6000}{5 + 3}$$

$$= 750$$

- The average speed for the round trip is 750 km/h.

Look back

- Is the average speed for the round trip somewhere between 600 km/h and 1000 km/h?
- Should the average speed for the round trip be closer to 600 km/h or to 1000 km/h?
- Does 750 km/h seem to be a reasonable answer?

Solve each problem

1. A boat has an average speed of 50 km/h on the first lap of a two-lap race. On the second lap it averages only 30 km/h. What is its average speed for the whole race?

2. If Mr. Swan drives at an average speed of 100 km/h he arrives at work at 09:00. If he leaves home at the same time but averages 80 km/h, he arrives at 09:06. How far does he live from work?

3. A car's cooling system contains a 25% solution of antifreeze. Half the system is drained and then topped up with pure antifreeze. What is the strength of the antifreeze in the system now?

4. Dianne rode her dirt bike up a hill and down the same distance on the other side. She rode down the hill at 4 times the speed she rode up the hill. If the entire trip took 20 min, how many minutes did it take her to ride down the hill?

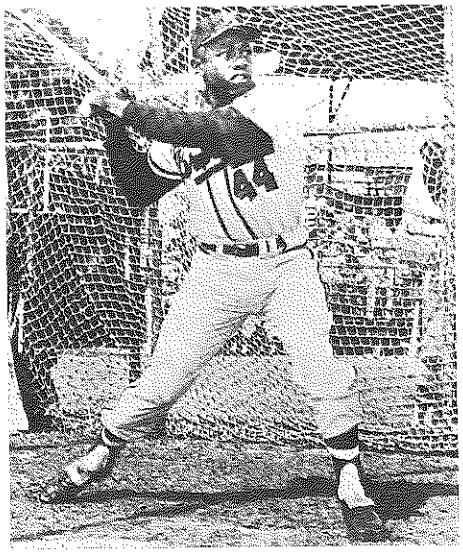
5. Marie drove her 18 wheeler 1280 km from Calgary to Winnipeg in 15.2 h. Part of the trip she drove in a snow storm at an average speed of 60 km/h. The rest of the time she drove at 100 km/h. How far did she drive in the storm?

6. Two people, 60 km apart, start cycling towards each other at the same time. One person cycles at 18 km/h. How fast must the other person cycle if the people meet in 1.5 h?

7. Car A and car B leave Halifax on the same road, 1 h apart. Car A leaves first and travels at a steady 80 km/h. How fast must car B travel to overtake car A in 4 h?

MATHEMATICS AROUND US

Election to the Baseball Hall of Fame



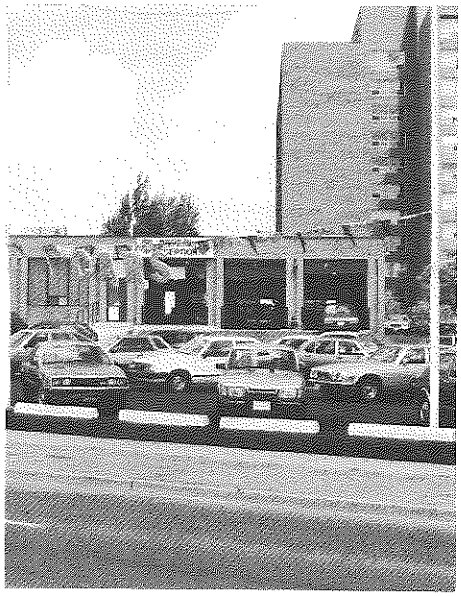
To be elected to the Baseball Hall of Fame a player must receive at least 75% of the votes cast by sportswriters who have had 10 or more years of experience. Each sportswriter can vote for up to 10 players.

QUESTIONS

1. In 1982 Henry Aaron was elected to the Hall of Fame with the highest percent of votes since 1936, the year the Hall of Fame started. He received 406 votes out of a total of 415 ballots.
   a) What percent of the votes did Henry Aaron receive?
   b) Frank Robinson, with 370 votes, was also elected in 1982. What percent of the votes did Frank Robinson receive?
   c) What was the minimum number of votes needed for election in 1982?
2. In 1986 there were 425 ballots. Willie McCovey received 81.4% of the votes. Billy Williams just missed being elected, with 74.1% of the votes.
   a) How many votes did each player receive?
   b) How many more votes did Billy Williams need to be elected?
3. Three players were elected to the Hall of Fame in 1936: Ty Cobb with 98.2%; and Babe Ruth and Honus Wagner, each with 95.1%. Ty Cobb received 222 votes.
   a) How many ballots were cast that year?
   b) How many votes did Babe Ruth and Honus Wagner receive?

6-6 APPLICATIONS OF PERCENT

Sita wants to buy a car. On the windshield of a car she sees a price of $8750 with an additional sign, "15% off sticker price". Sita calculates what the car will cost.



The reduction in price is 15%.

The sale price is 100% − 15%, or 85% of the sticker price of $8750.

$$85\% \text{ of } \$8750 = \frac{85}{100}(\$8750)$$
$$= \$7437.50$$

Sita knows that she will have to pay provincial sales tax which is currently 7%. The cost will be $7437.50 plus 7% of $7437.50.

$$\text{Total cost} = 107\% \text{ of } \$7437.50$$
$$= 1.07(\$7437.50)$$
$$= \$7958.13$$

Sita will have to pay $7958.13 for the car.

Example 1. a) Calculate 16% of 85.
b) 18% of a number is 50. What is the number?
c) What percent of 65 is 0.13?

Solution. a) $16\% \text{ of } 85 = \frac{16}{100}(85)$
$= 13.6$

b) 18% of a number is 54.

1% of the number is $\frac{54}{18}$.

100% of the number is $\frac{54}{18}(100)$ or 300.

c) To find what percent of 65 is 0.13, express 0.13 as a fraction of 65, that is, $\frac{0.13}{65}$.

Multiply the fraction by 100%.

$$\left(\frac{0.13}{65}\right)100\% = 0.2\%$$

0.13 is 0.2% of 65.

Example 2. Assume that the gas company gets the increase that it wants. A family's natural-gas bill last year was approximately \$860. Calculate the gas bill for this year if the same amount of gas is used.

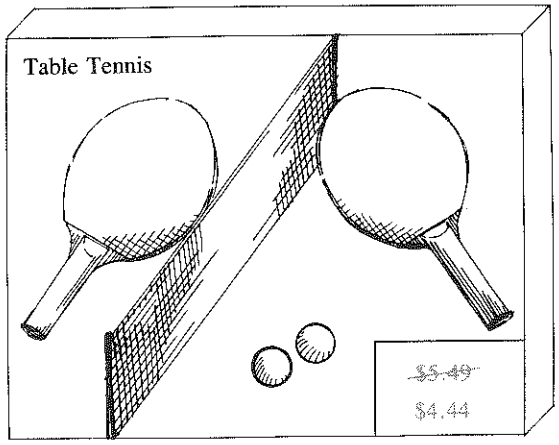
Gas company wants 6% increase in rates

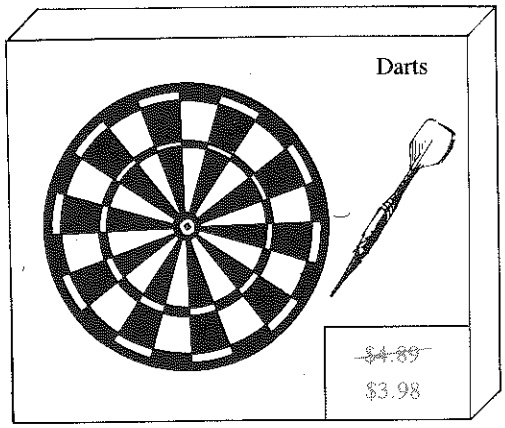
Solution This year's gas bill will be 106% of last year's bill.

$$\begin{aligned} 106\% \text{ of } 860 &= 1.06(860) \\ &= 911.60 \end{aligned}$$

This year's gas bill will be about \$910.

Example 3. Which game has the greater percent reduction?





Solution. To find the percent reduction, express the reduction as a percent of the original price.

$$\begin{aligned} \text{Reduction in price of table-tennis set} &= \$5.49 - \$4.44 \\ &= \$1.05 \end{aligned}$$

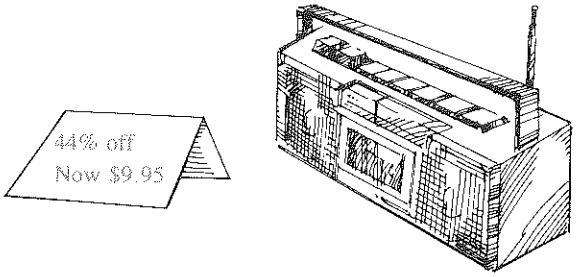
$$\begin{aligned} \text{Percent reduction of table-tennis set} &= \left(\frac{1.05}{5.49}\right)100\% \\ &\doteq 19.1\% \end{aligned}$$

$$\begin{aligned} \text{Reduction in price of dart game} &= \$4.89 - \$3.98 \\ &= \$0.91 \end{aligned}$$

$$\begin{aligned} \text{Percent reduction of dart game} &= \left(\frac{0.91}{4.89}\right)100\% \\ &\doteq 18.6\% \end{aligned}$$

The table-tennis set has the greater percent reduction.

Example 4. What was the original price of this radio?



Solution. The reduction in price is 44%.
So, \$9.95 must represent 100% − 44%, or 56% of the original price.

1% of the original price is $\frac{\$9.95}{56}$.

100%, the original price, is $\frac{\$9.95}{56}(100)$ or about \$17.77.

The radio was originally \$17.77.

Interest is the money paid for the use of money. If you have a savings account, the bank pays you interest for the use of your savings. If you borrow money from the bank, it charges you interest for the use of its money. The interest rate is always expressed as a percent.

Tara received \$150 from her grandparents as a birthday present. She deposited it into her daily-interest savings account, which pays interest at a rate of 8% per annum.

The interest is calculated daily and is added to the account at the end of each month. How much interest has the money earned in 30 days?

The yearly interest is 8% of \$150, or 0.08(150).

The daily interest is $\frac{1}{365}$ of this amount, or $\frac{1}{365}(0.08)(150)$.

After 30 days, the interest is $\frac{30}{365}(0.08)(150)$ or approximately 0.99.

After 30 days, \$150 has earned \$0.99 interest.

When interest is calculated in this way, it is called *simple interest*. Simple interest can be calculated using this formula.

$I = Prt$

I is the interest in dollars.
P is the principal, the money saved or borrowed.
r is the annual interest rate expressed as a decimal.
t is the time in years for which the money is saved or borrowed.

Example 5. Find the simple interest.
a) On a \$324 credit-card bill for 28 days at 21% per annum
b) \$6500 in a daily-interest savings account for 25 days at $11\frac{1}{2}\%$ per annum

Solution. a) Use the formula.
The rate is 21% or 0.21. The time is 28 days or $\frac{28}{365}$ years.

$$I = Prt$$
$$= 324(0.21)\left(\frac{28}{365}\right) \quad \text{Use a calculator.}$$

Key in: [3] [2] [4] [×] [·] [2] [1] [×] [2] [8] [÷] [3] [6] [5]
to display 5.2195068
The interest is \$5.22.

b) The rate is $11\frac{1}{2}\%$ or 0.115. The time is 25 days or $\frac{25}{365}$ years.

$$I = Prt$$
$$= 6500(0.115)\left(\frac{25}{365}\right)$$
$$\doteq 51.20$$

The interest is \$51.20.

EXERCISES 6-6

(A)

1. Find.
   a) 25% of 40 b) 20% of 40 c) 0.6% of 150 d) 5% of 35
   e) 109% of 75 f) 4% of 150 g) 0.7% of 95 h) 65% of 18

2. Determine the number in each statement.
   a) 50% of a number is 10. b) 20% of a number is 3.
   c) 40% of a number is 10. d) 75% of a number is 30.
   e) 60% of a number is 42. f) 15% of a number is 15.
   g) $66\frac{2}{3}\%$ of a number is 18. h) 10% of a number is 8.
   i) 104% of a number is 26. j) 130% of a number is 91.

3. a) What percent of 80 is 16? b) What percent of 135 is 15?
   c) What percent of 75 is 125? d) What percent of 50 is 45?
   e) What percent of 144 is 18? f) What percent of 81 is 270?
   g) What percent of 1900 is 1.9? h) What percent of 6000 is 3?

(B)

4. Find the simple interest.
   a) On \$1200 at 16% per annum for 3 months
   b) On \$8500 at 18% per annum for 6 months
   c) On \$3000 at 15% per annum for 30 days

5. A ten-speed bicycle regularly sells for $227.50. What will it cost during a "15% off" sale?

6. Skis are being sold at a discount of 45%. What will be the cost of a pair of skis that regularly sells for $180?

7. Food costs for the coming year are estimated to rise by 11.7%. What will be the coming year's food costs for a family who spent $8400 last year?

8. This year, AKA Ltd. hopes that its sales income will be 160% of last year's sales of $2 500 000. If the company succeeds, what will its sales income be?

9. In April, the unemployment figure was 800 000 people. In May, there was a modest 0.16% decrease. How many people apparently found work in May?

10. In all of William Shakespeare's works, he used 31 534 different words. Of these, 14 356 were used only once. What percent of the words did he use only once?

11. During a "20% off" sale, a clock radio is priced at $29.95. What is its regular price?

12. A calculator is priced at $9.98 during a "25% off" sale. What is its regular price?

13. Express each reduction as a percent, to 1 decimal place, of the original price.
    a) A TV set regularly priced at $540 is selling for $499
    b) An overcoat regularly priced at $195 is selling for $156

14. Sterling silver is an alloy of silver and copper in the ratio 37 : 3.
    a) What percent of a sterling silver bracelet is pure silver?
    b) If the bracelet has a mass of 30 g, how much silver does it contain?

15. Long distance telephone rates are reduced by 35% between 6 P.M. and 11 P.M., and by 60% between 11 P.M. and 8 A.M.
    a) A 5 min call from Ottawa to Charlottetown at 10 A.M. costs $4.30. Calculate the cost of a 5 min call at: i) 8 P.M. ii) 7 A.M.
    b) A 5 min call from Montreal to Vancouver at 9 P.M. costs $3.06. Calculate the cost of a 5 min call at: i) noon ii) 2 A.M.

16. a) In the news item, what percent, to 1 decimal place, of the cars stopped yesterday had defective equipment?
    b) Estimate the number of cars found with defective equipment since the campaign began. What assumption are you making?

Police checks nab 187

Police spot-check crews stopped 860 vehicles yesterday and found 187 with defective equipment, in the holiday safety traffic blitz across the city.

A total of 28 759 cars and trucks have been stopped for safety checks since the campaign began.

17. A pair of skis is priced at $185. Find the cost:
    a) when a 7% sales tax is added b) with a 15% discount and a 7% sales tax.

18. A dress that sells for $80 is placed on sale at "15% off". After two weeks on the rack, the current selling price is reduced another 10%. What is the new selling price?

19. The specifications by mass for the alloy, phosphor bronze, are 85% copper, 7% tin, 0.06% iron, 0.2% lead, 0.3% phosphorus, and the remainder zinc.
    a) What percent of phosphor bronze is the zinc content?
    b) What mass of each element is needed to make 1 t of phosphor bronze?

20. If a worker receives a cut of 20% in salary, what percent increase must he get to regain his original salary?

21. The perimeter of a rectangle doubles such that its length and width remain in the same ratio. By what percent has the area increased?

22. In a recent year, Statistics Canada reported that 8.9 million persons were employed and 810 000 were unemployed.
    a) What was the rate of unemployment, to 1 decimal place?
    b) If the size of the work force does not change, how many of the unemployed need to find jobs to bring the unemployment rate down to 4%?

23. Energy-conservation experts report that there is a 4.5% reduction in home-heating costs for every 1°C reduction in house temperature. A house is kept at a constant 20°C day and night. What percent reduction in heating costs should occur if the house were kept at 18°C from 07:00 to 22:00 and 15°C from 22:00 to 07:00?

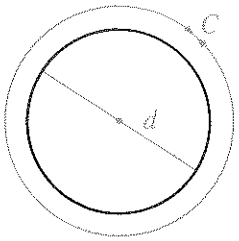
24. A company's profit is 5.4% of its sales. It must pay 48% corporate taxes on its profit. It always pays 60% of its after-taxes profit to its stockholders as dividends, and retains the balance as a reserve. How much did the company retain as a reserve in a year when its sales were $10 000 000?

1. a) Draw a rectangle and measure its dimensions.
   b) Calculate the area of the rectangle.
   c) Calculate the change in area after each change in the dimension(s) listed below.
   d) Express each change in area as a percent of the original area.
      i) Increase the length by 100%.
      ii) Decrease the width by 50%.
      iii) Increase the length by 50% and increase the width by 50%.
      iv) Decrease the length by 25% and decrease the width by 25%.
      v) Increase the length by 75% and decrease the width by 75%.
      vi) Increase the length by 75% and decrease the width by 25%.

2. If you knew the percent change(s) in the dimension(s) of a rectangle, how could you find the percent change in its area?

6-7 THE MOST FAMOUS RATIO IN MATHEMATICS

One ratio, above all others, has captured the interest of mathematicians through the ages. This is the ratio of the circumference C of a circle to its diameter d. The value of this ratio is the same for all circles and is denoted by the Greek letter π.



For any circle, $C : d = \pi : 1$ or $\frac{C}{d} = \pi$ or $C = \pi d$

In January 1986, a Cray 2 supercomputer calculated π to 29 360 128 decimal places. The calculation took 28 h of computer time. Calculations of π are done to check the calculating ability of a computer. A $\boxed{\pi}$ key on a calculator will give its value to 8 (or 10) decimal places.
Here is the value of π to 50 decimal places.
3.141 592 653 589 793 238 462 643 383 279 502 884 197 169 399 375 10
If your calculator does not have a $\boxed{\pi}$ key, use the approximate value of 3.14.

Example 1. A tree trunk with a circular cross section has a circumference of 172 cm. Find the diameter of the trunk, to the nearest centimetre.

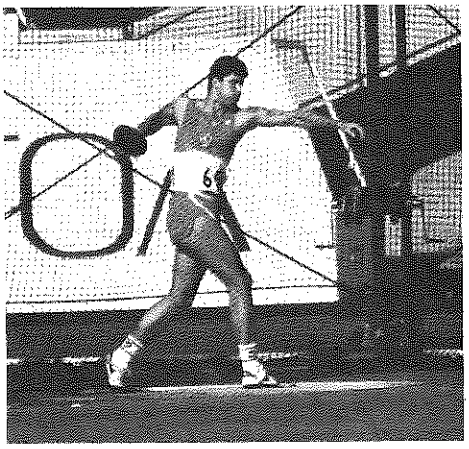
Solution. $C = \pi d$
Divide both sides of the equation by π.
$\frac{C}{\pi} = d$
Substitute 172 for C.
$d = \frac{172}{\pi}$
$\doteq 54.7$
The diameter of the trunk is about 55 cm.

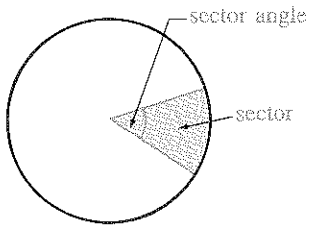
For a circle of radius r and area A, $A : r^2 = \pi : 1$ or $\frac{A}{r^2} = \pi$ or $A = \pi r^2$

Example 2. A pipe has a circular cross section of 18 cm^2. Find the radius of the cross section, to the nearest tenth of a centimetre.

Solution. $A = \pi r^2$
Divide both sides by π.
$\frac{A}{\pi} = r^2$
Substitute 18 for A.
$r^2 = \frac{18}{\pi}$
$r \doteq \pm \sqrt{\frac{18}{\pi}}$
$\doteq 2.4$ r cannot be negative.
The radius of the pipe is about 2.4 cm.

When throwing a discus, an athlete must stand inside a circle of radius 1.25 m. The region into which the discus is thrown is defined by two lines, at an angle of 40°, diverging from the centre of the circle. That part of the region that lies inside the throwing circle is called a *sector* of the circle. The angle between the lines defining the sector is called the *sector angle*.





The area of a sector is proportional to its sector angle.
The area of a circle corresponds to a sector angle of 360°.
These statements can be expressed as a proportion.

$$\frac{\text{area of sector}}{\text{sector angle}} = \frac{\text{area of circle}}{360°}$$

This proportion can be rearranged.

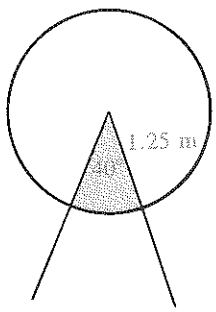
$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{sector angle}}{360°}$$

This proportion can be used to find the area of a sector, given its sector angle.

Example 3. When throwing the discus, what area of the region into which the discus is thrown lies inside the throwing circle? Give the answer to two decimal places.

Solution. Draw a diagram.
The required area is a sector of a circle with radius 1.25 m.
The sector angle is 40°. Since the areas are proportional to the corresponding sector angles,

$$\frac{\text{sector area}}{\text{circle area}} = \frac{\text{sector angle}}{360°}$$



The circle area is $\pi(1.25)^2$.

$$\frac{\text{sector area}}{\pi(1.25)^2} = \frac{40°}{360°}$$

$$\text{sector area} = \left(\frac{40}{360}\right)(\pi)(1.25)^2$$

$$\doteq 0.545$$

The sector area is about 0.55 m^2.

EXERCISES 6-7

(A)

1. For each circumference given below, calculate the corresponding diameter. Express the answers in fractional form, where appropriate.

   a) 5 cm b) 3π cm c) π^2 cm d) $\frac{1}{\pi}$ cm

2. Find the radius of each circle with the given area. Give the answer to 1 decimal place, where necessary.

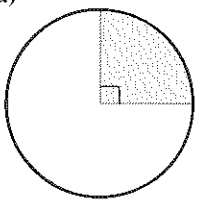
   a) 78.5 cm^2 b) 300 mm^2 c) 144π mm^2 d) $\frac{64}{\pi}$ mm^2

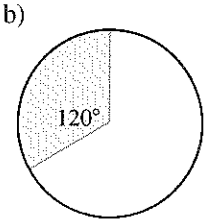
3. Find the sector angle for each sector.

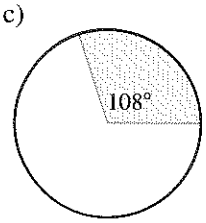
   a) $\frac{1}{6}$ of a circle b) $\frac{2}{3}$ of a circle c) $\frac{5}{8}$ of a circle

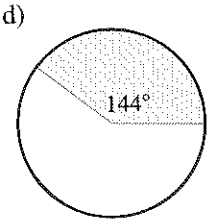
(B)

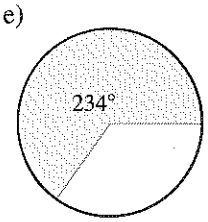
4. Write the area of each shaded sector as a percent of the area of the circle. Give the answer to 1 decimal place, where necessary.

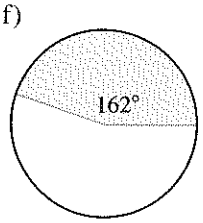
a) 

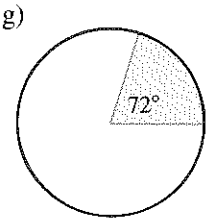
b) 

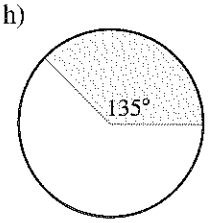
c) 

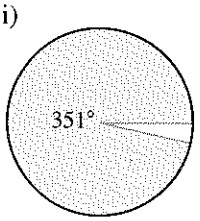
d) 

e) 

f) 

g) 

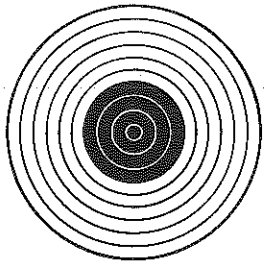
h) 

i) 

5. In the shot put event, the shot is thrown from within a circle of diameter 2.14 m.
   a) Find the area of the shot put circle.
   b) The shot must be thrown within a sector having a sector angle of 40°. What area of the region into which the shot is put lies inside the throwing circle?

6. In the sport of pistol shooting, the target consists of ten concentric rings, as shown. The chart shows the outside diameter of each ring.

Ring	Diameter
10	50 mm
9	100 mm
8	150 mm
7	200 mm
6	250 mm
5	300 mm
4	350 mm
3	400 mm
2	450 mm
1	500 mm

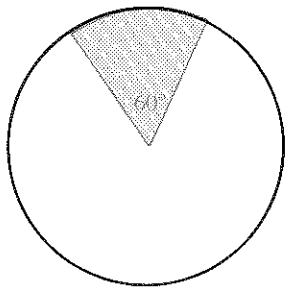


a) Find the area of the 10 ring, to the nearest square millimetre.
b) Find the total target area, to the nearest square centimetre.
c) Express the area of the 10 ring as a percent of the total target area.
d) Find the area of the 1 ring, to the nearest square centimetre.
e) Express the area of the 1 ring as a percent of the total target area.
f) Express the area of the 10 ring as a percent of the area of the 1 ring.

7. A thread of length 36 m is wound around a cylindrical spool of circular cross-section with diameter 2 cm. How many complete revolutions of the spool are needed to unravel all the thread?

8. Through what angle does the hour hand of a clock turn between noon and 05:00?

9. The length of an arc contained in a sector is proportional to the sector angle. The sector angle of the colored sector in the diagram is 60°. Since 60° is $\frac{1}{6}$ of 360°, the colored arc is $\frac{1}{6}$ of the circumference.



a) Express the arc length corresponding to each sector angle, as a fraction of the circumference.
i) 90° ii) 45° iii) 270° iv) 120°
b) Calculate each arc length in part a) for a circle with radius 12 cm. Give the answers to the nearest tenth of a centimetre.
c) What sector angle corresponds to an arc length that is $\frac{3}{8}$ of the circumference of the circle?
d) If you knew the sector angle and the circumference, how could you find the length of the arc?

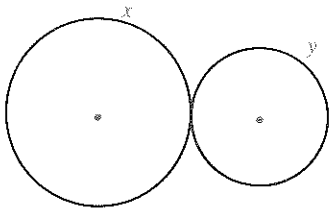
10. The diameter of a spherical weather balloon increases to 1.5 times its initial size by the time it ascends to the cloud base. Find the factors by which these measurements have increased.
    a) the circumference b) the cross-sectional area

11. Which would drain a tank faster — one drain 4 cm in diameter or two drains each 2 cm in diameter? How many times faster would it drain?

12. Calculate these ratios concerning the volumes of a cylinder, a sphere, and a cone.
    a) $\frac{V\text{ (cylinder)}}{V\text{ (sphere)}}$ when the radius of both the cylinder and the sphere is r and the height of the cylinder is $2r$
    b) $\frac{V\text{ (cylinder)}}{V\text{ (cone)}}$ when the cylinder and the cone have the same base and the same height
    c) $\frac{V\text{ (sphere)}}{V\text{ (cone)}}$ when the radius of both the sphere and the cone is r and the height of the cone is $2r$

13. Find the angle between the hands of a clock at each time.
    a) 01:00 b) 02:20 c) 03:30 d) 04:45

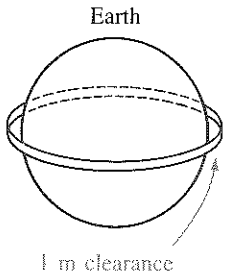
14. Two circles, with circumferences x and y, are touching. What is the distance between their centres?



15. A ball just fits into a can, touching the bottom, the sides, and the lid. If it is inserted into the can when the can is full of water, what percent, to 1 decimal place, of the water is displaced?

16. One of Jupiter's moons is three times the diameter of the Earth's moon. How many times greater is the mass of Jupiter's moon than the Earth's? What assumption are you making?

17. Imagine that a steel ring circles the Earth, just touching it at the equator. What length would need to be added to the ring to raise it 1 m from the surface all the way around?



Review Exercises

1. In a bracelet of 18 K gold, the ratio of gold to copper is 18 : 6.
   a) Write this ratio in lowest terms.
   b) Find the mass of gold in a bracelet with a mass of 72 g.

2. State which is the greater ratio.
   a) $\frac{7}{9}$ or $\frac{10}{13}$ b) 5 : 6 or 29 : 34 c) 8 : 17 or 11 : 25 d) 21 : 8 or 24 : 9

3. Find the value of each letter.
   a) $\frac{7}{10} = \frac{m}{15}$ b) $\frac{n}{4} = \frac{13}{8}$ c) $\frac{39}{a} = \frac{13}{3}$ d) $\frac{17}{21} = \frac{51}{b}$

4. Bill contributes \$3.50 and Laura \$6.50 for the purchase of a \$10 lottery ticket. It is drawn for a prize of \$125 000. How should the money be divided?

5. The partners in a business agree to share the profits in the ratio 3 : 8 : 4. How much does each partner receive from a total profit of \$45 000?

6. The shadow of a tree is 26.5 m long when that of a vertical metre rule is 58 cm long. How high is the tree?

7. On a map, two towns are 85 mm apart. If the scale of the map is 1 : 1 500 000, what is the actual distance between the towns?

8. The ferry crossing of the St. Lawrence River at Baie Comeau is a distance of 61 km. What will this distance be on a map drawn to a scale of 1 : 200 000?

9. A person's heart beats 13 times in 10 s.
   a) How many times does it beat in 1 h?
   b) How long does it take to beat: i) 1000 times ii) 1 000 000 times?

10. A car's rate of fuel consumption is 8.5 L/100 km.
    a) How much fuel is needed to travel 350 km?
    b) If the car's tank holds a maximum of 58 L of fuel, will one tankful be enough to travel 687 km from Corner Brook to St. John's in Newfoundland?

11. A person's rate of pay increases from \$5.00/h to \$7.00/h. What is the percent increase?

12. What is the sale price of a \$165 bicycle selling at a 15% discount?

13. The price of oil is \$150/m^3 and the price is raised 10% every year. What will be its price at the end of three years?

14. Find the simple interest on each amount.
    a) A \$215 credit-card bill for 1 month at 18% per annum
    b) \$150 in a daily-interest savings account for 20 days at $12\frac{1}{4}$% per annum

15. A tire has a diameter of 58 cm. Find its circumference to the nearest centimetre.

16. A sector of a circle of radius 14.5 cm has a sector angle of 75°. Calculate:
    a) the arc length of the sector
    b) the area of the sector.

1. Solve.
   a) $7 + y = -6$ b) $9 - t = -13$ c) $-18 = -6p$
   d) $11 = 2.25w$ e) $\frac{1}{7}x = -2$ f) $\frac{r}{3} = \frac{1}{5}$
   g) $-2.75x = 16.5$ h) $5x - 2 = 13$ i) $3 - 3y = -3$

2. Solve.
   a) $5x - 3 = 2x + 6$ b) $-3y + 5 = 2y - 10$
   c) $3(r - 1) = -2(r + 8)$ d) $5(7x - 3) = 17x - (2 - 5x)$
   e) $\frac{4}{3} - \frac{1}{2}x = \frac{2}{3}x$ f) $1.25 - 0.8x = 0.4x - 0.19$
   g) $2(3a - 5) = 7(2a + 3) - 3$ h) $\frac{3}{2}p + \frac{1}{4} = \frac{3}{4}p - \frac{7}{8}$

3. Solve and check.
   a) $12 - 4a + 7 = 2a + 31$ b) $-4(x - 3) = 2(x + 9)$
   c) $7(2s - 3) + 11 = 2(4s + 7)$ d) $1.4x - 3.6 = 0.4(2x + 1.5)$

4. The cost C dollars of printing a school yearbook is given by this formula.
   $C = 2750 + 9.5n$, where n is the number of books printed
   a) Find the cost of printing:
      i) 600 copies ii) 940 copies iii) 1050 copies.
   b) How many copies can be printed for \$13 010.00?

5. Express each statement as an equation and solve it.
   a) A number multiplied by seven equals fifty-six.
   b) A number divided by fifteen equals seventy-five.
   c) When twenty-nine is subtracted from a number, the result is negative two.

6. A father is three times as old as his daughter. In 12 years time, he will be only twice as old. What are their ages now?

7. Two numbers differ by 3. The sum of the larger and one-fourth the smaller is 13. Find the numbers.

8. Simplify.
   a) $2^3 + 2^4$ b) $2^3 \times 2^4$ c) $(-3)^2 + (-3)^3$ d) $(-2)^7 \div (-2)^4$
   e) $\left(\frac{3}{4}\right)^{-1} + \left(\frac{3}{4}\right)^{0}$ f) $2^{-3} \div 2^{-4}$ g) 3^{-2} h) $\left(-\frac{2}{3}\right)^{-2}$

9. If $m = 3$ and $n = -2$, find the value of each expression.
   a) $2m^2$ b) $-4n^3$ c) $8m^2 - 5n^2$ d) $(m + n)^3$
   e) $(m - n)^3$ f) $\frac{5m^2 - 3n^2}{m - n^3}$ g) $\frac{3(m - n)^2}{m^2 + n}$ h) $5n^3 - 2m^2$

10. Simplify.
    a) $x^3 \times x^7$ b) $x^{14} \div x^8$ c) $-3p^2 \times 7p^{-9}$
    d) $24m^{-3} \div (-8m^{11})$ e) $(3x)^2 \times (2x)^3$ f) $(4y^3)^2$
    g) $35a^3 \div 5a^{-7}$ h) $3x^5 \times 12x^{-7} \div 4x^{-3}$ i) $\frac{18s^4 \times 4s^{11}}{12s^7}$

11. Write in scientific notation.
a) 15 000 b) 2 700 000 c) 21 d) 0.000 016 e) 0.000 37 f) 0.19

12. Simplify.
a) $\sqrt{2500}$ b) $-\sqrt{1.96}$ c) $\sqrt{2\ 250\ 000}$
d) $2\sqrt{49} - 3\sqrt{16}$ e) $9\sqrt{64} + 2\sqrt{9}$ f) $7\sqrt{81} - 5\sqrt{100}$

13. Evaluate, to 2 decimal places, if $x = -3$ and $y = 2$.
a) $\sqrt{-14x}$ b) $\sqrt{2x^2 + 6y^3 - 2}$ c) $-\sqrt{7x^2 - 4y^2 + 8}$

14. The value of a \$3200 investment certificate is given by this formula.
$V = 3200(1.08)^n$, where n is the number of years after purchase
Find the value of the certificate after 5 years.

15. Solve each proportion.
a) $\frac{r}{3} = \frac{8}{15}$ b) $\frac{16}{25} = \frac{48}{x}$ c) $\frac{12}{39} = \frac{s}{13}$ d) $\frac{19}{t} = \frac{57}{42}$

16. Jacques contributes \$4.00 and Jeanne \$6.00 for the purchase of a lottery ticket. It is drawn for a prize of \$25 000. How should the money be divided?

17. On a map, two cities are 120 mm apart. The scale of the map is 1 : 1 250 000. What is the actual distance between the cities?

18. In an orienteering exercise, Sharon's location is 3.2 cm from town A on the map. If the actual distance is 24 km, what is the scale of the map?

19. a) 24% of a number is 18. What is the number?
b) What percent of 1210 is 484?

20. A calculator regularly priced at \$15.75 is on sale for \$12.60. What is the percent of discount?

21. A television that lists for \$480 is on sale at a 15% discount. What is the sale price?

22. Find the simple interest on:
a) a loan of \$650 for 3 months at 14% per annum
b) a deposit of \$125 for 90 days at 6.5% per annum.

23. A patio table has a circumference of 353 cm. Find its diameter.

24. A pizza measuring 36 cm in diameter is cut into 6 equal sections.
a) Find each sector angle.
b) Find the arc length of each sector.
c) Find the area of each sector.

25. One sector of a circle graph has a sector angle of 80°. The graph has a radius of 12.5 cm.
a) Find the arc length of the sector.
b) Find the area of the sector.