4 Solving Equations

If a car's rate of fuel consumption is known, how can the fraction of highway driving be determined? (See Section 4-5.)

4-1 SOLVING SIMPLE EQUATIONS

The amount of sleep, n hours, that a 15-year-old person needs is given by this equation.

$15 = 34 - 2n$

To find how much sleep is needed at this age, a value must be found for n which *satisfies* this equation. There is exactly one value of n that will make both sides of the equation the same. Finding this value is called *solving* the equation.

The right side of the equation must equal the left side.
So, 34 minus twice a number must equal 15.
We know that 34 minus 19 equals 15.
Hence, twice the number must equal 19.
So, the number must be $\frac{19}{2}$ or 9.5.

A 15-year-old person needs 9.5 h sleep.
When an equation is solved in this way, it is called *solving by inspection.*

Example 1. Solve by inspection.
a) $56 - n = 21$ b) $7d = 28$

Solution. a) $56 - n = 21$
This means, "56 less some number is 21."
The number that is subtracted from 56 to give 21 is 35.
Hence $n = 35$

b) $7d = 28$
This means, "7 times some number is 28."
The number that is multiplied by 7 to give 28 is 4.
Hence $d = 4$

Only very simple equations should be solved by inspection. Another method of solving equations is by *systematic trial*. Systematic trial means substituting values for the variable until the value which satisfies the equation is found.

Example 2. Solve by systematic trial. $54 - 7y = 26$

Solution. A value of y must be found such that $54 - 7y$ equals 26.

Suppose $y = 5$, $54 - 7y = 54 - 7(5)$
$= 54 - 35$
$= 19$

When $y = 5$, the value of $54 - 7y$ is less than 26.

Try $y = 3$. $54 - 7y = 54 - 7(3)$
$= 54 - 21$
$= 33$

When $y = 3$, the value of $54 - 7y$ is greater than 26.

Try $y = 4$. $54 - 7y = 54 - 7(4)$
$= 54 - 28$
$= 26$

The solution is $y = 4$.

The method of systematic trial can be a long one. It is the method used by computers to solve more complicated equations.

EXERCISES 4-1

(A)

1. Solve by inspection.
   a) $x + 17 = 32$ b) $29 - x = 12$ c) $x - 7 = 27$ d) $x + 26 = 61$
   e) $43 + x = 79$ f) $11 - x = 11$ g) $15 - x = 0$ h) $20 + x = 15$
2. Solve by inspection.
   a) $4z = 24$ b) $7s = -63$ c) $12q = 132$ d) $\frac{1}{3}y = 12$
   e) $8v = 4$ f) $0.5w = 25$ g) $\frac{1}{4}x = \frac{1}{2}$ h) $0.7t = 3.5$
3. Solve by inspection.
   a) $m - 3.5 = 5$ b) $-8 + x = 12$ c) $t + 2.4 = 5.4$ d) $-9 + q = 7$
   e) $w - 3.6 = 5$ f) $1.3 + z = 3.3$ g) $2.5 - a = 1.5$ h) $1.1 - x = 0.6$
4. Solve by systematic trial.
   a) $3 + 2n = 11$ b) $7 + 3m = 13$ c) $7x - 5 = 30$ d) $4c - 1 = 23$
   e) $24 - 3y = 15$ f) $9k - 27 = 36$ g) $7m - 99 = 6$ h) $7 + 13d = 72$

(B)

5. Solve.
   a) $2x + 7 = 17$ b) $28 - 5m = 18$ c) $6a - 4 = 20$
   d) $9 + 3y = 57$ e) $8s - 7 = 153$ f) $11t + 9 = 130$
   g) $40 - 13v = 1$ h) $110 - 9u = 2$ i) $15 = 3 - 2x$
   j) $-27 = 4a + 1$ k) $31 = -2 - 3y$ l) $-40 = -4 + 4z$

6. The number of hours of sleep that an 18-year-old person needs is given by the value of n in the equation $18 = 34 - 2n$. Solve the equation for n.
7. The rate, in chirps per minute, at which a cricket chirps at 30°C is given by the value of r in the equation $210 = r + 28$. Solve the equation for r.
8. The distance, in kilometres, that a taxi travels for a fare of \$9.65 is given by the value of d in the equation $9.65 = 1.40 + 0.75d$. Solve the equation for d.

4-2 ISOLATING THE VARIABLE

An equation is a mathematical sentence that uses an equals sign to relate two expressions.

To solve an equation, it is helpful to think of a level balance. The masses in each pan can be changed but as long as the total masses on both sides are the same, the balance remains level. The same rule applies to equations.

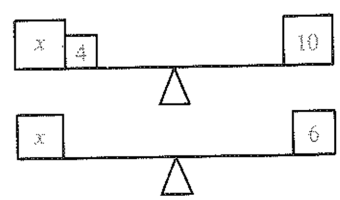
> Whatever change is made to one side of an equation must also be made to the other side.

These changes are made to reduce an equation to its solution in the form $x = a$. In this form, the variable is said to be *isolated*.

Example 1. Solve. $x + 4 = 10$

Solution. $x + 4 = 10$ means $x + 4$ balances 10.
Subtract 4 from both sides to isolate x.

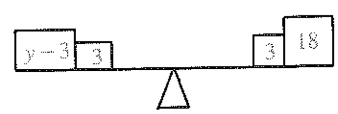
$$x + 4 - 4 = 10 - 4$$
$$x = 6$$



Example 2. Solve. $y - 3 = 18$

Solution. $y - 3 = 18$ means $y - 3$ balances 18.
Add 3 to both sides to isolate y.

$$y - 3 + 3 = 18 + 3$$
$$y = 21$$



> To solve an equation in the form $x + a = b$, where a and b are numbers, isolate x by subtracting a from both sides.
>
> To solve an equation in the form $x - a = b$, where a and b are numbers, isolate x by adding a to both sides.

Example 3. Solve.

a) $\frac{z}{3} = 10$ b) $-2p = 12$

Solution.

a) $\frac{z}{3} = 10$

Multiply both sides by 3 to isolate z.

$\frac{z}{3}(3) = 10(3)$

$z = 30$

b) $-2p = 12$

Divide both sides by -2 to isolate z.

$\frac{-2p}{-2} = \frac{12}{-2}$

$p = -6$

> To solve an equation in the form $\frac{x}{b} = c$, where b and c are numbers ($b \neq 0$), isolate x by multiplying both sides by b.
>
> To solve an equation in the form $bx = c$, where b and c are numbers ($b \neq 0$), isolate x by dividing both sides by b.

EXERCISES 4-2

(A)

1. Solve.
   a) $x + 5 = 11$ b) $z - 3 = 10$ c) $y + 7 = 16$ d) $m - 4 = 9$
   e) $a - 11 = 25$ f) $x + 17 = 23$ g) $w - 23 = 61$ h) $p + 19 = 47$
2. Solve.
   a) $\frac{x}{5} = 2$ b) $3x = 21$ c) $\frac{z}{3} = -4$ d) $5w = 35$ e) $\frac{1}{9}m = 5$
   f) $6x = 54$ g) $11a = 88$ h) $\frac{y}{4} = -8$ i) $\frac{1}{3}p = 18$ j) $7b = 98$
   k) $8a = 128$ l) $-7a = 91$ m) $\frac{c}{10} = 80$ n) $\frac{x}{12} = 132$ o) $\frac{1}{8}x = -9$
3. Solve.
   a) $m + 13 = 9$ b) $x - 4 = -10$ c) $a - 8 = 2$ d) $p + 11 = 5$
   e) $s + 19 = 14$ f) $w - 7 = -18$ g) $y - 3 = 0$ h) $m - 14 = -37$

B

4. Solve.

a) $\frac{w}{4} = 13$ b) $\frac{m}{7} = 4$ c) $5x = -35$ d) $7y = 91$ e) $\frac{1}{5}p = 3$

f) $\frac{1}{13}x = \frac{2}{13}$ g) $\frac{x}{7} = -9$ h) $-4s = -28$ i) $-56 = 8p$ j) $\frac{x}{-7} = -8$

5. Solve.

a) $4 + y = -9$ b) $-8 = 2 + x$ c) $3 = y - 5$ d) $20 = 10 + z$

e) $-11 = n + 21$ f) $-2 = x - 14$ g) $13 + x = 13$ h) $5.2 = 3.7 + p$

6. Solve.

a) $9.3 = a - 2.7$ b) $z - \frac{1}{4} = \frac{7}{4}$ c) $12 = m - 45$ d) $\frac{5}{2} = x + \frac{3}{4}$

e) $4.5 = -2.3 + x$ f) $-52 = 27 + y$ g) $q - \frac{9}{4} = \frac{43}{8}$ h) $\frac{2}{5} = a + \frac{3}{10}$

7. Solve.

a) $48 = -6y$ b) $-12 = \frac{x}{2}$ c) $13x = 169$ d) $5 = \frac{n}{15}$

e) $8.5 = 1.7m$ f) $2.5y = -10$ g) $72 = 18m$ h) $\frac{9}{4} = \frac{x}{12}$

8. A daytime, operator-assisted telephone call from Toronto to Calgary costs \$18.40. The time, in minutes, for the call is given by the value of t in this equation.
$18.40 = 0.80 + 1.10t$
Find how long the call lasted.

9. A car is rented for one day and the charge at the end of the day is \$172.95. The distance driven, in kilometres, is given by the value of d in this equation.
$172.95 = 28.50 + 0.15d$
Find how far the car was driven.

10. A loan company charges a flat rate of \$50 to process a loan and charges interest at the rate of 27.5% per annum on the principal. After 1 year, the amount a person has to pay back is \$2600.00. The principal, in dollars, is given by the value of p in this equation.
$2600.00 = 50.00 + 1.275p$
Find how much the person borrowed.

C

11. Solve for x.

a) $x + a = b$ b) $x - c = d$ c) $3x = m$

d) $\frac{1}{4}x = w$ e) $a - b = x + 2b$ f) $\frac{x}{4} = c - d$

g) $2y + x = z$ h) $b - 2a = c - x$ i) $-ax + b = c$

j) $n = m - kx$ k) $-b - cx = d$ l) $-p - \frac{x}{q} = -r$

4-3 SOLUTIONS REQUIRING SEVERAL STEPS

Many equations require more than one step to isolate the variable. To solve these equations, isolate the term that contains the variable first.

Example 1. Solve.

a) $-5w + 9 = 21$ b) $7.3 = 6.6y - 15.8$

Solution. a) $-5w + 9 = 21$

Subtract 9 from both sides.

$$-5w + 9 - 9 = 21 - 9$$

$$-5w = 12$$

Divide both sides by -5.

$$\frac{-5w}{-5} = \frac{12}{-5}$$

$$w = -\frac{12}{5}$$

b) $7.3 = 6.6y - 15.8$

Add 15.8 to both sides.

$$7.3 + 15.8 = 6.6y - 15.8 + 15.8$$

$$23.1 = 6.6y$$

Divide both sides by 6.6.

$$\frac{23.1}{6.6} = \frac{6.6y}{6.6}$$

$$3.5 = y$$

It may be necessary to expand, using the distributive law, to solve an equation.

Example 2. Solve. $23.98 = 11(1.10w + 2.07)$

Solution. $23.98 = 11(1.10w + 2.07)$

Expand the right side.

$$23.98 = 12.10w + 22.77$$

Subtract 22.77 from both sides.

$$23.98 - 22.77 = 12.10w + 22.77 - 22.77$$

$$1.21 = 12.10w$$

Divide both sides by 12.10.

$$\frac{1.21}{12.10} = \frac{12.10w}{12.10}$$

$$0.1 = w$$

Example 3. The cost, C dollars, of taking n students on a weekend trip to Ottawa is given by this formula.
$C = 180 + 35n$
The cost was \$1685. How many students went on the trip?

Solution. $C = 180 + 35n$
Substitute 1685 for C.
$1685 = 180 + 35n$
Solve for n. Subtract 180 from both sides.
$1685 - 180 = 180 + 35n - 180$
$1505 = 35n$
Divide both sides by 35.
$$\frac{1505}{35} = \frac{35n}{35}$$
$43 = n$
43 students went on the trip.

EXERCISES 4-3

Ⓐ

1. Solve.
   a) $2w - 5 = 11$ b) $5n - 8 = 12$ c) $8 - 2u = 12$
   d) $0 = 7p - 35$ e) $-9p - 81 = 0$ f) $10 = -3x - 5$
   g) $-11z - 2 = 20$ h) $3 - 2y = -7$ i) $-9 = 8b - 1$
   j) $17 = 5q + 2$ k) $8 - 3z = -1$ l) $-13 = 4p - 1$

2. Solve.
   a) $8t + 7 = -10$ b) $9p - 2 = 6$ c) $-5r + 6 = 8$
   d) $4x + \frac{3}{4} = \frac{7}{4}$ e) $3x - \frac{1}{4} = 2$ f) $-1 = 7x - \frac{5}{12}$
   g) $10t - \frac{2}{5} = \frac{3}{5}$ h) $7x + \frac{5}{4} = -3$ i) $11x - \frac{1}{2} = \frac{3}{2}$
   j) $9t - \frac{3}{5} = -\frac{6}{5}$ k) $1 = 8s - \frac{1}{3}$ l) $\frac{7}{6} + 8t = \frac{13}{6}$

Ⓑ

3. Solve.
   a) $\frac{1}{3}r - 3 = -6$ b) $\frac{1}{4}x + 6 = 10$ c) $\frac{1}{7}x - 1 = \frac{9}{7}$
   d) $1.5x - 3 = -12$ e) $2.5y + 3 = -8$ f) $1 = 3.8x - 0.9$
   g) $4.4y + 3 = 5.64$ h) $1.3w + 65 = 26$ i) $12.5z - 36 = 64$

4. Solve.
   a) $12 = 9 - 2t$ b) $23 = 11 - 3r$ c) $8 - 3z = -19$
   d) $5y + \frac{3}{5} = 3$ e) $7x - \frac{5}{9} = 3$ f) $\frac{5}{16} = \frac{11}{16} - \frac{1}{4}x$
   g) $0.2x + 4 = 7$ h) $-1 = 5 - 0.3t$ i) $0.25p + 0.25 = 0.5$

5. Solve.

a) $\frac{2}{5}t + 3 = 11$ b) $\frac{n}{3} + 4 = -6$ c) $-\frac{3}{4}z + 5 = -1$

d) $1.2(2x - 3) = 7.2$ e) $-3.5(1 + 3r) = 7$ f) $3\left(5.6 + \frac{x}{3}\right) = 0$

6. Solve.

a) $4(x - 2) = 9$ b) $-3 = 5(z + 7)$ c) $-\frac{2}{3}(x + 1) = 6$

d) $-3\left(y - \frac{1}{2}\right) = \frac{1}{2}$ e) $0.02 = 0.8(y + 0.1)$ f) $1.2(t + 2.3) = 3.96$

g) $0.6(5s - 6) = 2.4$ h) $1.21 = 11(0.51 + 0.4x)$ i) $2.4(6.7 + 1.2x) = 24.72$

7. If $3w = 6 - 9z$, find:
   a) the value of w for each given value of z
      i) 1 ii) -2 iii) 0.5 iv) -1.4 v) 0
   b) the value of z for each given value of w.
      i) 0 ii) -1 iii) 2.2 iv) -19 v) 110

8. To determine how far away the centre of a storm is, count the number of seconds, t, between a flash of lightning and the sound of thunder. Substitute this value for t in the formula $d = \frac{8t}{25}$ to find d, the distance in kilometres.
   a) Find how far away the storm is when the time lapse is:
      i) 5 s ii) 10 s iii) 3.5 s.
   b) Find the time lapse when the storm is:
      i) 8 km away ii) 6 km away.

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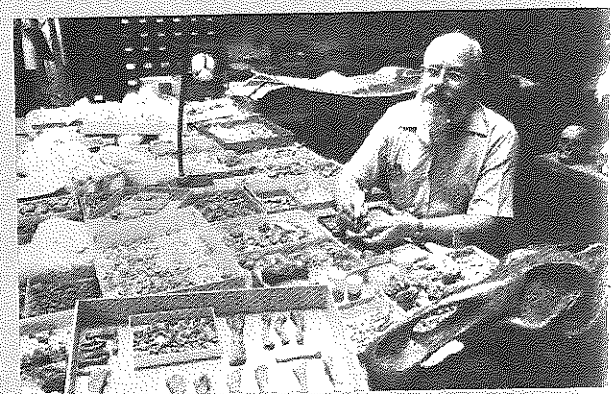
9. Typing speed, S, in words per minute, is calculated with the formula $S = \frac{w - 10e}{5}$, where w is the number of words typed in 5 min and e is the number of errors made in the same period.
   a) Find how many words must be typed in 5 min if, when 5 errors are made, the typing speed is 40 words/min.
   b) Find how many errors are made for the typing speed to be 30 words/min when 180 words are typed in 5 min.

10. Weekly mathematics tests have 15 questions. Each question has either one, two, or three parts.
    a) If, on one test, 9 questions have one part, 4 questions have two parts, and 2 questions have three parts, how many parts are there altogether?
    b) If a test has a total of 29 parts, 6 questions having one part and 5 questions having three parts, how many questions have two parts?
    c) One test has 8 questions, each with only one part and there is a total of 24 parts. How many questions have two parts and how many have three?

PROBLEM SOLVING

Write an Equation



An anthropologist discovered the bones of a huge fish with a head that was 9 m long.
The tail was as long as the head, plus half the length of the body.
The body was as long as the head and tail combined.
How long was the fish?

Understand the problem

- How long was the head?
- How long was the tail?
- What are we asked to find?

Think of a strategy

- Write an equation to relate what we are asked to find, to the information given.

Carry out the strategy

- Choose a variable l to represent the length of the body in metres.
- The length of the tail is $\left(9 + \frac{1}{2}l\right)$ metres.
- We know that: length of head + length of tail = length of body

  that is, $$9 + \left(9 + \frac{1}{2}l\right) = l$$

- Solve the equation for l.

$$18 + \frac{1}{2}l = l$$

$$18 = \frac{1}{2}l$$

$$l = 36$$

- The body is 36 m long.
  Length of fish = length of head + length of body + length of tail

$$= 9 + 36 + 9 + \frac{1}{2}(36)$$

$$= 72$$

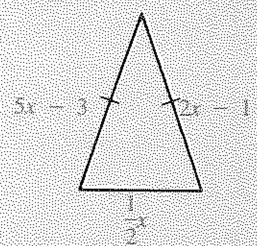
  The fish was 72 m long.

Look back

- Calculate the length of the tail in two ways. Are the answers the same?

Solve each problem

1. The lengths of the sides of an isosceles triangle are shown in the diagram (below left). Find the value of x.



		19
51	50	
		82

2. All rows, columns, and diagonals of a magic square have the same sum. Complete the magic square (above right).

3. Kim lives at the bottom of a mountain. It took her 6 h to ride to the top of the mountain and back. Her average speed up the mountain was 6 km/h and her average speed down the mountain was 24 km/h. How long did it take Kim to ride down the mountain?

4. Peanuts worth \$2.80/kg are mixed with pecans worth \$4.20/kg. How many grams of each should be mixed to produce a 1 kg mixture worth \$3.36?

5. A rectangular walk is a line of 9 identical square cement tiles. The perimeter of the walk is 20 m. What is the area of each cement tile?

6. The total mass of a can and the paint it contains is 5 kg when half full, and 4 kg when one-third full. What is the total mass when the can is:
   a) empty
   b) full of paint?

7. A shopkeeper sets retail prices at $p\%$ above cost. On a special sale he reduces these prices by 20%. At these prices he makes no profit. What is the value of p?

8. If a block weighs 7 kg plus half a block, what is the mass of a block and a half?

9. Eric and Lois are trading hockey cards. If Eric gives one card to Lois, they will have the same number. If Lois gives one card to Eric, he will have twice as many as she. How many cards does each person have to start with?

10. The mass of a candy-bar wrapper is $\frac{1}{11}$ the mass of the wrapped bar. If the candy bar alone has a mass of 75 g, what is the mass of the wrapper?

11. The sum of the digits of a two-digit number is 9. When the digits are interchanged, the number is decreased by 45. What is the number?

4-4 COMBINING TERMS CONTAINING THE VARIABLE

To solve an equation, it is necessary to isolate the variable on one side of the equation. The numerical terms are combined on the other side of the equation. When several terms contain the variable they must be combined, too.

Example 1. Solve.

a) $6x - 5 = 2x + 7$ b) $2 - 3y = 7 + y$

Solution. a) $6x - 5 = 2x + 7$

Subtract $2x$ from both sides.

$$6x - 5 - 2x = 2x + 7 - 2x$$
$$4x - 5 = 7$$

Add 5 to both sides.

$$4x - 5 + 5 = 7 + 5$$
$$4x = 12$$

Divide both sides by 4.

$$\frac{4x}{4} = \frac{12}{4}$$
$$x = 3$$

b) $2 - 3y = 7 + y$

Add $3y$ to both sides.

$$2 - 3y + 3y = 7 + y + 3y$$
$$2 = 7 + 4y$$

Subtract 7 from both sides.

$$2 - 7 = 7 + 4y - 7$$
$$-5 = 4y$$

Divide both sides by 4.

$$\frac{-5}{4} = \frac{4y}{4}$$
$$-\frac{5}{4} = y$$

These examples illustrate that it doesn't matter on which side of the equation the variable is isolated.

Sometimes it is necessary to expand before combining the terms and isolating the variable.

Example 2. Solve.

a) $5(y - 1) = 7(3 + y)$ b) $3(y - 1) - 5y = 2y - (y - 2)$

Solution. a)

$$\begin{aligned} 5(y - 1) &= 7(3 + y) \\ 5y - 5 &= 21 + 7y \\ 5y - 5 - 5y &= 21 + 7y - 5y \\ -5 &= 21 + 2y \\ -5 - 21 &= 21 + 2y - 21 \\ -26 &= 2y \\ \frac{-26}{2} &= \frac{2y}{2} \\ -13 &= y \end{aligned}$$

b)

$$\begin{aligned} 3(y - 1) - 5y &= 2y - (y - 2) \\ 3y - 3 - 5y &= 2y - y + 2 \\ -2y - 3 &= y + 2 \end{aligned}$$

Add 3 to, and subtract y from, both sides.

$$\begin{aligned} -2y - 3 + 3 - y &= y + 2 + 3 - y \\ -3y &= 5 \\ \frac{-3y}{-3} &= \frac{5}{-3} \\ y &= -\frac{5}{3} \end{aligned}$$

EXERCISES 4-4

(A)

1. Solve.
   a) $7x - 3 = 4x + 3$
   b) $5y + 9 = 2y - 3$
   c) $-4m + 2 = 6m + 12$
   d) $-8t - 5 = -9t - 7$
   e) $3r - 2 = -5r + 14$
   f) $6p - 7 = -6p - 7$
   g) $5 - y = 3 - 2y$
   h) $9 - 2x = 6 - x$
   i) $3 - 2t = 5 - 5t$
   j) $4 - p = 5 - 3p$
   k) $7 - 5p = 6 + p$
   l) $8 - 3r = -6 + r$
   m) $-11 + 6v = -6v + 11$
   n) $-8w = -4 - 6w$

(B)

2. Solve.
   a) $3(x - 1) = 12$
   b) $5(x + 2) = 10$
   c) $-14 = x - 3$
   d) $x - 2 = 2(x - 1)$
   e) $3y - 2 = y + 4$
   f) $4y = -2(9 - y)$
   g) $y + 7 = 3y - 9$
   h) $9y - 3 = 3(y - 4)$

3. Solve.
   a) $2(t - 3) = -3(t - 1)$
   b) $-3(r + 2) = -4(r - 1)$
   c) $7(z + 3) = 5(z - 1)$
   d) $4(2y - 1) = 5(3y + 1)$
   e) $-3(4p + 2) = 4(2p - 2)$
   f) $-2(1 - x) = -3(2 - x)$
   g) $6(-2 - x) = -5(2x + 4)$
   h) $2.5(2 - 3x) = 1.5(3x - 2)$

4. Solve.
   a) $6y - 2 = 5y + 4$
   b) $3p + 2 = 5p - 7$
   c) $4 - r = 3 - 2r$
   d) $9 - 2p = -8 - p$
   e) $5(x - 1) = 8(1 - x)$
   f) $7(y - 2) = 13$
   g) $-2(x - 1) = 3(x + 2)$
   h) $-3(y + 1) = -2(y - 1)$
   i) $r - 1 = 5r - 7$
   j) $19t - 13 = 2t + 4$
   k) $17y + 3 = 15y - 3$
   l) $z - 2 = 2z - 2$
   m) $x + 4 = 11x + 4$
   n) $t - 8 = -12t + 18$

5. Solve.
   a) $4(x - 2) + 5 = 3 + 2(x - 3)$
   b) $1 + 5(x - 1) = 4(x - 3) + 6$
   c) $-2(m + 4) = 3(5 - m) - 8$
   d) $y + 3(y - 6) = 2(3 - y) + 3y$
   e) $3(n - 2) + 12 = 6n - 3(4 - n)$
   f) $11 - 2(5 + 3x) = 2(x - 6) + 14$

4-5 EQUATIONS WITH FRACTIONAL COEFFICIENTS

Consider the problem posed at the beginning of the chapter. A car uses gasoline at an average rate of 10.9 L/100 km. The fraction of driving that the car does on the highway is given by the value of f in this equation.

$$10.9 = \frac{-36f}{5} + \frac{29}{2}$$

To find the fraction of highway driving, solve the equation for f.

$$10.9 = \frac{-36f}{5} + \frac{29}{2}$$

Multiply both sides by the common denominator 10.

$$\begin{aligned}
10(10.9) &= 10\left(\frac{-36f}{5} + \frac{29}{2}\right) \\
109 &= 10\left(\frac{-36f}{5}\right) + 10\left(\frac{29}{2}\right) \\
109 &= -72f + 145 \\
109 - 145 &= -72f + 145 - 145 \\
-36 &= -72f \\
\frac{-36}{-72} &= \frac{-72f}{-72} \\
\frac{1}{2} &= f
\end{aligned}$$

Half the car's driving is done on the highway.

When an equation contains fractions, multiply both sides of the equation by a common denominator of the fractions to obtain an equivalent equation without fractions.

Example 1. Solve. $\frac{a}{3} - 3 = \frac{3}{4}a + \frac{1}{2}$

Solution.

$$\frac{a}{3} - 3 = \frac{3}{4}a + \frac{1}{2}$$

Multiply both sides by the common denominator 12.

$$12\left(\frac{a}{3} - 3\right) = 12\left(\frac{3a}{4} + \frac{1}{2}\right)$$
$$12\left(\frac{a}{3}\right) - 12(3) = 12\left(\frac{3a}{4}\right) + 12\left(\frac{1}{2}\right)$$
$$4a - 36 = 9a + 6$$
$$4a - 36 - 4a - 6 = 9a + 6 - 4a - 6$$
$$-42 = 5a$$
$$-\frac{42}{5} = \frac{5a}{5}$$
$$-\frac{42}{5} = a$$

Example 2. Solve. $\frac{3x + 2}{2} - \frac{x + 1}{3} = x$

Solution.

$$\frac{3x + 2}{2} - \frac{x + 1}{3} = x$$

Multiply both sides by 6.

$$6\left(\frac{3x + 2}{2} - \frac{x + 1}{3}\right) = 6(x)$$
$$\frac{6(3x + 2)}{2} - \frac{6(x + 1)}{3} = 6x$$
$$9x + 6 - 2x - 2 = 6x$$
$$7x + 4 = 6x$$
$$7x + 4 - 7x = 6x - 7x$$
$$4 = -x$$
$$\frac{4}{-1} = \frac{-x}{-1}$$
$$-4 = x$$

EXERCISES 4-5

(A)

1. Solve.

a) $\frac{a}{4} = \frac{1}{2}$ b) $\frac{x}{5} = -\frac{2}{3}$ c) $\frac{1}{3} = \frac{-2x}{5}$

d) $\frac{1}{2}x + \frac{1}{3}x = 10$ e) $\frac{1}{4}y - \frac{1}{2}y = 4$ f) $\frac{y}{3} - \frac{2}{3} = 4$

g) $\frac{2}{5}a + \frac{a}{2} = a - 2$ h) $\frac{1}{2}n - \frac{2}{3}n + \frac{3}{4}n = -7$ i) $\frac{x}{3} + \frac{1}{2} = -2x$

2. Solve.

a) $-\frac{1}{3}x + \frac{3}{4}x = 10$ b) $\frac{3}{5}x - \frac{3}{2}x = 10$ c) $\frac{2a}{3} = \frac{3a}{5} + 4$

d) $\frac{2}{3}x + 9 = \frac{3}{4}x - 6$ e) $\frac{5x}{2} - 3 = 8 + \frac{2x}{3}$ f) $5 - \frac{4}{3}x = \frac{3}{4}x + \frac{5}{2}$

3. Nicole's car uses gasoline at a rate of 11.3 L/100 km. The fraction of driving that Nicole does on the highway is given by the value of f in this formula.

$$11.3 = -\frac{25}{4}f + \frac{69}{5}$$

Find the fraction of driving that Nicole does on the highway.

B

4. Solve.

a) $\frac{a}{5} - a = \frac{1}{2}$ b) $\frac{2x}{3} = \frac{x}{2} - \frac{1}{4}$

c) $\frac{m}{6} - 5 = \frac{1}{2}m$ d) $\frac{3k}{4} + \frac{1}{2} = \frac{k}{3}$

e) $\frac{1}{3}(x + 1) = \frac{1}{2}(x - 2)$ f) $\frac{1}{5}(2n + 1) = \frac{2}{3}(n - 1)$

g) $\frac{a + 5}{3} = \frac{3 - a}{7}$ h) $\frac{1}{5}\left(\frac{1}{2}x + 4\right) = \frac{1}{3}\left(\frac{1}{4}x + 3\right)$

i) $\frac{1}{4}(2 - x) + \frac{1}{2} = \frac{1}{2}\left(\frac{1}{3}x + 7\right)$ j) $-\frac{1}{2}(x - 2) + \frac{1}{4} = \frac{2}{5}(2 - x)$

5. Solve.

a) $\frac{4x}{5} - \frac{3}{2} = \frac{2}{3} + \frac{1}{3}x$ b) $\frac{1}{4}(x - 3) + \frac{1}{3}(3 + x) = 1$

c) $-\frac{x}{3} + \frac{x}{4} - \frac{x}{6} = \frac{1}{10}$ d) $-\frac{1}{6}(3 - 5x) = \frac{2}{3}(5x + 3)$

e) $-\frac{3}{4}x - \frac{4x}{5} + \frac{7x}{10} = \frac{-1}{20}$ f) $-\frac{3}{7} = \frac{5}{14}(4 - 6x) + \frac{2x}{7}$

g) $\frac{7}{8}(-x - 6) + \frac{3}{4}(2x + 3) = \frac{-3}{8}$ h) $\frac{3x}{10} - \frac{2x}{5} = \frac{3x}{2} + \frac{1}{2}$

i) $\frac{11}{2}(7x - 6) = -\frac{1}{4}x + \frac{9}{2}(3 - 2x)$ j) $-\frac{3}{2}(7 - 4x) = \frac{2}{7}x - \frac{1}{2}(-3x + 4)$

6. If $y = -\frac{3}{5}x - \frac{1}{4}$, find:

a) the value of y for each given value of x

i) 0 ii) $\frac{1}{3}$ iii) $-\frac{1}{3}$ iv) 2 v) -2

b) the value of x for each given value of y.

i) 0 ii) $\frac{1}{2}$ iii) $-\frac{1}{2}$ iv) 1.5 v) -1.5

4-6 CHECKING SOLUTIONS

Solving an equation may require several steps. As the number of steps increases, the greater is the chance of an error. For this reason it is wise to check that the solution is correct. To do this

- Substitute the solution for the variable in each side of the original equation.
- Simplify each side of the equation independently.

The solution is correct if each side of the equation simplifies to the same number.

Example 1. Check that $y = -3$ is the solution of the equation $6y + 5 = 4y - 1$.

Solution. $6y + 5 = 4y - 1$

Substitute -3 for y in each side of the equation and simplify each side independently.

Left side	Right side
$= 6y + 5$	$= 4y - 1$
$= 6(-3) + 5$	$= 4(-3) - 1$
$= -18 + 5$	$= -12 - 1$
$= -13$	$= -13$

Each side simplifies to the same number.
Hence, $y = -3$ is correct.

Example 2. Solve and check. $3(x - 2) = 5(x + 6)$

Solution.

$$3(x - 2) = 5(x + 6)$$
$$3x - 6 = 5x + 30$$
$$3x - 6 + 6 - 5x = 5x + 30 + 6 - 5x$$
$$-2x = 36$$
$$\frac{-2x}{-2} = \frac{36}{-2}$$
$$x = -18$$

Check. Substitute -18 for x in each side of the equation.

Left side	Right side
$= 3(x - 2)$	$= 5(x + 6)$
$= 3(-18 - 2)$	$= 5(-18 + 6)$
$= 3(-20)$	$= 5(-12)$
$= -60$	$= -60$

Since the left side equals the right side, $x = -18$ is correct.

EXERCISES 4-6

(A)

1. Solve and check.

a) $2(x + 1) = 3x$ b) $3(2 - m) = m + 2$ c) $3 - y = y - 7$

d) $4a + 6 = 2a - 2$ e) $2 - x = 5x + 8$ f) $3m - 5 = 2m + 1$

g) $3k - 5 = k - 4$ h) $\frac{1}{2}a + 3 = \frac{2}{3}a$ i) $\frac{1}{4}(c + 2) = \frac{1}{3}(c - 1)$

(B)

2. Solve and check.
   a) $8(y - 5) = 7y$
   b) $-6(3 - x) = -9x - 6$
   c) $9 - v = v - 9$
   d) $8w - 4 = 4 - 8w$
   e) $-5t - 2 = 11t + 16$
   f) $11 - p = 3p - 21$
   g) $6(q + 1) = 3(q - 1)$
   h) $3 - (6s - 15) = 4s + 8$
   i) $-3(2x - 1) - 7 = -41 - 2(x + 0.5)$
   j) $\frac{1}{5}(y + 3) - \frac{5}{4} = \frac{1}{4}(y - 1)$

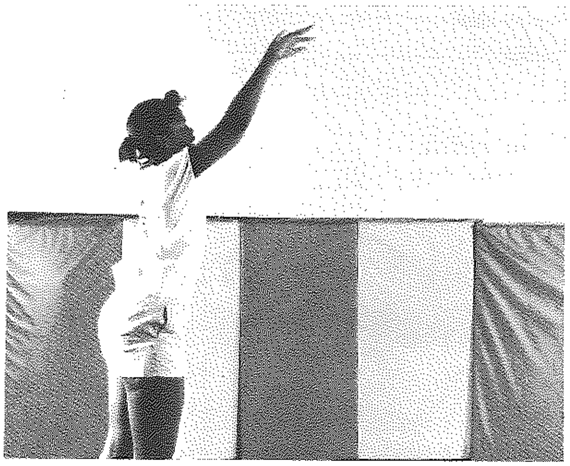
3. Solve and check.
   a) $3(y - 2) + 6 = 2(y - 3) - 5$
   b) $\frac{1}{2}(m + 3) - 4 = \frac{1}{3}(4 - m) + 7$
   c) $-(a - 7) + 6 = 5(3 - a) - 8$
   d) $3(x - 6) + 2 = 4(x + 2) - 21$
   e) $\frac{1}{4}(8m + 4) - 17 = -\frac{1}{2}(4m - 8)$
   f) $-\frac{1}{3}(6a + 24) = 20 - \frac{1}{4}(12a - 72)$
   g) $0.2t - 0.4 + 0.4t = -0.1t + 0.6$
   h) $0.25(8y + 4) - 17 = -0.5(4y - 8)$
   i) $-1.03 - 0.62m = 0.71 - 0.22m$
   j) $0.125w - 8(3.75 - 0.375w) = -0.875w$

INVESTIGATE

1. Consider the equation $3(x + 2) = x + 2(x + 3)$.
   a) Check that $x = 5$, $x = 8$, and $x = -1$ are all solutions of the equation.
   b) Choose any other number and check that it also is a solution of the equation.
   c) Attempt to solve the equation. Can you suggest why every number is a solution of this equation?
   d) Give another example of an equation like this one, which has infinitely many solutions.

2. Consider the equation $3(x + 2) = x + 2(x + 4)$
   a) Choose any number and show that it is *not* a solution of the equation.
   b) Attempt to solve the equation. Can you explain why it does not have a solution?
   c) Give another example of an equation like this one, which has no solution.

3. *Without solving* these equations, can you tell which have infinitely many solutions, which have no solution, and which have exactly one solution?
   a) $5(n + 2) = n + 6(n - 3)$
   b) $2(p + 1) - 3 = 4(2p + 1) - 11$
   c) $2(y + 1) = y + (1 + y)$
   d) $5(x - 2) = x + 4(x - 3) + 2$
   e) $4(q - 1) = q + 3(q + 3)$
   f) $6(r + 2) = 3r + 3(r - 4)$



4-7 WORKING WITH FORMULAS

After working out, Michelle wonders if her pulse rate of 150 beats per minute exceeds the maximum for her age. She uses the formula $a = 220 - m$, where a is a person's age in years and m is the maximum desirable pulse rate in beats per minute.

Michelle substitutes 15 for a and solves the equation.

$$15 = 220 - m$$
$$-205 = -m$$
$$m = 205$$

The maximum desirable pulse rate for a 15-year-old is 205 beats per minute. Michelle's rate of 150 beats per minute is well below this maximum.

Recall that formulas are equations which relate two or more variables using the basic operations of arithmetic. Science, engineering, and industry use many formulas. Often, the values of all but one of the variables are known. It is necessary to substitute the known values into a formula, and then solve the equation to find the value of the unknown variable.

Example 1. The annual simple interest, I dollars, on a principal, P dollars, is given by this formula. $I = 0.095P$
Find the principal that earned \$807.50 interest in one year.

Solution. $I = 0.095P$

Substitute 807.50 for I.

$807.50 = 0.095P$

Divide both sides by 0.095.

$$\frac{807.50}{0.095} = P$$

$$8500 = P$$

The principal is \$8500.

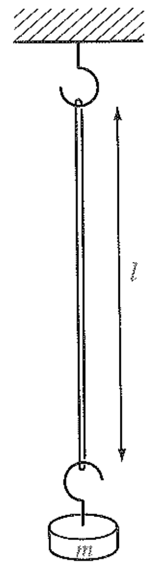
Example 2. A scientific experiment illustrates that a rubber band stretches according to this formula.

$l = 9.2 + 0.17m$

l is the length of the band in centimetres, and m is the mass in grams, suspended on one end of the band.

a) Calculate the length, to the nearest centimetre, of the rubber band when the mass on the end is 25 g.

b) Calculate the mass, to the nearest gram, that will stretch the band to 86 cm.



Solution. a) $l = 9.2 + 0.17m$

Substitute 25 for m.

$l = 9.2 + 0.17(25)$

$= 13.45$

The band is about 13 cm long.

b) $l = 9.2 + 0.17m$

Substitute 86 for l.

$86 = 9.2 + 0.17m$

Solve for m.

$76.8 = 0.17m$

$\frac{76.8}{0.17} = m$

$m \doteq 452$

A mass of 452 g will stretch the band to 86 cm.

To find the masses which correspond to several lengths of the rubber band, it is more efficient to solve the equation for m before substituting.

$$l = 9.2 + 0.17m$$
$$l - 9.2 = 0.17m$$
$$\frac{l - 9.2}{0.17} = m$$

For other types and thicknesses of rubber, the constant term and the coefficient in this formula would be different. Thus, the general formula for stretching any type of rubber band might be given by this formula.

$l = a + bm$

a and b are constants with values depending on the type of rubber band.

An equation of this type, in which the constants are represented by letters, is called a *literal* equation.

Example 3. a) Solve for F. $C = \frac{5}{9}(F - 32)$

b) Solve for x. $ax + 3c = d$

Solution. a)

$$C = \frac{5}{9}(F - 32)$$
$$9C = 5(F - 32)$$
$$9C = 5F - 160$$
$$9C + 160 = 5F$$
$$F = \frac{9C + 160}{5}$$

b)

$$ax + 3c = d$$
$$ax = d - 3c$$
$$x = \frac{d - 3c}{a}$$

EXERCISES 4-7

(A)

1. The monthly interest, I dollars, on a loan is given by the formula $I = 0.0175P$, where P is the principal. Find the first month's interest on a principal of $1250.

2. The yearly interest, I dollars, is given by the formula $I = 0.11P$, where P is the principal.
   a) Find the interest for one year on each principal.
      i) $100 ii) $1000 iii) $2500 iv) $9000
   b) Find the principal which yields each interest, in one year.
      i) $22 ii) $55 iii) $132 iv) $159.50

3. Solve for x.

   a) $mx + n = p$ b) $\frac{1}{2}x - c = d$ c) $ax - b = d$

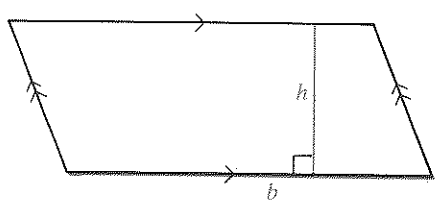
   d) $\frac{2}{3}x + 3 = k$ e) $wx + 1 = v$ f) $a - bx = d$

4. Solve for the variable indicated.

   a) $I = Prt$, for t b) $P = 2l + 2w$, for l c) $A = \frac{1}{2}bh$, for b

   d) $C = 2\pi r$, for r e) $l = a + bm$, for m f) $A = 50 + 1.275P$, for P

5. The area, A, of a parallelogram is given by the formula $A = bh$, where b is the length of the base and h is the height.



   a) Find the area of a parallelogram with base 10.3 cm and height 13.6 cm.
   b) Find the height of a parallelogram with area 25.2 cm² and base 5.6 cm.

B

6. A person's maximum desirable pulse rate, m beats per minute, can be found from the formula, $m = 220 - a$ if a, the person's age, is known.
   a) Find the maximum desirable pulse rate for a person of each age.
      i) 20 years ii) 37 years iii) 63 years
   b) Find the age of a person corresponding to each maximum desirable pulse rate.
      i) 170 beats/min ii) 192 beats/min iii) 141 beats/min

7. When an object falls freely from rest, its approximate speed, v metres per second after t seconds, is given by the formula $v = 9.8t$.
   a) Solve the formula for t.
   b) Find the time, to the nearest tenth of a second, it will take the object to reach these speeds.
      i) 54 m/s ii) 81 m/s iii) 343 m/s iv) 1 km/s

8. When the temperature at sea level is t°C, the approximate temperature at an altitude h metres is T°C, where $T = t - 0.0065h$.
   a) It is 15°C at sea level. Find the temperature, to the nearest degree, at each altitude. i) 1600 m ii) 4000 m iii) 12 000 m
   b) The temperature at sea level is 15°C. At what altitude, to the nearest metre, will the temperature be −8°C?

9. A company determines the age at which an employee can retire with full pension, from the formula $a + b = 90$, where a is the employee's age and b is the number of years of service.
   a) Find the years of service for retirement with full pension at each age.
      i) 60 years ii) 65 years iii) 70 years
   b) Find the minimum age for retirement with full pension after each period.
      i) 30 years of service ii) 20 years of service iii) 35 years of service

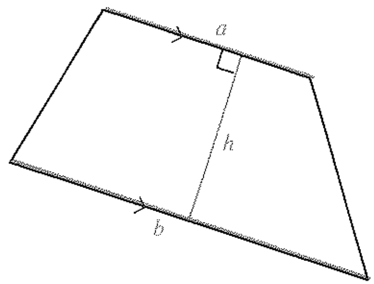
10. Solve for the variable indicated.
    a) $V = \pi r^2 h$, for h b) $F = \frac{Mm}{d}$, for m c) $I = \frac{100m}{P}$, for m
    d) $A = \frac{1}{2}h(a + b)$, for h e) $n = \frac{v}{4l}$, for l f) $\frac{1}{3}x + b = c$, for x

11. Solve for the variable indicated.
    a) $S = \frac{w - 10e}{5}$, for w b) $n = 17 - \frac{1}{2}a$, for a
    c) $L_2 = L_1(1 + at)$, for t d) $C = 3.7 + 0.99(n - 3)$, for n
    e) $S = \frac{n}{2}(a + l)$, for l f) $A = \frac{1}{2}h(a + b)$, for b

12. The length, l centimetres, of a rubber band suspending a mass of m grams is given by the formula $l = 14.3 + 0.27m$.
    a) Solve the formula for m.
    b) Find the mass, to the nearest gram, that stretches the band to each length.
       i) 98 cm ii) 103 cm

13. When an object on the moon falls freely from rest, its approximate speed, v metres per second after t seconds, is given by the formula $v = 1.63t$.
    a) Find its speed, to one decimal place, after each time period.
    i) 1 s ii) 5 s iii) 8 s iv) 10 s
    b) Find the time, to the nearest tenth of a second, the object takes to reach each speed.
    i) 5 m/s ii) 32 m/s iii) 57 m/s iv) 0.6 km/s

14. The area, A, of a trapezoid is given by the formula, $A = \frac{1}{2}h(a + b)$, where a and b are the lengths of the parallel sides and h is the distance between them.



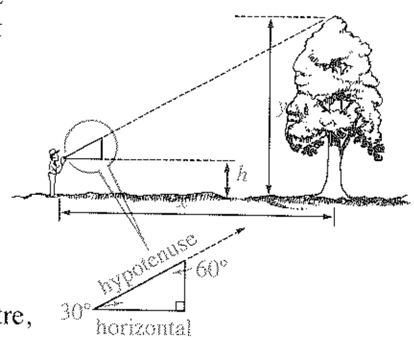
    a) Find the distance between the parallel sides if their lengths are 9 cm and 23 cm, and the area is 256 cm^2.
    b) Find the length of one parallel side if the other parallel side is 3.4 cm, the distance between them is 6.0 cm, and the area is 23.7 cm^2.

15. The approximate mass, m grams, of a volume of air is given by the formula $m = 1.29V$, where V is the volume in litres. Find the volume, to the nearest litre, of air that has each mass.
    a) 20 g b) 300 g c) 22.575 kg

(C)

16. To find the height, y, of a tall object
    - Sight the top of the object along the hypotenuse of a 30° − 60° − 90° triangle.
    - Measure the distance, x, from the object.
    - Substitute the measured value for x in the formula $y = 0.577x + h$. h is the height of the eye above the ground. (x, y, and h are measured in the same units.)

    Assume h is 1.4 m.

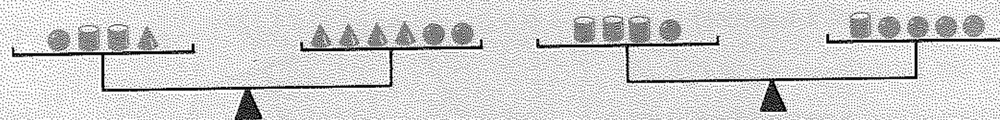


    a) Calculate the height of an object, to the nearest tenth of a metre, for each value of x.
    i) 30 m ii) 40 m
    iii) 15 m iv) 17.5 m
    b) Calculate the distance, to the nearest metre, of a viewer from an object which, when correctly sighted, has each of these heights.
    i) 40 m ii) 55 m
    iii) 100 m iv) 81 m

PROBLEM SOLVING

Choose the Strategy

1. All solids of the same shape have the same mass. How many cones have the same mass as 4 cylinders?



2. There were 6 more students in Mrs. Zovak's class who did not have scientific calculators than in Mr. Alvi's class. Five of these students transferred from Mrs. Zovak's class into Mr. Alvi's class. Now Mr. Alvi has twice as many students without scientific calculators as Mrs. Zovak. How many students in Mr. Alvi's class do not have scientific calculators?

3. A shuttle bus picked up a group of people at the airport. The bus travelled to the Hyatt Hotel where it let off 14 people and picked up 3. At the Holiday Inn, it let off half of those who were remaining and picked up 13 people. There were now 16 people on the bus. How many people were picked up at the airport?

4. Jack is now 4 times as old as his dog. In 6 years he will be only twice as old as his dog. How old is Jack? How old is his dog?

5. An 18 m log is cut into 2 pieces. The longer piece is 3 m shorter than twice the shorter piece. How long is the shorter piece?

6. Mrs. Richards divided \$45 among her four children: Amanda, Betty, Carol, and Dan. When the children complained that the shares were not equal, she instructed Betty to give Amanda \$2. Then she doubled Carol's share and cut Dan's share in half. Now all of the children have the same amount. How much money do they have in total?

7. a) Explain why every integer can be expressed in exactly one of the forms, $6n$, $6n + 1$, $6n + 2$, $6n + 3$, $6n + 4$, $6n + 5$ for some integer n.
   b) In which of these 5 forms can the prime numbers be expressed?
   c) Show that all prime numbers (except 2 and 3) when divided by 6 leave a remainder of either 1 or 5.

8. Raymond has a box of candy bars. He gave Monique half of what he had plus half a bar. Then he gave Claude half of what he had left plus half a bar. After which he gave Laura half of what he had left plus half a bar. And, finally, he gave Alfred half of what he had left plus half a bar. Then he had no bars left. How many candy bars did Raymond have to start?

4-8 WRITING EQUATIONS

When problems are solved with algebra, the first step is to translate the given facts into the language of algebra. The kinds of problems in this section give two facts. We use one fact to express each quantity in terms of the variable. The other fact enables us to write an equation.

Example 1. A number is 4 times another number. The sum of the numbers is 59. Write an equation with the smaller number as its solution.

Solution. The facts of the problem are

① A number is 4 times another number.

② The sum of the numbers is 59.

Method A. Use fact ① to express the larger number in terms of the smaller number.

Let the smaller number be represented by x.

Then, the larger number is $4x$.

Use fact ② to write the equation.

The sum of the numbers is 59.

$$x \quad + \quad 4x \quad = \quad 59$$

Method B. Use fact ② to relate the two numbers.

Let the smaller number be represented by x.

Then, the larger number is $59 - x$.

Use fact ① to write the equation.

The larger number is 4 times the smaller number.

$$59 - x = 4x$$

In *Example 1*, the two methods produce two forms of the same equation. This illustrates that the equation is independent of the fact that was used to write it. However, when one fact is more complicated than the other, use the complicated fact to write the equation.

Example 2. The sum of two numbers is 117. Five times the smaller number is seven less than three times the larger. Write an equation with the larger number as its solution.

Solution. The simple fact is: The sum of two numbers is 117.

Let the larger number be represented by x.

Then, the smaller number is $117 - x$.

Use the complicated fact to write the equation.

Five times the smaller | is | three times the larger | less seven

$$5(117 - x) = 3x - 7$$

Example 3. Find two consecutive odd numbers with a sum of 352.
Write an equation with the smaller number as its solution.

Solution. Let the smaller number be represented by x.
Then, the larger number is $x + 2$.
Write the equation.
The sum of
the numbers is 352

$$x + x + 2 = 352$$

EXERCISES 4-8

Ⓐ

In Exercises 1 to 4, write algebraic expressions to complete parts a) and b). Then write an equation for part c).

1. Ravi is 8 years older than Natasha.
   a) Let Natasha's age be represented by ▒ years.
   b) Then, Ravi's age is ▒ years.
   c) The sum of their ages is 42.
2. Gayle's mass is 2.5 kg less than Maria's.
   a) Let Maria's mass be represented by ▒ kilograms.
   b) Then, Gayle's mass is ▒ kilograms.
   c) The sum of their masses is 97.5 kg.
3. A 12 m tree trunk is cut into two pieces.
   a) Let the length of the shorter piece be represented by ▒ metres.
   b) Then, the length of the longer piece is ▒ metres.
   c) The shorter piece is one-third the length of the longer.
4. The ages of Paul and Judy total 27 years.
   a) Let Paul's age be represented by ▒ years.
   b) Then, Judy's age is ▒ years.
   c) Judy's age plus twice Paul's age is 43.

Ⓑ

Write an equation the solution of which will solve the problem for each of Exercises 5 to 15.

5. Brian ran 2 km less than Tom. They ran a total distance of 12 km. Find how far each boy ran.
6. Marie ran twice as far as Brenda. They ran a total distance of 12 km. Find how far each girl ran.
7. The length of a rectangle is 5 cm longer than the width. The perimeter is 68 cm. Find the dimensions of the rectangle.
8. Find two consecutive numbers with a sum of 263.
9. Find four consecutive numbers with a sum of 234.

10. Find two consecutive even numbers with a sum of 170.

11. One number is one-fifth of another number. The two numbers total 18. Find the numbers.

12. The combined mass of a dog and a cat is 24 kg. The dog is three times as heavy as the cat. Find the mass of each animal.

13. In a class of 33 students, there are 9 fewer boys than girls. Find how many girls there are.

14. Millie is four times as old as Marty. The sum of their ages is 65 years. Find how old the people are.

15. Find two numbers that
    a) are consecutive and have a sum of 83.
    b) differ by 17 and have a sum of 39.

In Exercises 16 to 18, write algebraic expressions to complete parts a) and b). Then write an equation for part c).

16. The sum of two numbers is 54.
    a) Let the smaller number be represented by ▒.
    b) Then, the larger number is ▒.
    c) Twice the smaller is 9 more than the larger.

17. The sum of Millie's and Marty's ages is 65.
    a) Let Millie's age be represented by ▒ years.
    b) Then, Marty's age is ▒ years.
    c) Three times Millie's age is 15 years less than twice Marty's age.

18. Adrienne is twice as old as René.
    a) Let René's age be represented by ▒ years.
    b) Then, Adrienne's age is ▒ years.
    c) The sum of their ages 3 years ago was 48.

Write an equation the solution of which will solve the problem for each of Exercises 19 to 24.

19. The sum of two numbers is 63. Three times the smaller number is 14 more than twice the larger number. Find the numbers.

20. The sum of Julio's and Ramona's ages is 35. Twice Ramona's age is 7 more than Julio's age. Find how old they are.

21. Susan is twice as old as Lana. The sum of their ages 4 years ago was 37. Find how old they are now.

C

22. The sum of three numbers is 33. The second number is 7 less than the first, and the third number is 2 more than the second. Find the numbers.

23. The sum of three numbers is 75. The second number is 5 more than the first, and the third is three times the second. Find the numbers.

24. James has two-fifths the amount that Lorna has, and Muriel has seven-ninths the amount that James has. Together, they have $770. Find how much each person has.

4-9 SOLVING PROBLEMS USING EQUATIONS – PART ONE

Alexia sees a package deal for skis and boots costing \$225. The salesman tells Alexia that the skis cost \$60 more than the boots. Alexia wants to know what the skis cost.

She writes an equation, after listing the information in algebraic terms.

Let b dollars represent the cost of the skis.
Since the skis cost \$60 more than the boots, the boots cost $(b - 60)$ dollars.
The total cost is \$225, so the equation is

$$\begin{aligned} b + (b - 60) &= 225 \\ 2b - 60 &= 225 \\ 2b &= 285 \\ b &= 142.5 \end{aligned}$$

The skis cost \$142.50.

Solving a problem using an equation involves three steps.

1		2		3
Write an equation.	→	**Solve the equation.**	→	**Answer the question.**

The previous sections prepared you for Step 1. Earlier sections in this chapter prepared you for Step 2. Step 3 is merely taking the solution of the equation and answering the question the problem asks.

When you check the solution, do *not* substitute in the equation, substitute in the problem. You could have made a mistake in writing the equation.

Example 1. When four times a number is increased by 25, the result is 77. Find the number.

Solution. Step 1. Let the number be represented by x.
Then, 4 times the number, increased by 25 is $4x + 25$.
The equation is $4x + 25 = 77$.

Step 2.
$$\begin{aligned} 4x + 25 &= 77 \\ 4x &= 52 \\ x &= 13 \end{aligned}$$

Step 3. The number is 13.

Check. The number is 13.
Four times the number is 4(13) or 52.
52 increased by 25 is 77. The solution is correct.

Example 2. In a fishing derby, the mass of fish that Yvonne caught was four times the mass of Michael's catch. Their total catch was 59 kg. How much fish did each person catch?

Solution. Step 1. Let m kilograms represent the mass of Michael's fish.
Then, $4m$ kilograms is the mass of Yvonne's fish.
The equation is $m + 4m = 59$.

Step 2.
$$m + 4m = 59$$
$$5m = 59$$
$$m = 11.8$$

Step 3. Michael's catch was 11.8 kg.
Yvonne's catch was 4(11.8) kg or 47.2 kg.

Check. The total catch was 11.8 kg + 47.2 kg or 59 kg.
The solution is correct.

In *Step 1* of this solution, the unit of mass was included with the variable to represent the quantity of fish caught. It is important to state the unit because a variable represents a number, not a quantity.

In reading the problem, look for the simple fact. Use it to express each quantity in terms of a variable. Use the more complicated fact to write the equation. Solve the equation, and answer the question the problem asks. Check the answer by substituting in the problem.

EXERCISES 4-9

(A)

1. Six times a number increased by 7 is 103. Find the number.
2. When 19 is added to one-quarter of a number the result is 40. Find the number.
3. When 13 is subtracted from three-eighths of a number the result is 11. Find the number.
4. Find two numbers with a difference of 5 and a sum of 27.
5. Find two consecutive numbers with a sum of 45.
6. Mr. Zaluski is 4 years older than Mrs. Zaluski. Their total age is 76 years. How old is each person?
7. A ribbon 22 m long is cut into two pieces. One piece is 10 m longer than the other. How long is each piece?
8. Bruce is 10 years older than Cindy. The sum of their ages is 52. How old is each person?

(B)

9. Ian's mass is 2.5 kg less than that of Sean. The sum of their masses is 121.5 kg. What is the mass of each person?
10. The ages of John and Mary total 27 years. Mary's age plus twice John's age is 40. How old is each person?
11. Joan's jump was longer than Enid's jump by 15 cm. Joan's jump was 1.04 times as long as Enid's jump. How far did each person jump?

12. For two consecutive integers, the sum of the smaller and twice the larger is 38. What are the integers?

13. For two consecutive integers, the sum of twice the larger and three times the smaller is 242. Find the integers.

14. A Jaguar travelled 1.2 times as fast as a Mercedes. The difference in their speeds was 24 km/h. Find the speed of each car.

15. Marie ran twice as far as Brenda. They ran a total distance of 18 km. How far did each person run?

16. One number is five times another number. If the two numbers total 36, find the numbers.

17. The length of a rectangle is 5 cm longer than the width. The perimeter is 54 cm. Find the dimensions of the rectangle.

18. Find two consecutive numbers with a sum of 285.

19. Find three consecutive numbers with a sum of 159.

20. Find four consecutive numbers with a sum of 198.

21. Find two consecutive even numbers with a sum of 226.

22. In a cross-country marathon, Jack and Ted ran a total of 81 km. Ted ran 5 km farther than Jack. How far did each boy run?

23. In a class of 33 students, there are 7 more girls than boys. How many girls are there?

24. Maria is four times as old as Marty. The sum of their ages is 55 years. How old are they?

25. The difference between two numbers is 96. One number is nine times the other. What are the numbers?

26. One number is 0.25 less than another number. The sum of the numbers is 7.25. Find the numbers.

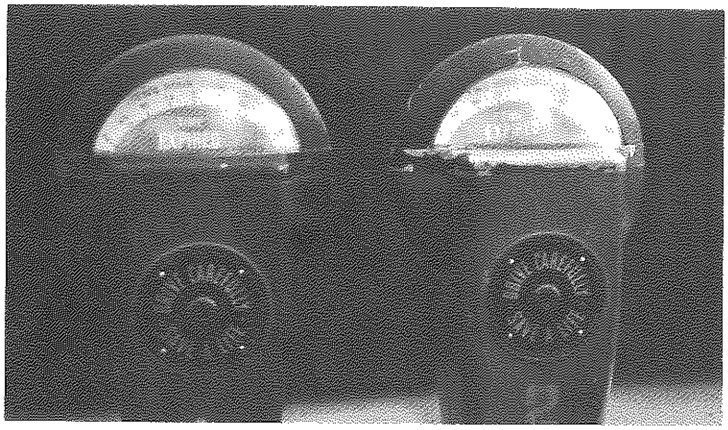
27. The sum of two numbers is 36. Four times the smaller is 1 less than the larger. What are the numbers?

Ⓒ

28. The sum of three numbers is 33. The second number is 7 less than the first, and the third is three times the second. What are the numbers?

29. The sum of three numbers is 75. The second number is 5 more than the first, and the third is three times the second. What are the numbers?

30. The least of three consecutive integers is divided by 10, the next is divided by 17, the greatest is divided by 26. What are the numbers if the sum of the quotients is 10?



4-10 SOLVING PROBLEMS USING EQUATIONS – PART TWO

In some problems where two facts are given, the statement of the second fact may seem quite complicated. It may not be immediately obvious how to write the equation in terms of the chosen variable. In such cases, it is often helpful to use a table to organize the information.

Example 1. A parking meter contains $36.85 in dimes and quarters. If there is a total of 223 coins, how many quarters does the meter contain?

Solution. Step 1. Let x represent the number of quarters.
Then, the number of dimes is $(223 - x)$.
Before writing the equation, write the value of the quarters and the dimes.

	Number of Coins	Value in Cents
Quarters	x	$25x$
Dimes	$223 - x$	$10(223 - x)$

The total value of the coins is $36.85 or 3685 cents.
The equation is $25x + 10(223 - x) = 3685$.

Step 2.
$$25x + 10(223 - x) = 3685$$
$$25x + 2230 - 10x = 3685$$
$$15x = 1455$$
$$x = 97$$

Step 3. The parking meter contains 97 quarters.

Check. Since there are 223 coins, the meter contains 126 dimes.
The value, in cents, of 97 quarters and 126 dimes is
$$(25)(97) + (10)(126) = 2425 + 1260$$
$$= 3685$$
This is $36.85. The solution is correct.

A table can be particularly useful in problems involving ages.

Example 2. A mother is three times as old as her daughter. Six years ago, she was five times as old. How old are the mother and the daughter now?

Solution. Step 1. Let x represent the daughter's age now in years. Then, the mother's age now is $3x$ years.

	Now	6 years ago
Daughter's age in years	x	$x - 6$
Mother's age in years	$3x$	$3x - 6$

The equation is $3x - 6 = 5(x - 6)$.

Step 2.

$$3x - 6 = 5(x - 6)$$
$$3x - 6 = 5x - 30$$
$$24 = 2x$$
$$12 = x$$

Step 3. The daughter is 12 years old and the mother is 36.

Check. Six years ago, the daughter's age was $12 - 6$, or 6.
The mother's age was then $36 - 6$, or 30.
Since $5(6) = 30$, the solution is correct.

EXERCISES 4-10

Ⓐ

1. A vending machine contains nickels and dimes only. There is a total of 80 coins. Copy and complete the table to show the value of each kind of coin.

	Number of Coins	Value in Cents
Nickels	x	
Dimes	$80 - x$	

Write an algebraic expression for the total value of the coins.

2. A pay telephone contains twice as many dimes as nickels and four times as many quarters as nickels. Copy and complete the table.

	Number of Coins	Value in Cents
Nickels	x	$5x$
Dimes		
Quarters		

Write an algebraic expression for the total value of the coins.

3. Debbie is now twice as old as Sandra. Copy and complete the table to show algebraic expressions for their ages at different times.

	Now	Last Year	Next Year	4 Years Ago	3 Years From Now
Debbie's age in years	$2x$				
Sandra's age in years	x				

4. Tracey is 8 years older than Trevor. Copy and complete the table to show algebraic expressions for their ages at different times.

	Now	Last Year	Next Year	5 Years Ago	8 Years From Now
Trevor's age in years	x				
Tracey's age in years	$x + 8$				

5. Jeanne is twice as old as Michel. The sum of their ages three years ago was 45 years. What are their ages now?

6. Yvonne has equal numbers of nickels, dimes, and quarters. Their total value is $2.00. How many of each kind of coin does she have?

(B)

7. Tanya is 12 years older than Leah. Three years ago, Tanya was five times as old as Leah. How old is Leah?

8. Find three consecutive odd numbers with a sum of 267.

9. The length of a rectangular pool is 12 m greater than its width. What is the length if the perimeter of the pool is 96 m?

10. Bill is twice as old as his brother Dan. In 7 years, Bill will be only one and one-half times as old as Dan. How old is Bill now?

11. A collection of nickels and dimes has a total value of $8.50. How many coins are there if there are 3 times as many nickels as dimes?

12. Roberta is three years younger than Rebecca. Eight years ago, Roberta was one-half of Rebecca's age. How old is each girl now?

13. A piggy bank contains 91 coins which are nickels, dimes, and quarters. There are twice as many quarters as dimes, and half as many nickels as dimes. How much is in the piggy bank?

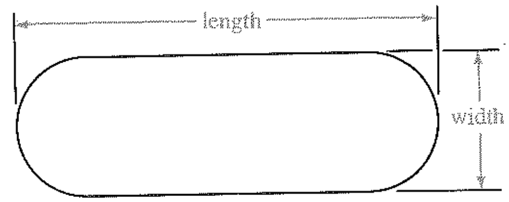
C

14. Sophia's age is four years less than twice Beryl's age. In two years, Beryl's age will be three-quarters of Sophia's age. How old is each girl now?

15. A piece of string, 60 cm long, is cut into three pieces. The middle-sized piece is 2 cm longer than the shortest piece and 2 cm shorter than the longest piece. What is the length of each piece?

16. Girish had $2 more than three times the amount that Joseph had. He gave Joseph $5 who then had one-half as much as Girish. How much did each person have at first?

17. A 500 m track has semicircular ends. If the length of the track is three times its width, what is its width?



18. At Happy Snack, a milk shake costs twice as much as an order of french fries. If two milk shakes and three orders of french fries cost $4.20, what is the cost of a milk shake?

INVESTIGATE

An inequality is a mathematical sentence that uses the sign > or the sign <.

Investigate whether the rules for solving equations can be used to solve inequalities.

1. Write an inequality that is true, for example, $4 < 8$.
2. Apply each operation listed below. Each time, ask the question, "Is the inequality statement still true?"
   - Add the same integer to both sides.
   - Subtract the same integer from both sides.
   - Multiply both sides by the same positive integer.
   - Multiply both sides by the same negative integer.
   - Divide both sides by the same positive integer.
   - Divide both sides by the same negative integer.
3. What appears to be true?
4. Do the rules for solving equations apply to inequalities?

4-11 SOLVING INEQUALITIES

Boyd sees this sign on a rack of jackets for sale. He calculates that if a jacket has a ticket price of \$100, then $\frac{1}{4}$ of \$100 is \$25. Since the saving is *more than* $\frac{1}{4}$, Boyd realizes that the saving would be more than \$25.

If S represents the saving in dollars, then $S > 25$.

Similarly, the sale price would be *less than* \$75.

If P represents the sale price in dollars, then $P < 75$.

$S > 25$ and $P < 75$ are examples of *inequalities*.

Inequalities may also be written using these signs.

$\geqslant$, meaning "is greater than or equal to", and

$\leqslant$, meaning "is less than or equal to".

In the previous investigation, you should have discovered that when both sides of an inequality are multiplied or divided by the same negative number, the statement is no longer true. To keep the statement true, the inequality sign must be reversed, For example,

$$4 < 8$$

$$\text{but } 4(-2) > 8(-2)$$

$$\text{since } -8 > -16$$

$$\text{Also } \frac{4}{-2} > \frac{8}{-2}$$

$$\text{since } -2 > -4$$

> When both sides of an inequality are multiplied or divided by a negative number, the inequality sign must be reversed.

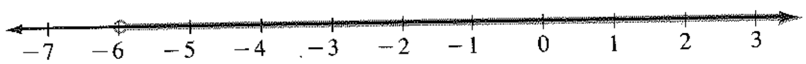
Example 1. Solve. $3x < 5x + 12$

Solution.

$3x < 5x + 12$	or	$3x < 5x + 12$
$-12 < 2x$		$-2x < 12$
$-6 < x$		$x > -6$
$x > -6$		

In the alternative solution, the inequality sign was reversed because of the division by -2.

The solution $x > -6$ means that any number greater than -6 satisfies the inequality. This solution can be illustrated on a number line.



An arrow is drawn in the direction "greater than -6". The open dot at -6 indicates that -6 is not part of the solution.

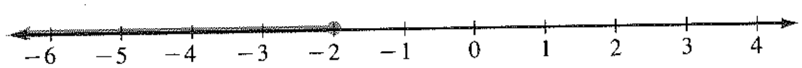
It is not possible to check the limitless number of solutions to an inequality. However, select one solution and substitute it into the inequality. If the resulting statement is correct, a reasonable conclusion is that the solution is correct.

Example 2. Solve, graph, and check. $3 - 2a \geqslant a + 9$

Solution.

$$\begin{aligned} 3 - 2a &\geqslant a + 9 \\ 3 &\geqslant 3a + 9 \\ -6 &\geqslant 3a \\ -2 &\geqslant a \\ a &\leqslant -2 \end{aligned}$$

Graph the solution on a number line.



The solid dot at -2 indicates that it is part of the solution.

Check. Since the solution is less than or equal to -2, this includes -5. Substitute $a = -5$ in the inequality.

Left side $= 3 - 2a$	Right side $= a + 9$
$= 3 - 2(-5)$	$= -5 + 9$
$= 3 + 10$	$= 4$
$= 13$	

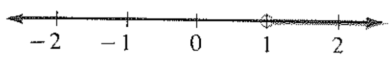
Since $13 > 4$, the left side *is* greater than the right side, and $a = -5$ satisfies the inequality. This suggests that $a \leqslant -2$ is the correct solution.

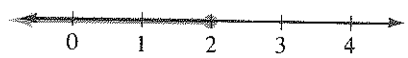
EXERCISES 4-11

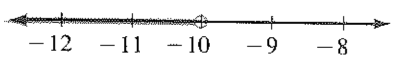
Ⓐ

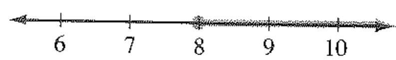
1. Solve and graph.
   a) $x + 1 < 4$ b) $x - 1 \leqslant 3$ c) $x + 3 > 2$
   d) $4 > 9 - x$ e) $-13 \geqslant x - 11$ f) $9 \leqslant 15 - x$

2. Write the inequality represented by each graph.

   a)


   b)


   c)


   d)


Ⓑ

3. Solve and graph.
   a) $7x < 14$ b) $5x \geqslant 10$ c) $9 > -2x$
   d) $3y + 8 > 17$ e) $21 - 5z > 11$ f) $13.5 + 2y \leqslant 18.5$
   g) $13 \leqslant 1 - \frac{3}{4}x$ h) $61 < 13w - 4$ i) $18 \geqslant 4.5 - 1.5a$

4. Solve, graph, and check.
   a) $4x - 7 \geqslant 2x + 5$ b) $13 - 2y \leqslant 4y - 14$
   c) $-25 + 11z \leqslant 30 - 11z$ d) $39 + 4w \geqslant 13 - 6w$
   e) $3(x + 2) < 11$ f) $2(x + 8) \leqslant 4(3 + x)$
   g) $5(2 - x) \leqslant 2(x + 7)$ h) $\frac{2}{3}(15 - 3x) > \frac{1}{2}(2 + 5x)$

5. Solve and check.
   a) $-3y + 8 < 5 - 7y$ b) $18 + 10z \geqslant -12 - 2z$
   c) $14x - 9 \leqslant 17 + x$ d) $-35 - 8a > 6a - 7$
   e) $-4(7 + 2b) \geqslant 3b + 5$ f) $6(-2c - 11) < -5(3c + 8)$
   g) $3(-8 + 2x) > 4 - 2(3x + 5)$ h) $\frac{1}{4}(-3y + 7) \leqslant \frac{2}{5}(8 - 3y)$

INVESTIGATE

Which of these statements is false?

- If $\frac{a}{b} = \frac{c}{d}$ and $a > c$, then $b > d$.
- If $x < y$, then $x < 2y$.

Review Exercises

1. Solve.
   a) $5 + x = -11$
   b) $y - 14 = 83$
   c) $14 - z = -14$
   d) $8 - t = -2$
   e) $w + 21 = -13$
   f) $17 = 19 - t$
   g) $15 = x + 23$
   h) $31 = y - 11$
   i) $-p - 16 = -21$

2. Solve.
   a) $5x = 45$
   b) $-15 = -3n$
   c) $\frac{1}{5}t = -3$
   d) $\frac{n}{14} = -7$
   e) $\frac{s}{5} = \frac{1}{3}$
   f) $8r = 56$
   g) $1.3x = 9.1$
   h) $\frac{x}{17} = \frac{39}{51}$

3. Solve.
   a) $6p - 3 = 15$
   b) $13 = 4 + 3x$
   c) $-6 - 2r = 8$
   d) $5 - 5y = 1$
   e) $8p - 3 = 7$
   f) $3x - \frac{1}{5} = 4$
   g) $3y - 7 = 14$
   h) $\frac{1}{4}x - \frac{2}{3} = 2$
   i) $2.7y - 3.1 = 5$
   j) $\frac{4}{5} - \frac{1}{3}x = \frac{1}{2}x$
   k) $1.69 - 1.3x = 0$
   l) $-64.5 + 2.5x = -2$

4. The cost, C cents, of making copies on a copying machine is given by the formula $C = 90 + 3n$, where n is the number of copies.
   a) What is the cost of making 200 copies?
   b) How many copies can be made for \$6.00?

5. The cost, C dollars, of a telephone call from Vancouver, B.C., to St. John's, Newfoundland, is given by the formula $C = 1.20 + 0.95(n - 1)$, where n is the time in minutes, for the call, and $n \geqslant 1$.
   a) Find the cost of a call that lasts:
      i) 1 min ii) 3 min iii) 5 min.
   b) The charge for one call was \$24. How long was the call?

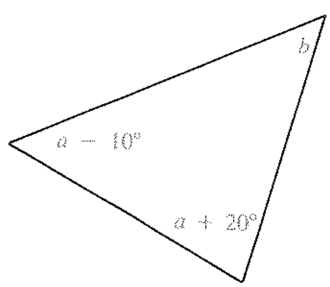
6. Solve.
   a) $5y - 2 = 3y + 4$
   b) $-7x + 6 = 2x - 3$
   c) $r - 3 = 2r + 4$
   d) $11 - 1.3x = 4.7x - 7$
   e) $4(x - 3) = -2$
   f) $-5(y + 3) = 14$
   g) $\frac{3}{8}(2 - 4x) = -\frac{5}{4} + \frac{x}{2}$
   h) $0.5(5x - 3) = 1.2$
   i) $3(1 - x) = -2(2 - x)$
   j) $-0.3(0.2a - 0.7) = 0.4(1.1a - 1.2)$
   k) $\frac{3}{4}(2x - 3) = \frac{5}{6}(-2 - 4x)$
   l) $-\frac{2}{3}(7 + 5a) = 1 + \frac{3}{2}(-4 + 5a)$
   m) $-2 + \frac{1}{5}(-4a + 6) = \frac{1}{10}(3 + 2a)$
   n) $\frac{5}{8}(-7c + 3) = -3 - \frac{3}{4}(3 - 7c)$
   o) $3 + \frac{1}{4}(5x + 3) = \frac{3}{8}(x - 4)$
   p) $-\frac{5}{2}(-4 + 3x) = 1 - \frac{3}{4}(3x + 4)$

7. Solve.
   a) $-2z - 3 + 5z = 6 - z - 9$
   b) $-8t + 7 - 5t = 9 - t - 11$
   c) $-13 - q - 9 = 5q - 9 - q$
   d) $-19 - 3r + 6 = 7r - 11 - r$
   e) $4(w - 5) - w = -9 - w + 7$
   f) $-7(p - 3) + 11 = 3p - 12 - p$
   g) $t + 17 - 2t = -3(t - 1) - 3$
   h) $9 - 3(1 - q) = 7 - 4(2 - q)$
   i) $-9(r + 3) - 9r = -3r - (3 - r) + 8$
   j) $3(7v + 8) - 9 = 14 - 2v + 6(3v - 4)$
   k) $5(6w - 3) - 7 = 17 + 3w - 5(2w + 1)$
   l) $-2(5x - 1) - 4 = 22 - 7x + 8(3x - 1)$

8. Solve and check.
   a) $3(x + 1) = 2x$
   b) $4n - 2 = n + 1$
   c) $8 - x = x - 8$
   d) $12 - y = 3y - 14$
   e) $4(t + 1) = 2t + (1 - t)$
   f) $6(w - 2) = 3w + 2(w + 1)$
   g) $7.2x - 7.5 - 1.7x = 4.6 + 4.4x$
   h) $15(0.3 - z) + 14.5z = 2(0.5z - 10)$

9. The speed, s metres per second, of an object is given by the formula $s = \frac{d}{t}$, where t seconds is the time taken to travel a distance d metres. Find the distance travelled for each speed and time.
   a) 15 m/s for 3 s
   b) 2 m/s for 12 s
   c) 160 m/s for 15 s
   d) 180 m/s for 6 s
   e) 19 m/s for 13 s
   f) 80 km/h for 1.5 h

10. The sum of the angles of a triangle is 180°. For the triangle shown
    a) Find the value of a when b is:
       i) 30° ii) 60°.
    b) Find the value of b when a is:
       i) 20° ii) 50°.



11. A ball, dropped from a height of d centimetres, bounces to a height of b centimetres where $b = \frac{3}{4}d$.
    a) Find the height from which the ball was dropped if the height of the bounce is:
       i) 90 cm ii) 75 cm iii) 60 cm iv) 39 cm
    b) The ball is dropped from a height of 160 cm. Find the height of:
       i) its second bounce ii) its third bounce.

Write an equation the solution of which will solve the problem for each of Exercises 12 to 16.

12. An airplane travels eight times as fast as a car. The difference in their speeds is 420 km/h. Find how fast each vehicle is travelling.

13. Find three consecutive numbers with a sum of 141.

14. In a cross-country marathon, Jack and Jill ran a total of 73 km. Jill ran 5 km farther than Jack. Find how far each person ran.

15. Jeanne is twice as old as Michel. The sum of their ages 3 years ago was 45 years. Find their ages now.

16. Five times a number decreased by 8 is 17. Find the number.

17. Jason is three times as old as Mark. The sum of their ages is 20 years. How old is Mark?

18. Jackie ran 2 km farther than Pat. They ran a total distance of 14 km. How far did each person run?

19. The combined mass of a dog and a cat is 21 kg. The dog is two-and-one-half times as heavy as the cat. What are their masses?

20. Roger has some dimes and quarters with a total value of $2.50. If he has three more quarters than dimes, how many of each kind of coin does he have?

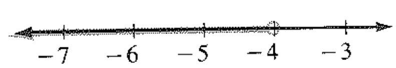
21. Mrs. Jenkins is three times as old as her son, Jerry. In 12 years, Mrs. Jenkins will only be twice as old. How old is Jerry now?

22. One number is seven times one-half of another number. The numbers differ by 35. What are the numbers?

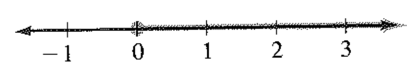
23. Donna's average mark out of three tests was 84 out of 100. Her highest mark was one-and-one-quarter times her lowest mark. The middle mark was 81. What were Donna's marks on the three tests?

24. Write the inequality represented by each graph.

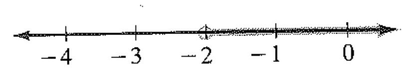
a)



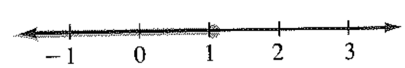
b)



c)



d)



25. Solve and graph.

a) $5x - 17 < 19 - 4x$

b) $\frac{3}{4}y + \frac{1}{3} \geqslant \frac{1}{2}y + \frac{1}{4}$

c) $7(3 - 2z) \leqslant -2(7 + 2z)$

d) $-0.6(3a - 7) > -0.7(-4 + 2a)$