3 From Arithmetic to Algebra

A 4 m log is cut into 2 pieces. Suppose you knew the length of one piece. How could you find the length of the other piece? (See Section 3-5, *Example 1*.)

3-1 WHAT IS ALGEBRA?

A group of grade 9 students decided to participate in Junior Achievement. This involves setting up and running a business. The students decided to make trivets. After a few weeks, the students had a meeting to look at their sales figures. Here is a chart showing the sales for the first 3 weeks.

Trivets Sold	50	70	40
Profit	\$300	\$420	\$240

How could the students calculate their profit if they knew how many trivets they sold?

We need to find a general rule relating the number of trivets sold and the profit. Notice that the profit in dollars is six times the number of trivets sold.

If we let t represent the number of trivets sold, then the profit is $6t$ dollars. This profit of $6t$ dollars represents many different amounts of money. Each amount depends on a value of t, which can vary. For this reason, t is called a *variable*.

The use of a letter to represent a number is the basis of *algebra*. Algebra is used to express patterns in arithmetic, in a general way.

Example 1. Suppose this pattern were continued.

a) How many toothpicks would be needed to make 10 triangles?
b) How could the number of toothpicks be found if the number of triangles were known?

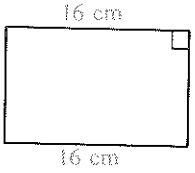
Solution. a) Identify the pattern. The number of triangles in a figure coincides with its position in the pattern. For example, the third figure has 3 triangles. Every triangle has 3 sides but adjacent triangles have a common side. In any figure, there is 1 toothpick to begin the pattern and each triangle adds 2 toothpicks. Therefore, to make 10 triangles, we would need $1 + 2(10)$ or 21 toothpicks.

b) For any number of triangles, we need to *generalize* the pattern found in part a). That is, the number of toothpicks is 1 more than twice the number of triangles.

If the pattern were to continue, how many toothpicks would there be on the 150th figure?

Example 2. A rectangle has a length of 16 cm.
a) Calculate the perimeter of the rectangle if its width is:
i) 10 cm ii) 8 cm iii) 5 cm.
b) How could the perimeter of the rectangle be found if its width were known?
c) Write an expression for the perimeter of the rectangle in terms of a variable.

Solution. Draw a diagram.



a) The perimeter is the distance around a rectangle.
i) The width is 10 cm so the perimeter is $[2(10) + 2(16)]$ cm or 52 cm.
ii) The width is 8 cm so the perimeter is $[2(8) + 32]$ cm or 48 cm.
iii) The width is 5 cm so the perimeter is $[2(5) + 32]$ cm or 42 cm.

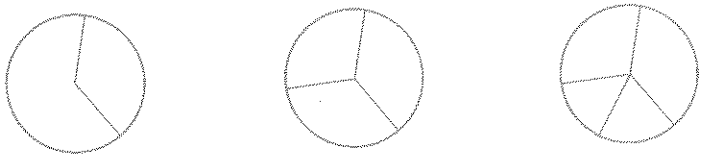
b) To find the perimeter for any width, look for a pattern in part a) of the solution.
The perimeter is the sum of twice the width, and 32.

c) Let w represent the width of the rectangle. Then, the perimeter is $(2w + 32)$ centimetres.

EXERCISES 3-1

(A)

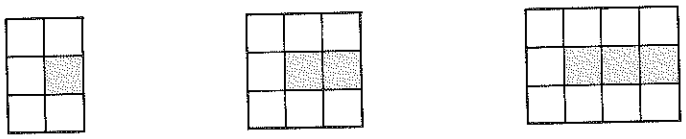
1. Suppose this pattern were continued.



a) How many regions would there be in the 14th circle? 15

b) If the position of the circle in the pattern were known, how could the number of regions be found?

2. Suppose this pattern were continued.



a) On the 8th figure, how many squares are: i) shaded ii) unshaded?

b) How could the number of unshaded squares be found if the number of shaded squares were known?

3. Suppose this pattern were continued.

X OX OOX OOOX
XO OXO OOXO
XOO OXOO
XOOO

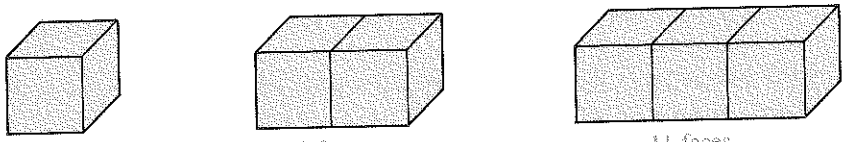
a) On the 20th diagram, how many:
i) Xs are there ii) Os are there?

b) On any diagram, how could the number of Os be found if the number of Xs were known?

c) Let a represent the number of Xs. Write an expression for the corresponding number of Os.

(B)

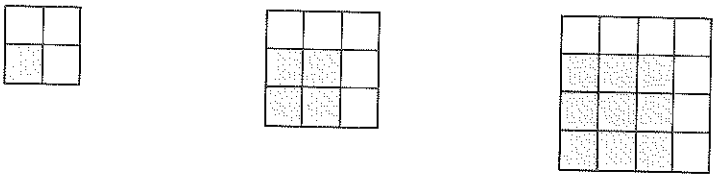
4. A series of cubes are placed together as shown. The total number of faces that show are counted.



5 faces 8 faces 11 faces

Suppose this pattern were to continue.
a) How many faces would show on the 12th diagram?
b) How many faces would show on the 30th diagram?
c) If the number of cubes were known, how could the number of faces be found?
d) Write an expression for the number of faces in terms of the number of cubes.

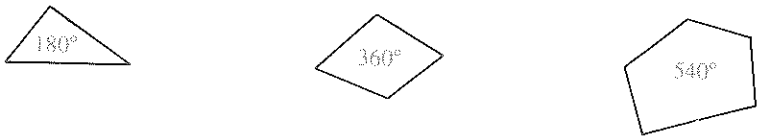
5. Suppose this pattern were continued.



a) How many shaded squares would there be on the 12th diagram?
b) How many unshaded squares would there be on the 12th diagram?
c) If the position of the diagram in the pattern were known, how could:
   i) the number of shaded squares be found
   ii) the number of unshaded squares be found?
d) i) Write an expression for the number of shaded squares in terms of the position of the diagram.
   ii) Write an expression for the number of unshaded squares in terms of the position of the diagram.

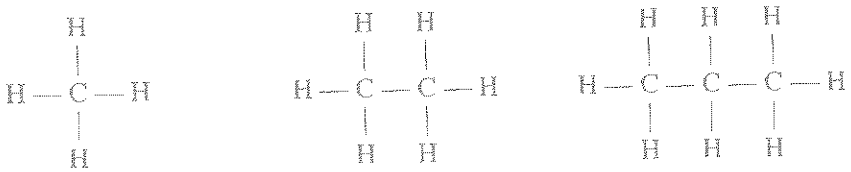
6. A cow is milked twice a day. Each time, she gives 11 kg of milk.
a) Calculate the total milk production after:
   i) 16 days ii) 49 days iii) 35 weeks.
b) If the number of days that the cow is milked is known, how can the amount of milk be found?
c) Write an expression for the amount of milk produced in terms of the number of days that the cow is milked.
d) If the total amount of milk is known, how can the number of milking days be found?
e) Write an expression for the number of days that the cow has been milked, in terms of the amount of milk produced.

7. The sum of the interior angles of each polygon is shown. Suppose this pattern were continued.



a) Calculate the sum of the interior angles of:
   i) a decagon ii) a 15-sided polygon.
b) If the number of sides of a polygon is known, how can the sum of the interior angles be found?
c) Write an expression for the sum of the interior angles of a polygon in terms of the number of sides of the polygon.

8



methane ethane propane

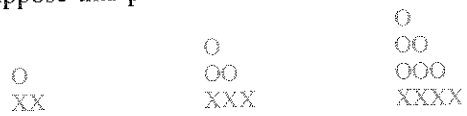
The diagrams show the molecular structures of some fuels. C represents a carbon atom and H represents a hydrogen atom. How can the number of hydrogen atoms in a molecule of octane be found, if the number of carbon atoms is 8?

9. Suppose this pattern were continued.

a) Which way will:
i) the 20th arrow point ii) the 33rd arrow point iii) the 47th arrow point?

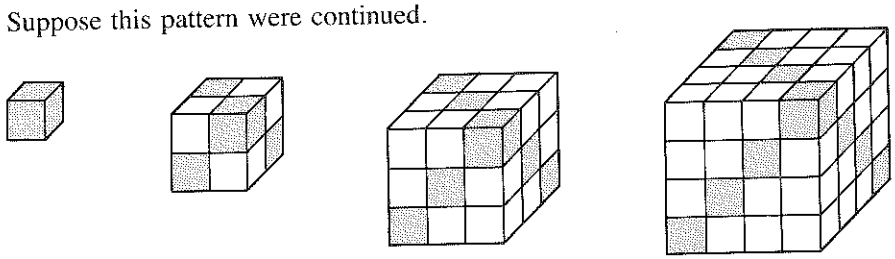
b) If the position of the arrow in the pattern is known, how can the direction of the arrow be found?

10. Suppose this pattern were continued.



a) How many Xs would there be on the 10th diagram?
b) How many Os would there be on the 10th diagram?
c) If the number of Xs were known, how could the number of Os be found?
d) Write an expression for the number of Os in terms of the number of Xs.

11. Suppose this pattern were continued.



The cubes along one diagonal of each face of a cube are colored, as shown, including the diagonals of the faces that can't be seen.

a) How many cubes are colored on the 5th diagram?
b) How many cubes are plain on the 10th diagram?
c) If the position of the diagram in the pattern were known, how could:
i) the number of colored cubes be found
ii) the number of plain cubes be found?

3-2 SUBSTITUTING IN ALGEBRAIC EXPRESSIONS

In hockey standings, 2 points are given for a win and 1 point is given for a tie. Suppose you knew the numbers of wins and ties that a hockey team had in a season. How could you find the total points?

Let w represent the number of wins and t represent the number of ties. The total points scored can be expressed as $2w + t$. This is an example of an *algebraic expression*; $2w$ and t are called the *terms* of this expression.

A term usually has a variable (or variables). The term $2w$ has the variable w. A term always has a *coefficient*. For example, $2w$ has the coefficient 2 and t has the coefficient 1, that is, t represents $1t$. When a term has the coefficient 1, it is not usually written as part of the term.

In the first half of the 1986/87 season, the Maple Leafs had 17 wins and 5 ties. The team's total points can be calculated by *substituting* into the algebraic expression $2w + t$.

Substitute 17 for w and 5 for t.

$$\begin{aligned} & 2w + t \\ = \ & 2(17) + 5 \\ = \ & 34 + 5 \\ = \ & 39 \end{aligned}$$

The Maple Leafs had a total of 39 points for the first half of the 1986/87 hockey season.

Example 1. Copy and complete.

Expression	Variables	Terms	Coefficients
$2m - 9n$			
$35x + 17y$			
$5a - 4b + 6c$			

Solution.

Expression	Variables	Terms	Coefficients
$2m - 9n$	m, n	$2m, 9n$	2, 9
$35x + 17y$	x, y	$35x, 17y$	35, 17
$5a - 4b + 6c$	a, b, c	$5a, 4b, 6c$	5, 4, 6

Example 2. Copy and complete.

a)

+	3	-1	x	w	$3d$
5	8	4	$x + 5$		
1				$w + 1$	
7					

b)

×	3	-1	x	w	$3d$
5	15	-5	$5x$		
1				w	
7					

Solution. a)

+	3	-1	x	w	$3d$
5	8	4	$x + 5$	$w + 5$	$3d + 5$
1	4	0	$x + 1$	$w + 1$	$3d + 1$
7	10	6	$x + 7$	$w + 7$	$3d + 7$

b)

×	3	-1	x	w	$3d$
5	15	-5	$5x$	$5w$	$15d$
1	3	-1	x	w	$3d$
7	21	-7	$7x$	$7w$	$21d$

Example 3. Evaluate.

a) $5x + 9$ for $x = 6$

b) $3a - 7b$ for $a = -9$ and $b = 2$

c) $2.6m$ for $m = 2.5$

Solution. a) When $x = 6$, $5x + 9 = 5(6) + 9$
$= 39$

b) When $a = -9$ and $b = 2$, $3a - 7b = 3(-9) - 7(2)$
$= -41$

c) When $m = 2.5$, $2.6m = (2.6)(2.5)$
$= 6.5$

EXERCISES 3-2

(A)

1. Copy and complete.

	Expression	Variables	Terms	Coefficients
a)	$6p - 2q$			
b)	$a - 2b + 9c$			
c)	$1.8C + 32$			
d)	$2\pi r$			

2. Copy and complete.

a)

+	3	7	a	$2b$	ab
4					
9					
x					
y					
xy					

b)

×	3	7	a	$2b$	ab
4					
9					
x					
y					
xy					

3. The students at the John Cabot Secondary School write their examinations in the gymnasium. How many students can write at one time if:
   a) there are 11 rows and 32 desks in each row
   b) there are 11 rows and d desks in each row
   c) there are r rows and 32 desks in each row
   d) there are r rows and d desks in each row?

4. At a track and field meet, points are awarded as follows:
   first place — 5 points, second place — 3 points, third place — 1 point.
   How many points would be awarded for:
   a) 4 firsts, 2 seconds, and 6 thirds
   b) x firsts
   c) y seconds
   d) x firsts, y seconds, and z thirds?

5. Evaluate.
   a) $2x + 7$ for $x = 5$
   b) $28 - 5m$ for $m = -3.5$
   c) $9x - 4y$ for $x = -8$ and $y = 7$
   d) $8a + 19b$ for $a = 28$ and $b = -8$
   e) $8a - 4b - c$ for $a = 12$, $b = 18$, and $c = -3$
   f) $-3m + 5n - 6p$ for $m = -3$, $n = 4$, and $p = -5$

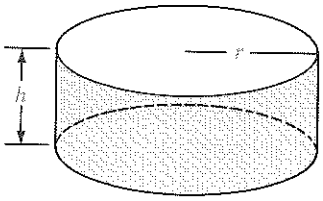
B

6. Evaluate.
   a) $2.3x + 0.7y$ for $x = 4$ and $y = -8$
   b) $0.27j - 3k$ for $j = 2.3$ and $k = 0.09$
   c) $3.7a - 2.1b$ for $a = 4.8$ and $b = 3.7$
   d) $5m - 9.2n$ for $m = -2.8$ and $n = 0.6$
   e) $8.3r - 1.27s + 0.6t$ for $r = 0.8$, $s = -5$, and $t = -0.5$
   f) $-4.8d + 2.3f - 2.8g$ for $d = 1.1$, $f = -2.2$, and $g = 0.7$
   g) $-0.6p - 1.3q - 2.5r$ for $p = 2.5$, $q = 1.2$, and $r = 0$

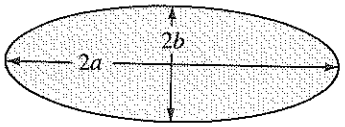
7. Evaluate.
   a) $\frac{3}{4}c + \frac{5}{7}d$ for $c = 12$ and $d = -14$
   b) $\frac{5}{6}m - \frac{2}{9}n$ for $m = \frac{2}{5}$ and $n = \frac{3}{8}$
   c) $\frac{2}{5}x + \frac{1}{3}y$ for $x = \frac{3}{4}$ and $y = -\frac{4}{5}$
   d) $\frac{2}{5}p + \frac{2}{3}q$ for $p = \frac{1}{2}$ and $q = \frac{9}{22}$
   e) $\frac{3}{8}w + \frac{5}{6}y - \frac{3}{4}z$ for $w = \frac{1}{3}$, $y = -\frac{8}{15}$, and $z = \frac{2}{3}$
   f) $-\frac{1}{8}a - \frac{2}{7}b - \frac{3}{4}c$ for $a = \frac{4}{3}$, $b = -14$, and $c = -4$
   g) $-\frac{7}{10}d + \frac{3}{5}e + \frac{2}{15}f$ for $d = -\frac{5}{2}$, $e = -\frac{10}{7}$, and $f = \frac{5}{2}$

8. The cost, C dollars, of installing a steel-panel fence is given by this formula.
$C = 7l + 15p + 80$
l is the length of the fence in metres and p is the number of posts required. Find the total cost when:
a) $l = 120$ and $p = 41$
b) $l = 32$ and $p = 12$
c) the fence is 65 m long and requires 25 posts
d) 85 posts are required for a 250 m fence.

9. The formula for the curved surface area, A, of a cylinder is $A = 2\pi rh$ where r is its radius and h is its height. Find the curved surface area of a cylinder that has a radius 4 cm and a height 15 cm.



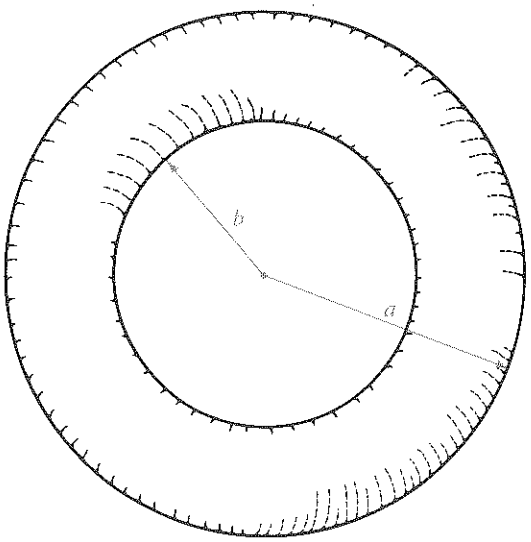
10. The area, A, of an ellipse is given by the formula $A = \pi ab$. Calculate the area when a is 31 cm and b is 19 cm.

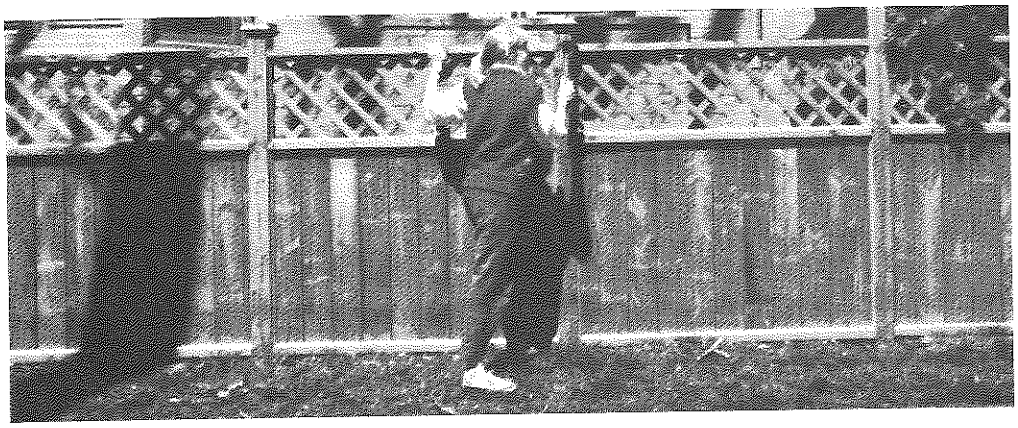


11. The intelligence quotient (IQ) is a measure of a student's intellectual ability. The formula is $\text{IQ} = \dfrac{100m}{p}$, where m is the mental age and p the physical age. Calculate the IQ of a student who is:
a) 14 years old with a mental age of 16
b) 15 years old with a mental age of 13.

C

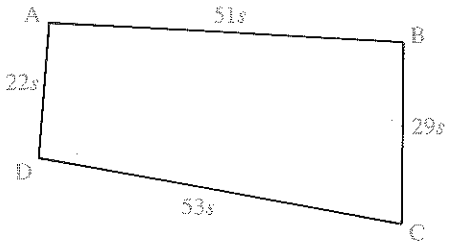
12. A Canadian astronaut, Steve MacLean, predicts that there will be a permanent space station in orbit in the near future. The volume, V, of one proposed station is given by this formula.
$V = \dfrac{1}{4}\pi^2(a^3 - a^2b - ab^2 + b^3)$
a and b are the external and internal radii in metres. Find the volume of the space station if $a = 500$ and $b = 400$.





3-3 LIKE TERMS

Mrs. Zak wanted to fence her yard. She did not have a measuring tape long enough, so she used a long piece of wood to measure each side. Then, Mrs. Zak drew a plan of the yard. If she knew the length of the wood, how could she find the length of fencing she needed?



On the plan, the letter s represents the length of the wood.

For side AB, the wood was placed 51 times.
For side BC, the wood was placed 29 times.
For side CD, the wood was placed 53 times.
For side DA, the wood was placed 22 times.

The total number of times the wood was placed is 51 + 29 + 53 + 22 or 155.

Mrs. Zak measured the piece of wood and it was 83 cm long. She calculated the length of fencing as 155(0.83) m or 128.65 m. She ordered 130 m of fencing.

Algebraic terms that have the same variable (or variables) such as $51s$, $29s$, $53s$, and $22s$ are called *like terms*.

The terms $3a$, $4b$, and $10c$ are *unlike terms* because they have different variables. Similarly, $2p$, $7q$, and 9 are unlike terms.

Like terms can be combined into a single term by adding (or subtracting) their coefficients.

Example 1. Simplify.

a) $2x + 3x$ b) $5a - a$ c) $12y + 6 + 3y$

Solution. a) Since $2x + 3x$ means $x + x + x + x + x$, or $5x$, then $2x + 3x = 5x$

b) Since $5a - a$ means $a + a + a + a + a - a$, or $4a$, then $5a - a = 4a$

c) $12y + 6 + 3y = 12y + 3y + 6$
$= 15y + 6$
$15y$ and 6 are unlike terms and cannot be combined.

Example 2. Simplify. $26x - 11y - 12x + 3x - 5y$

Solution. $26x - 11y - 12x + 3x - 5y$ can be written as
$(+26x) + (-11y) + (-12x) + (+3x) + (-5y)$
$= (+26x) + (+3x) + (-12x) + (-11y) + (-5y)$
$= (+29x) + (-12x) + (-16y)$
$= (+17x) + (-16y)$
$= 17x - 16y$

With practice, algebraic expressions can be simplified more directly.

Example 3. Simplify. $-6a + 14b - 13a + 7b + 10a - 11b$

Solution. $-6a + 14b - 13a + 7b + 10a - 11b$
$= -6a - 13a + 10a + 14b + 7b - 11b$
$= -19a + 10a + 21b - 11b$
$= -9a + 10b$

At the beginning of this section, the lengths of the sides of a yard were represented by like terms.

The perimeter of the yard can be found in two ways.

Method 1.

Perimeter $= 51s + 29s + 53s + 22s$
$= 155s$
Substitute 0.83 for s.
Perimeter $= 155(0.83)$
$= 128.65$

Method 2.

Perimeter $= 51s + 29s + 53s + 22s$
Substitute 0.83 for s.
Perimeter $= 51(0.83) + 29(0.83) + 53(0.83) + 22(0.83)$
$= 42.33 + 24.07 + 43.99 + 18.26$
$= 128.65$

These two methods illustrate that when substituting into an algebraic expression, it is more efficient to simplify by collecting like terms first.

Example 4. Simplify then evaluate.

a) $14p - 5p$ for $p = -5$ b) $16y - 29y - 15$ for $y = 3.7$

Solution. a) $14p - 5p = 9p$

When $p = -5$,

$9p = 9(-5)$

$= -45$

b) $16y - 29y - 15 = -13y - 15$

When $y = 3.7$,

$-13y - 15 = -13(3.7) - 15$

$= -48.1 - 15$

$= -63.1$

EXERCISES 3-3

(A)

1. Simplify.

a) $4a + 7a$ b) $19m - 6m$ c) $-42x + 29x$

d) $14p - 5p$ e) $-21g - 16g$ f) $12b + 37b$

g) $6r + 47r - r$ h) $13w + w - 9w$ i) $36p - 29p - 24p$

2. Simplify.

a) $-7x + 5x + 8y - 3y$ b) $-18m + 7m + 6p - 11p$

c) $9a - 23b - 4a - 11b$ d) $52x + 31y - 31x - 2y$

e) $44u + 17v - 4v + 41u$ f) $-7j + 13k + 5j - k$

g) $4s + 5t - 19t - 37s$ h) $28x + 15y - 19x - 11y$

i) $-6a + 9b - 7c + 5b - 3a - c$ j) $14x - 17y - 5x - 11z - 6y - 2z$

(B)

3. Simplify where possible.

a) $4m + 5 - 3m$ b) $2c + d - 3c - d$ c) $5a + 3b + 5a$

d) $3x + 2y$ e) $8u + 3v - 11v - 7$ f) $5m + 4$

g) $-7x + y - 2x$ h) $15x - 3y - 9x + z$ i) $10p - 5 + 8q - 3p - 2$

4. Simplify.

a) $23a - 42b - 18b + 17a$ b) $12x - 10y - 6x - 6y + x$

c) $45m + 15n - 7 - 5m - 5n$ d) $-32c + 10 - 15c + 4d - 3$

e) $23a + 7a - 13 - 2a$ f) $16x - 17y + x - y$

g) $-2a - 3b - a + 4$ h) $48p - 16q - 3r - 18p - 3r$

5. Simplify.

a) $16a - 3b + 5a + b$ b) $-3 + 2m - 13n - 3n + 2$

c) $-2z + 12 + 10y - 15 + 4z - 15y$ d) $13s - 16r - 4s - 6 + 10 + 3r$

e) $4 + 7x - 3y - 7 - y + 18x$ f) $-4x + 7m - 11x - 10m + 8$

g) $-q + 8q - 11p - 7q + 11p$ h) $-13c - 15d - 18d - 3$

6. Simplify.

a) $-4p + 7s - 8r - 2s - 18s + 10r + 2p$

b) $-8z + 13p + 10x - 12z - x - 23p$

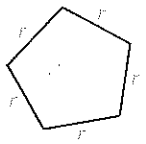
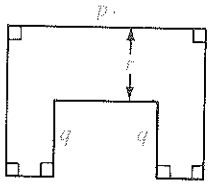
c) $5q - 7m + 6q - 11n + m - 8n + 7q$

d) $-10 + a - 13b + 8 - 15a + 2b - 17$

e) $0.3 + 1.5x - 7.0 + 2.4z - 1.0 - 1.3x + 8 - 1.4z$

f) $0.8 + 2.3m - 11.2n - 1.4 - 14.7n + 3.2m$

7. Simplify then evaluate.
   a) $4a + 7a$ for $a = 3$
   b) $19m - 6m$ for $m = -2.5$
   c) $42x - 29x$ for $x = -7$
   d) $-14p + 5p$ for $p = \frac{2}{3}$
   e) $23b + 17b$ for $b = 5$
   f) $-64k - 44k$ for $k = -\frac{3}{4}$
   g) $4x - 7x - 11x$ for $x = 4$
   h) $16y - 29y - 15y$ for $y = -\frac{1}{2}$

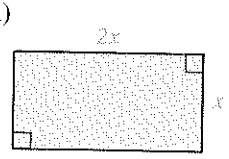
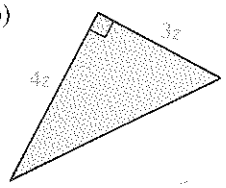
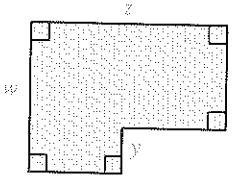
8. Write an expression for the perimeter of each figure.
   a)
   
   b)
   
   c)
   

9. Simplify.
   a) $3x + 7y - 2z - 6y - 5x - 4z + 12x - 5y$
   b) $-10m + 3n - 4p - 7n - 5m - 8p + 17m$
   c) $7c - 2a - 5c - 3b + 8a + 6b - 10c$
   d) $4x - 6x + 5y - 7y + 3x - 4y - 12y - 7x$

10. Simplify.
    a) $3x^2 + 2x - 5x - 4x^2$
    b) $6x - x^2 - 4x - 3x^2$
    c) $8a - 7b - 6a - 2b + 3ab$
    d) $-9p - q^2 + 3r^2 + 4q^2 - 7r$
    e) $4y^2 - 3y + 7 - 2y^2$
    f) $-6b - 3c + 4c^2 - 6 - 6b$
    g) $6x - 3x - 5x + x^2 - 5$
    h) $7x^2 - 9b + 5m^2 - 8b - 7x + 1$
    i) $-w^2 + 2v - 3w - 9v + 4w^2$
    j) $-3a^2 - 3ab + 3b - 4a - 7ab + 3$

11. Simplify.
    a) $3xy - 2yz + 5xz - 6yz + 4xy - 10xz$
    b) $-4ab - 5bc + 3ac + 8ab - 6bc - 9ac$
    c) $-3xyz + 4xyw - 5yzw + 8xyz + 5xyw - 6yzw$

12. Evaluate each expression for $x = 3$, $y = 2$, and $z = -1$.
    a) $-3xy + 5yz - 2xz + 6xy + yz - 5xz$
    b) $4x^2yz + 5xy^2z - 3xyz + 5xy^2z - 3x^2yz + 6xyz$

13. Write expressions for the perimeter and the area of each figure.
    a)
    
    b)
    
    c)
    

PROBLEM SOLVING

Use a Variable

How old is Janet?

Understand the problem

- How old is Janet's father?
- Is Janet's father more than 3 times Janet's age?
- How many years older than 3 times Janet's age is her father?
- What are we asked to find?

Think of a strategy

- Use a variable to represent Janet's age.
- Express Janet's father's age in terms of the variable.

Carry out the strategy

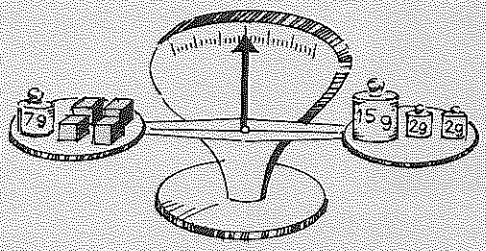
- Let n represent Janet's age.
- Then, 3 times Janet's age is $3n$.
- To obtain father's age, we add 6 to $3n$.
  $\boxed{3n} \rightarrow \boxed{+6} \rightarrow \boxed{42}$
- To calculate $3n$ we reverse the flow chart.
  $\boxed{36} \leftarrow \boxed{-6} \leftarrow \boxed{42}$
- From a comparison of the flow charts, $3n = 36$ so $n = 12$.
  Janet is 12 years old.

Look back

- What is three times Janet's age plus 6?
- Is that the same as her father's age?
- Is 12 the only solution to the problem?

Solve each problem

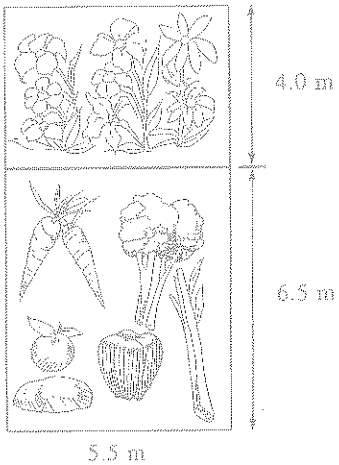
1. What is the mass of each colored cube?



2. One-eighth of a number is 28.5 more than 76. What is the number?
3. The rental cost of a videocassette recorder is $25 plus a fixed daily amount. Ms. Singh was charged $64 for a 4-day rental period. What is the fixed daily rental fee?
4. Sung Choi saved his allowance for eight weeks. When he added to this the $15.50 he received for his birthday, he had a total of $61.50. How much allowance does he receive each week?
5. Margaret handed the cashier a $10 bill for the purchase of 3 tennis balls. After 54¢ tax was added, the clerk gave her $2.71 change. What was the cost of each tennis ball?
6. A rectangular table is twice as long as it is wide. How long is the table if its perimeter is 15.6 m?
7. A gold pen and pencil set cost $69.98. The pen costs $6.50 more than the pencil. What is the cost of the pencil?
8. The sum of two consecutive integers is 25. What are the two integers?
9. In an 18-game hockey tournament between Canada and the U.S.S.R., Canada won 4 more games than they lost. Since there was no tie, how many games did Canada win?
10. Find two integers with a sum of 8 and a difference of 42.
11. When 28 is added to seven times a number, the result is the same as if 16 were subtracted from eleven times that number. What is the number?
12. Find 3 consecutive integers with a sum of 375.

3-4 FROM PRODUCTS TO SUMS AND DIFFERENCES

Mr. Ying grows flowers and vegetables in a rectangular-shaped garden. He needs to buy fertilizer for his garden. Each bag of fertilizer is labelled with the area, in square metres, that the fertilizer should cover. Mr. Ying can calculate the area of his garden in two ways.



Method 1.

$$\begin{aligned}\text{Total area} &= \text{width} \times \text{length}\\ &= 5.5(4.0 + 6.5)\\ &= 5.5(10.5)\\ &= 57.75\end{aligned}$$

Method 2.

$$\begin{aligned}\text{Total area} &= \text{area with flowers} + \text{area with vegetables}\\ &= 5.5(4.0) + 5.5(6.5)\\ &= 22.0 + 35.75\\ &= 57.75\end{aligned}$$

The area of Mr. Ying's garden is about 60 m^2.
Since both methods gave the same area, it follows that
$5.5(4.0 + 6.5) = 5.5(4.0) + 5.5(6.5)$
This is an example of the *distributive law*. It shows a product expanded into a sum.

It can be shown that this law is true for a product expanded into a difference.
For example, consider $5(9 - 4) = 5(9) - 5(4)$

$$\begin{aligned}5(9 - 4) &= 5(5)\\ &= 25\end{aligned} \qquad \begin{aligned}5(9) - 5(4) &= 45 - 20\\ &= 25\end{aligned}$$

This law can be written algebraically.

Distributive Law

$$a(b + c) = ab + ac$$
$$a(b - c) = ab - ac$$

Example 1. Use the distributive law to expand these products.

a) $6(x - 4)$ b) $3(4b + 8)$ c) $4(2x + y - 3)$

Solution. a) $6(x - 4) = 6(x) - 6(4)$
$= 6x - 24$

b) $3(4b + 8) = 3(4b) + 3(8)$
$= 12b + 24$

c) The distributive law can be extended to the sum or difference of more than two terms.
$4(2x + y - 3) = 4(2x) + 4(y) - 4(3)$
$= 8x + 4y - 12$

Some expressions must be expanded before they can be simplified.

Example 2. Simplify.

a) $12(3p + q) - 8(q + 2p)$
b) $1.5(2x - y) - 2.5(2y - x)$

Solution. a) Multiply both terms in the second bracket by -8.
$12(3p + q) - 8(q + 2p) = 36p + 12q - 8q - 16p$
$= 20p + 4q$

b) $1.5(2x - y) - 2.5(2y - x) = 3.0x - 1.5y - 5.0y + 2.5x$
$= 5.5x - 6.5y$

EXERCISES 3-4

(A)

1. Expand.
   a) $3(m - 8)$ b) $18(x + 5)$ c) $11(p + 7)$ d) $-23(a - 9)$
   e) $7(2p + 6)$ f) $4(a - b + 15)$ g) $-8(7a + b - 1)$ h) $6(2s + 11t - 5)$

2. Simplify.
   a) $5(m + 3) + 63$ b) $18(2x + 4) - 27$
   c) $14 - 3(6x + 7)$ d) $96 + 7(3a - 12)$
   e) $-17(3x + 5) - 2$ f) $7a - 3(2a - 9 - b)$
   g) $15e + 5(12 + e - 4f)$ h) $6t + 9(3t - 4) - 12t$
   i) $-2(5x - 7) - 3x - x$ j) $8w - 6(3w + 5) - 19$
   k) $-3(c + 4) + 2(2c - 3)$ l) $-12t + 3(5 - 2t) - 7$

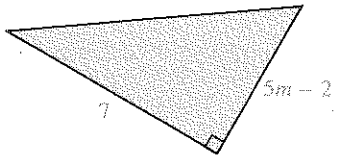
3. Simplify.
   a) $5m - 2 + 3(4m + 1) - 2m$ b) $-4(2c + 5d) - 2(3c - 7d)$
   c) $5(a + 3) + 2(a - 5) + (a - 1)$ d) $4(2a + 5b + 3) - 3(6b - a - 1)$

(B)

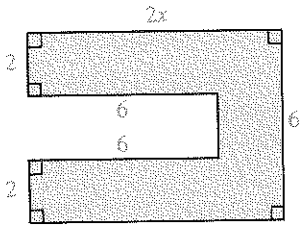
4. Simplify.
   a) $3(2x + 5y) + 7(4x - 2y)$ b) $5(7x + 2y) - 3(2y - x)$
   c) $-5(3m + 6n) - 8(9m - 2n)$ d) $5(4a - 16b) + 2(17a - 29b)$
   e) $11(8k + 4l) + 3(13l - 2k)$ f) $3(p + 2q) - 7(2p + q)$
   g) $-11(3r + 2s) + 7(2r + q)$ h) $6(4u + 7v) - 9(u - w)$
   i) $10(3a + 2b + c) - 5(a - b - c)$ j) $-8(12x + 5y + 4) + 3(2x - 4y + 2)$

5. Simplify.
   a) $0.5(4x + 6y) + 1.5(6x + 2y)$
   b) $-1.4(3x + 5y) - 2.8(5x + 2y)$
   c) $2.6(15x + 5y) - 5.2(5x - 3y)$
   d) $-3.8(5y + x) + 7.5(4x + y)$

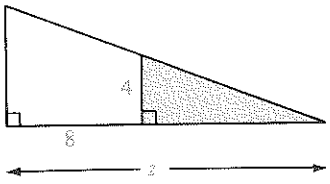
6. Write the area of each shaded region as a sum or difference of terms.
   a)



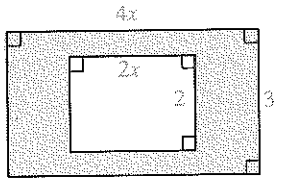
   b)



   c)

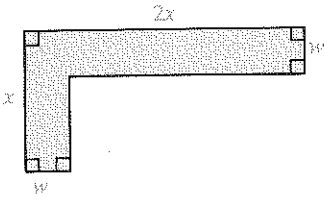


   d)

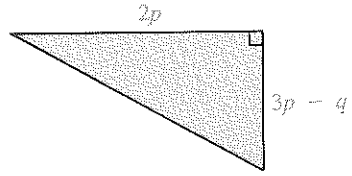


7. Simplify.
   a) $3a(x - y) - 5a(y - x)$
   b) $8c(b - 3a) - 2c(b - a)$
   c) $4x(x^2 - 2x + 3) - 5x^2(x^2 - x)$
   d) $2x(y - x) + (y - 2x^2)$
   e) $3x - 5y(x - y) + y^2$
   f) $4p(q - p) - (p^2 - q^2)$
   g) $x(x - y) - 3y(y - x)$
   h) $6a(a + b) + 3b(b - a)$

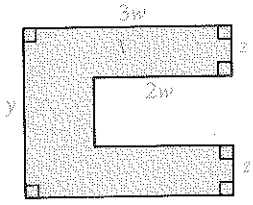
8. Write the area of each shaded region as a sum or difference of terms.
   a)



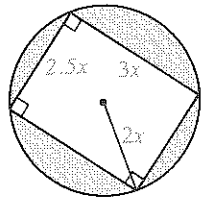
   b)



   c)



   d)



9. Evaluate for $x = -1.1$ and $y = 2.3$.
   a) $3(2x - y) + 2(x - 2y) - 2(x + y)$
   b) $4(x - 2y) - 3(3x - 4y) + 2(x + 2y)$
   c) $2(y - 3x) + 4(3x - 3y) - (3x + 2y)$
   d) $2x(y - x) - 3y(x - 2y) + x(3x - y)$

3-5 TRANSLATING WORDS INTO SYMBOLS

Five students in Mrs. Iverson's class each invested the same amount in stocks in September. They are now reviewing their gains and losses.

- Ian lost \$16.
- Linda gained \$7.
- Maria has two-and-a-half times her initial investment.
- Ravi gained \$9.
- Susan has half of what she started with.

Suppose you knew how much the students started with. How could you find out whether, collectively, they now have more than or less than that?

This problem is most easily solved with algebra. Let x dollars represent each student's initial investment. Then, an algebraic expression can be written for the amount each student has now.

Student	Verbal Expression	Algebraic Expression
Ian	A loss of \$16	$(x - 16)$ dollars
Linda	A gain of \$7	$(x + 7)$ dollars
Maria	2.5 times the initial investment	$2.5x$ dollars
Ravi	A gain of \$9	$(x + 9)$ dollars
Susan	Half of the initial investment	$0.5x$ dollars

The total amount the students have now is:

$$\begin{aligned}&(x - 16) + (x + 7) + 2.5x + (x + 9) + 0.5x\\ &= x - 16 + x + 7 + 2.5x + x + 9 + 0.5x\\ &= x + x + 2.5x + x + 0.5x - 16 + 7 + 9\\ &= 6x\end{aligned}$$

The students started with x dollars each or $5x$ dollars together. Now they have $6x$ dollars. Collectively, the students have more than they started with.

By translating verbal expressions into algebraic expressions, the problem was solved without knowing the initial investment.

Study these verbal expressions and their algebraic equivalents.

Verbal Expression	Algebraic Expression
four more than a number	$n + 4$
a number increased by eight	$p + 8$
nine less than a number	$a - 9$
twice a number	$2d$
one-sixth of a number	$\frac{1}{6}s$ or $\frac{s}{6}$
five more than four times a number	$4f + 5$
the product of one more than a number, and seven	$7(n + 1)$
three less than five times a number	$5l - 3$

Sometimes, two numbers are related. Suppose one of these numbers is known. How can the other number be found? One number can be represented by a variable and the other number expressed in terms of this variable.

Example 1. Choose a variable to represent one quantity. Express the other quantity in terms of this variable.

a) Two consecutive integers
b) Two numbers which differ by five
c) Mary's age now, and in six years
d) The lengths of the pieces if a 4 m log is cut in two

Solution.

a) Two consecutive integers
Let l represent the lesser integer.
Then, the greater integer is $(l + 1)$.
Alternatively, let j represent the greater integer.
Then, the lesser integer is $(j - 1)$.

b) Two numbers which differ by five
Let n represent the greater number.
Then, the lesser number is $(n - 5)$.
Can you think of an alternative way to describe the numbers?

c) Mary's age now, and in six years
Let a years represent Mary's age now.
Then in 6 years, Mary will be $(a + 6)$ years old.

d) The lengths of the pieces of a 4 m log cut into two
Let l metres represent the length of one piece.
Then, the length of the other piece is $(4 - l)$ metres.

Example 2. Express each quantity in terms of the variable.
a) The value in cents of n quarters
b) The value in dollars of x five-dollar bills
c) The number of centimetres in k metres
d) The number of minutes in t hours

Solution.
a) The value in cents of n quarters
1 quarter has a value of $25¢$.
n quarters have a value of $n(25¢)$ or $25n$ cents.

b) The value in dollars of x five-dollar bills
1 five-dollar bill has a value of \$5.
x five-dollar bills have a value of $x(\$5)$ or $5x$ dollars.

c) The number of centimetres in k metres
1 metre has a length of 100 cm.
k metres have a length of $k(100\text{ cm})$ or $100k$ centimetres.

d) The number of minutes in t hours
1 hour is 60 min.
t hours are $t(60\text{ min})$ or $60t$ minutes.

EXERCISES 3-5

(A)

1. Write an algebraic expression for each verbal expression.
   a) five more than a number
   b) six less than a number
   c) eight times a number
   d) one-fifth of a number
   e) the product of a number and eight
   f) four more than five times a number
   g) two less than eight times a number
   h) the product of two less than a number, and eight
   i) the sum of one-fourth of a number, and three
   j) one-fourth of the sum of a number and three

2. Express each quantity in terms of the variable.
   a) The number of seconds in m minutes
   b) The number of grams in k kilograms
   c) The value in cents of n nickels
   d) The value in dollars of x two-dollar bills
   e) The number of hours in m minutes
   f) The distance in metres of c centimetres

3. Write a verbal expression for each algebraic expression.
   a) $p + 6$ b) $q - 10$ c) $\frac{1}{4}r$ d) $10s$ e) $4 + \frac{3}{10}t$
   f) $3u + 2$ g) $4v - 5$ h) $2(w - 3)$ i) $\frac{1}{3}(x + 5)$ j) $x(x - 3)$

4. Choose a variable to complete the first statement. Using that variable, write an expression that completes the second statement.
   a) Clyde is 12 years older than Bonnie.
      Let Bonnie's age be represented by ▒ years. Then, Clyde's age is ▒ years.
   b) Ian's mass is 1.5 kg less than that of Sean.
      Let Sean's mass be represented by ▒ kilograms. Then, Ian's mass is ▒ kilograms.
   c) A 30 m log is cut into two pieces of unequal lengths.
      Let the length of the longer piece be represented by ▒ metres.
      Then, the length of the shorter piece is ▒ metres.

(B)

5. Write an algebraic expression to complete each statement.
   a) The heights, in centimetres, of Lim and David are consecutive integers. Lim is the taller of the two.
      Let David's height be represented by ▒ centimetres.
      Then, Lim's height is ▒ centimetres.
   b) The ages of Alan, Becky, and Carmen are three consecutive numbers. Alan is the oldest and Carmen is the youngest.
      Let Carmen's age be represented by ▒ years.
      Then, Becky's age is ▒ years. Alan's age is ▒ years.
   c) Two numbers have a product of 36.
      Let one number be represented by ▒. Then, the other number is ▒.
   d) Stefa's marks on two tests are consecutive even integers.
      Let one mark be represented by ▒. Then, the other mark is ▒.

6. Express each quantity in terms of the variable.
   a) The area in square centimetres of A square metres
   b) The area in square millimetres of B square centimetres
   c) The volume in cubic centimetres of V cubic metres
   d) The area in square centimetres of M square millimetres

7. Choose a variable to represent one quantity and express the other quantity (or quantities) in terms of the first.
   a) The ages of two brothers if one brother is twelve years older than the other
   b) Two numbers where one number is one-fifth of the other
   c) The ages of Angelo and Mary whose total age is 21 years
   d) The lengths of the jumps if Joan's jump was 15 cm longer than Enid's
   e) The speeds of the two cars if the Jaguar travelled 1.1 times as fast as the Mercedes
   f) Two numbers with a product of 76
   g) Three consecutive integers
   h) Three consecutive odd integers

(C)

8. Choose a variable to represent one quantity and express the other quantity in terms of the first.
   a) Anita's age doubled is 4 years less than Ron's age tripled.
   b) Five times one number is 7 more than eight times a smaller number.
   c) Gloria is now twice as old as Chad was 9 years ago.

Review Exercises

1. Suppose this pattern were continued.

   a) How many matchsticks would be needed to make 12 squares?
   b) How could the number of matchsticks be found if the number of squares were known?
   c) Let s represent the number of squares. Write an expression for the number of matchsticks needed to make s squares.

2. Suzy has a Saturday job where she works 8 h a day and is paid \$5/h.
   a) How much money does Suzy make in 1 week?
   b) How much money does Suzy make in 12 weeks?
   c) Suzy wants to buy a stereo system, which will cost \$160. How many weeks will she have to work to earn this money?
   d) If the amount of money earned is known, how could:
      i) the number of days worked be found
      ii) the number of hours worked be found?

3. For the expression $6x + 4y - 3z$, list:
   a) the terms b) the variables c) the coefficients.

4. Evaluate.
   a) $9 + 3y$ for $y = 16$
   b) $8s - 7$ for $s = 5.4$
   c) $5p + 8q$ for $p = 13$ and $q = -6$
   d) $6s - t$ for $s = -1.7$ and $t = 2.9$
   e) $14u + v - 9w$ for $u = -6$, $v = 29$, and $w = 12$
   f) $21s + 4t$ for $s = 0.4$ and $t = -1.2$

5. Simplify.
   a) $23x - 11x$
   b) $4m + 13m$
   c) $15x - 9x + 3x$
   d) $12a + 4b - 5a + 3b$
   e) $9x + 4y - x - 2y$
   f) $14c + d - 3d + 11c$
   g) $6m - 5m - 5n + 5n$
   h) $17w + 8x - 7x - 8w$
   i) $8.5e + 1.5e - 6.2f + 5.7f$
   j) $-3.6d + 4.7c - 5.8c - 1.2d$

6. Evaluate.
   a) $\frac{9}{4}x - 3y$ for $x = \frac{2}{3}$ and $y = \frac{3}{8}$
   b) $7y + 9y - 4y$ for $y = \frac{9}{4}$
   c) $3.2x + 4.1y - z$ for $x = 0.3$, $y = -1.1$, and $z = 2.4$
   d) $\frac{3}{8}a - \frac{2}{5}b + \frac{5}{2}c$ for $a = \frac{5}{3}$, $b = \frac{3}{8}$, and $c = -\frac{4}{25}$

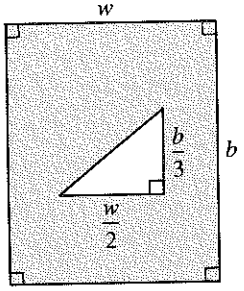
7. Simplify.
   a) $3x - 5x - 3$
   b) $-4a + 3b - 6a$
   c) $5p - 7q - 8p$
   d) $-11m - 9n - 2n + 6m$
   e) $16f - 3e + 10g - 6e$
   f) $-8d - 10b + 2c + 4d$

8. Simplify.
   a) $2(5a + 7) + 17$
   b) $2(3x + 5y) - 4x$
   c) $-7(3x - 2y) - 5(3y - 4x)$
   d) $5(2m + 7n) + 3(m - 4n)$
   e) $-3(2x + 4y) + 3(x - y)$
   f) $7(r + 3s) - 2(2r - 9s)$

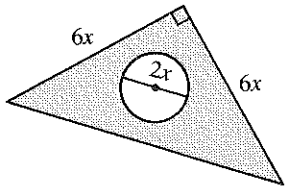
9. Simplify.
   a) $4m - 3n - 15 + 14n - 2 + 6n$
   b) $-11 + 3b - 7a + 13 - 8b + 10a$
   c) $10x - 3z + 14 - 11y + 2z - 22$
   d) $14d - 23e + 11d - 16 + 5e$
   e) $4.8x - 3.2y + 2.4z + 1.7y - 3.9x$
   f) $-5.7b + 4.1a - 1.1b + 3.5b - 9.2a$

10. Write an expression for the area of each shaded region.

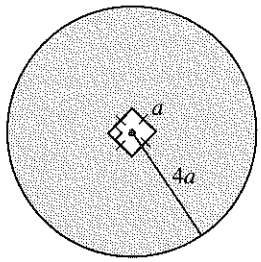
a)



b)



c)



11. Write an algebraic expression for each verbal expression.
   a) one-tenth of a number
   b) eleven times a number
   c) twenty more than a number
   d) thirty-one less than a number
   e) fourteen more than five times a number
   f) the sum of one-seventh of a number, and nineteen

12. Choose a variable to represent one quantity and express the other quantity in terms of the first.
   a) The cost of two cars if one is $1300 more than the other
   b) Two consecutive odd numbers
   c) Three consecutive even numbers
   d) The heights of two trees if one is 2.6 m shorter than the other
   e) Two numbers with a sum of 37.8
   f) The lengths of the pieces of wood if a 15 m log is cut into two
   g) Two numbers which have a difference of 0.4
   h) The ages of two boys if Raj's age is four years less than twice Egino's age

1. Write as an integer.
   a) a gain of \$17 b) a loss of 3 points c) a decrease of 12°C
   d) a credit of \$425 e) a drop of 12 m f) 210 m below sea level

2. Simplify.
   a) $(-14) + (-23)$ b) $(+9) - (-26)$ c) $(+31) + (-48)$
   d) $(-28) - (+17)$ e) $(-12) + (+35)$ f) $(-19) - (-11)$

3. Simplify.
   a) $(+7)(-9)$ b) $(-8)(-5)$
   c) $(-60) \div (-12)$ d) $(-21)(-7)$
   e) $(-225) \div (+25)$ f) $(+240) \div (-30) \div (-2)$

4. Simplify.
   a) $\dfrac{(+5)(-8)}{(-4)} + \dfrac{(-4)(-6)}{(+3)}$ b) $15 - (+8)(-3)$
   c) $(+12)(-5 + 9) - (-3 - 7)$ d) $14 + (-5) - 3(2 - 9)$
   e) $\dfrac{6 + 15}{-7} - \dfrac{-4 + 19}{3}$ f) $\dfrac{19 - 28}{-3} - \dfrac{(-11) + 27}{4} + \dfrac{6 - 32}{-2}$

5. Arrange in order from least to greatest.
   a) $-4, \frac{5}{2}, \frac{22}{3}, -17, 6, -\frac{5}{4}, -3$ b) $12, -4.5, -2.3, \frac{17}{5}, -7, -\frac{2}{3}, 1.25$

6. Express each rational number as a decimal.
   a) $\frac{2}{5}$ b) $-\frac{3}{8}$ c) $\frac{17}{-6}$ d) $\frac{11}{7}$ e) $\frac{-19}{-12}$

7. Express each decimal as a fraction in lowest terms.
   a) 3.25 b) −7.6 c) 1.875 d) −2.145 e) −11.03

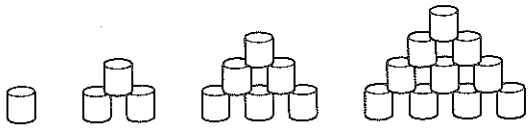
8. Simplify.
   a) $-\dfrac{7}{10} \div \dfrac{2}{-5}$ b) $\dfrac{-32}{15} \times \dfrac{-25}{-44}$ c) $-12.25 \div 3.5$
   d) $\dfrac{-21}{16} \div \dfrac{-7}{8} \times \dfrac{3}{-4}$ e) $(4.2)(-3.5) \div (-2.1)$ f) $\dfrac{18}{-5} \times \dfrac{10}{-27} \div \dfrac{-9}{2}$

9. Simplify.
   a) $-4.21 + 13.7$ b) $\dfrac{-9}{4} - \dfrac{-3}{8}$ c) $\dfrac{-16}{9} + \dfrac{5}{-6}$
   d) $\dfrac{-8}{-15} + \dfrac{3}{-10} - \dfrac{2}{3}$ e) $3.17 - 5.04 - 1.317$ f) $\dfrac{11}{6} - \dfrac{4}{3} - \dfrac{-15}{9}$

10. Simplify.
    a) $\dfrac{7}{8} + \left(-\dfrac{3}{4}\right)\left(\dfrac{5}{6}\right)$ b) $2.17 - \dfrac{9.6}{3.2}(7.6 - 4.3)$
    c) $\left(-\dfrac{5}{8} - \dfrac{1}{2}\right) \times \left(\dfrac{7}{9} - \dfrac{2}{3}\right)$ d) $\dfrac{17}{5} + \dfrac{-13}{10} - \dfrac{5}{8}\left(\dfrac{-4}{-15}\right)$
    e) $\dfrac{5.6}{-0.14} + \dfrac{7.2}{1.8} - \dfrac{-4.5}{-0.9}$ f) $\dfrac{11}{18} + \left(\dfrac{-7}{3}\right) - \left(\dfrac{2}{5}\right)\left(\dfrac{-3}{4}\right)$

11. The volume of a cylinder is given by the formula $V = \pi r^2 h$, where r is the radius of the cylinder and h is its height. Find the volume of corn in a full silo, which has a radius of 3.4 m and a height of 8.9 m.

12. A series of tins are stacked as shown. Suppose the pattern were continued.



a) How many tins would there be on the bottom row of:
   i) the 7th diagram ii) the 11th diagram?
b) How many tins would there be in:
   i) the 7th diagram ii) the 11th diagram?
c) Write an expression for the number of tins in terms of the position of the diagram.

13. Evaluate.
a) $\frac{3}{4}x - 5$ for $x = -12$
b) $3m + 7n - 11m - 5n$ for $m = -4$ and $n = 2$
c) $1.7x - 4.3y$ for $x = -3$ and $y = 5$
d) $\frac{-7}{8}a - \frac{5}{3}b + \frac{1}{2}a + \frac{2}{3}b$ for $a = \frac{4}{3}$ and $b = -\frac{2}{5}$
e) $\frac{5}{4}p + \frac{3}{2}q - \frac{4}{5}r$ for $p = -\frac{2}{3}$, $q = \frac{4}{3}$ and $r = -2$

14. Simplify.
a) $-7p - 4q - 12p - 5q$
b) $16x + 7y - 8z - 12x - 15y + 3z$
c) $3.6a - 2.5b - 1.4a + 3.8b - 7.2a$
d) $-4x + 11y - 17y + 9x - 3y$
e) $12m - 4n - 7m - 9n - 8m + 6n$

15. Simplify.
a) $3a - 2(5a + 4b) - 3b$
b) $5(4m - 3n) - 8m - 7n$
c) $-2(7x + 4y) + 6(2x - 5y)$
d) $8(-3p - 9q) - 3(4p - 12q)$
e) $2(5c - 3d + e) + 6(c + 2d - 4e) - (3c - d + 2e)$
f) $-11(3x - 2y + z) - 5(2x + 4y - 9z) + 4(x - 8y - 7z)$

16. Write an algebraic expression for each verbal expression.
a) the sum of a number and eleven
b) six times a number, decreased by four
c) the sum of two consecutive numbers
d) the difference between a number and twenty-one
e) eight, plus three times a number
f) three-quarters of a number, subtracted from ten