2 Rational Numbers

A car's fuel consumption depends on whether it is driven in the city or on the highway. How can the fuel consumption be calculated when it is driven under both conditions? (See Section 2-6, *Example 1*.)

2-1 WHAT ARE RATIONAL NUMBERS?

Just as integers are used to indicate change, so are positive and negative fractions.

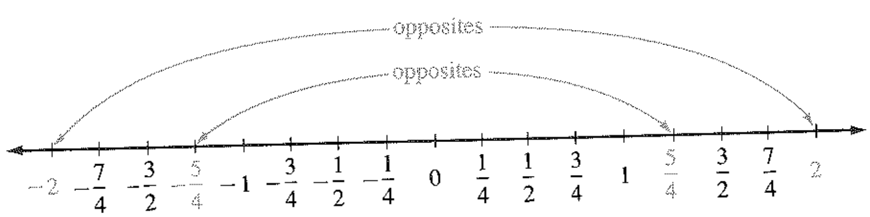
Some newspapers publish the prices of stocks.

TORONTO VALUE LEADERS

INDUSTRIALS	Volume	Value	Last	Change
Noranda	669,465	15,648,294	$23\frac{1}{2}$	$+\frac{1}{2}$
Alcan	312,146	13,334,646	43	$+\frac{7}{8}$
Shell Canada	431,543	12,074,983	$28\frac{1}{4}$	$+\frac{3}{4}$
Alberta Energy	641,158	10,830,158	$16\frac{3}{4}$	$+\frac{1}{4}$
BCE	254,686	10,039,940	$39\frac{3}{8}$	$+\frac{1}{4}$
Laidlaw B	488,262	9,733,485	20	
Cdn Pac Ltd	474,181	9,370,041	$19\frac{1}{2}$	$-\frac{3}{8}$
Bk of Montreal	226,259	8,089,602	$35\frac{5}{8}$	$-\frac{1}{4}$

On the stock market a change of $+\frac{3}{4}$ means that the price of the stock has *risen* by $\frac{3}{4}$ of a dollar, or \$0.75, from the day before. A change of $-\frac{1}{4}$ means that the stock has *dropped* by \$0.25 from the day before.

The stock market changes are examples of rational numbers. Rational numbers can be represented on a number line.



Each positive fraction has an opposite negative fraction, for example, $-\frac{1}{2}$ is the opposite of $\frac{1}{2}$. Similarly, each negative fraction has an opposite positive fraction, for example, $\frac{7}{8}$ is the opposite of $-\frac{7}{8}$.

A positive rational number like $\frac{1}{2}$ is usually written without the positive sign, but it could be written as $+\frac{1}{2}$ or as $\frac{+1}{+2}$. The rules for dividing integers suggest that it can also be written as $\frac{-1}{-2}$.

The rules for dividing integers also suggest that a negative rational number like $-\frac{1}{2}$ can be written as $\frac{-1}{+2}$ or as $\frac{+1}{-2}$. We can see why this is true by considering the product $(+2)\left(-\frac{1}{2}\right)$.

$$(+2)\left(-\frac{1}{2}\right) \text{ means } 2\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)$$
$$= -1$$

With whole numbers, we can write, for example,

Since $(3)(4) = 12$, then $\frac{12}{3} = 4$

Similarly, with rational numbers,

Since $(+2)\left(-\frac{1}{2}\right) = -1$, then $\frac{-1}{+2} = -\frac{1}{2}$

In a similar manner, it can be shown that

$\frac{+1}{-2} = -\frac{1}{2}$ and $\frac{-1}{-2} = +\frac{1}{2}$.

> Any number which can be written in the form $\frac{m}{n}$, where m and n are integers and $n \neq 0$, is called a *rational number*.

Example 1. State which of the following numbers are rational.
$\frac{3}{2}, \frac{-8}{20}, 3\frac{1}{4}, 5, \frac{-6}{-7}, 0, 2.5, -11.27, \frac{-60}{-12}$

Solution. Since $\frac{3}{2}, \frac{-8}{20}, \frac{-6}{-7}, \frac{-60}{-12}$ are in quotient form, we know that they are rational numbers.

Check to see if the other numbers can be written in quotient form.

$3\frac{1}{4} = \frac{13}{4}$; 5 can be written $\frac{5}{1}$; 0 can be written $\frac{0}{1}$.

Recall that 2.5 means 25 tenths or $\frac{25}{10}$.

-11.27 means -1127 hundredths or $-\frac{1127}{100}$.

All the given numbers are rational numbers.

A rational number can be reduced to lower terms by dividing the numerator and the denominator by a common factor. For example,

$$\frac{-8}{20} = -\frac{8}{20} = -\frac{8 \div 4}{20 \div 4} = -\frac{2}{5} \qquad \frac{-60}{-12} = \frac{60}{12} = \frac{60 \div 12}{12 \div 12} = 5$$

When a rational number cannot be further simplified, it is said to be in *lowest terms* (or in *simplest form*).

The rational numbers $\frac{-8}{20}$ and $-\frac{2}{5}$ are said to be *equivalent* because they represent the same quantity. Similarly, $\frac{-60}{-12}$ and 5 are equivalent rational numbers.

Example 2. Find which of these rational numbers are equivalent.
$\frac{18}{24}, \frac{-6}{8}, \frac{16}{-12}, \frac{-9}{-12}$

Solution. Reduce each rational number to lowest terms.
$\frac{18}{24} = \frac{3}{4}, \quad \frac{-6}{8} = -\frac{3}{4}, \quad \frac{16}{-12} = -\frac{4}{3}, \quad \frac{-9}{-12} = \frac{3}{4}$
Since $\frac{18}{24}$ and $\frac{-9}{-12}$ both reduce to $\frac{3}{4}$, they are equivalent.

A rational number can be raised to higher terms by multiplying the numerator and the denominator by the same number.

Example 3. Arrange these numbers in order from least to greatest.
$\frac{3}{4}, \frac{-2}{5}, \frac{-7}{-10}, -\frac{-9}{-20}$

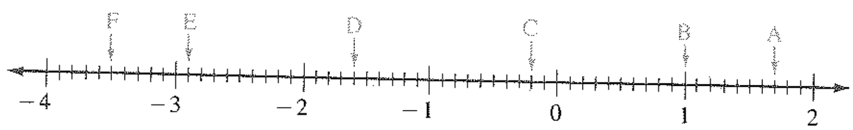
Solution. Express the rational numbers as fractions with a common denominator of 20.
$\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}, \quad \frac{-2}{5} = \frac{-2 \times 4}{5 \times 4} = \frac{-8}{20}, \quad \frac{-7}{-10} = \frac{-7 \times 2}{-10 \times 2} = \frac{14}{20}, \quad -\frac{-9}{-20} = \frac{-9}{20}$

From least to greatest: $\frac{-9}{20}, \frac{-8}{20}, \frac{14}{20}, \frac{15}{20}$ or $-\frac{-9}{-20}, \frac{-2}{5}, \frac{-7}{-10}, \frac{3}{4}$

EXERCISES 2-1

Ⓐ

1. Write the rational numbers for the points indicated.



2. Compare each pair of rational numbers. Replace the comma with > or <.
   a) $\frac{1}{2}, \frac{3}{2}$ b) $-2.8, 3.1$ c) $\frac{7}{4}, \frac{-5}{4}$ d) $-1.25, 0.75$
   e) $\frac{-13}{7}, \frac{5}{7}$ f) $0.01, -1.00$ g) $\frac{-10}{17}, \frac{-5}{17}$ h) $108.6, -116.8$

3. Reduce to lowest terms.
   a) $\frac{5}{-10}$ b) $\frac{10}{-15}$ c) $\frac{-12}{-30}$ d) $-\frac{6}{15}$ e) $-\frac{-6}{11}$ f) $-\frac{-6}{18}$
   g) $-\frac{4}{-14}$ h) $-\frac{-14}{-25}$ i) $-\frac{-15}{-35}$ j) $-\frac{-24}{-72}$ k) $-\frac{-42}{-28}$ l) $-\frac{54}{-81}$

4. Compare each pair of rational numbers. Replace the comma with > or <.
   a) $\frac{8}{6}, \frac{2}{3}$ b) $\frac{-7}{10}, \frac{-16}{20}$ c) $\frac{7}{5}, \frac{12}{5}$ d) $\frac{-8}{28}, \frac{-3}{7}$
   e) $\frac{-110}{121}, \frac{9}{11}$ f) $\frac{-27}{24}, \frac{-10}{8}$ g) $\frac{52}{16}, \frac{-11}{4}$ h) $\frac{19}{6}, \frac{110}{30}$

Ⓑ

5. Round each rational number to the nearest integer.
   a) -3.7 b) -5.2 c) -6.7 d) -0.4 e) 0.2 f) -0.6
   g) -0.97 h) -0.35 i) -7.52 j) $-\frac{22}{5}$ k) $-\frac{67}{4}$ l) $-\frac{37}{5}$

6. The depths reached by divers wearing scuba gear are shown. Arrange the depths in order from greatest to least.

Diver	Depth in metres
Damont	−64.0
Giesler	−99.1
Hilton	−104.9
Trouth	−97.5

7. Write the rational number which is
   a) 1 more than -3. b) 1 less than -4. c) 5 more than -6.
   d) 7 more than -7.8. e) 6 less than 3.5. f) 10 less than 6.2.
   g) $\frac{1}{2}$ more than $-\frac{17}{2}$. h) $\frac{1}{2}$ less than $-\frac{3}{2}$. i) $\frac{1}{2}$ less than $\frac{1}{4}$.

8. Compare each pair of rational numbers. Replace the comma with > or <.

a) $\frac{15}{6}, \frac{56}{16}$ b) $\frac{-35}{15}, \frac{-30}{9}$ c) $\frac{-72}{40}, \frac{72}{45}$ d) $\frac{24}{44}, \frac{45}{55}$

e) $\frac{-119}{70}, \frac{-57}{30}$ f) $\frac{77}{28}, \frac{78}{32}$ g) $\frac{126}{54}, \frac{-115}{45}$ h) $\frac{-75}{35}, \frac{-64}{28}$

9. Reduce each set of rational numbers to lowest terms. Then list them from greatest to least.

a) $\frac{28}{16}, \frac{30}{8}, \frac{33}{12}, \frac{15}{20}$ b) $\frac{-65}{25}, \frac{36}{10}, \frac{-27}{15}, \frac{28}{20}$

c) $\frac{34}{18}, \frac{-120}{54}, \frac{-91}{63}, \frac{-40}{36}$ d) $\frac{-154}{49}, \frac{-100}{28}, \frac{-45}{21}, \frac{-22}{14}$

10. In each set, express the rational numbers with a common denominator. Then list them from least to greatest.

a) $\frac{3}{2}, \frac{6}{5}, \frac{5}{4}, \frac{4}{3}$ b) $\frac{-11}{4}, \frac{-8}{3}, \frac{-5}{2}, \frac{-23}{8}$

c) $\frac{11}{8}, \frac{-3}{2}, \frac{-16}{10}, \frac{-7}{4}$ d) $\frac{-29}{6}, \frac{21}{4}, \frac{-88}{18}, \frac{55}{12}$

11. List these rational numbers from least to greatest.

$\frac{5}{-10}, \frac{-3}{2}, \frac{-7}{-28}, \frac{-5}{20}, -\frac{12}{16}, \frac{9}{-6}, \frac{10}{-8}, -\frac{-18}{-9}$

12. List these rational numbers from greatest to least.

$-\frac{3}{8}, \frac{-1}{3}, \frac{1}{-4}, -\frac{-2}{9}, \frac{-7}{-18}, -\frac{-13}{-36}$

13. List these rational numbers from least to greatest.

$\frac{4}{-8}, \frac{-3}{-15}, \frac{-16}{40}, -\frac{19}{-60}, \frac{14}{30}, -\frac{13}{20}$

14. Find which five of these rational numbers are equivalent.

$\frac{-2}{-3}, \frac{4}{-6}, \frac{-3}{4}, \frac{-12}{-20}, \frac{15}{-20}, \frac{12}{-16}, \frac{6}{-10}, -\frac{-6}{-8}, -\frac{-8}{-12}, \frac{9}{-12}$

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15. If two numbers have a common factor, it is also a factor of their difference. Use this fact to determine which of these fractions are in lowest terms.

$\frac{169}{182}, \frac{171}{188}, \frac{200}{201}, \frac{209}{247}$

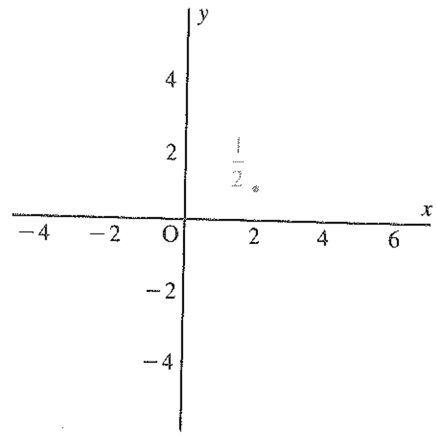
16. a) Write a sentence to explain why the sum of two or more rational numbers is a rational number.

b) Use the result in part a) to explain why any number that can be expressed as a decimal with a finite number of decimal digits is a rational number.

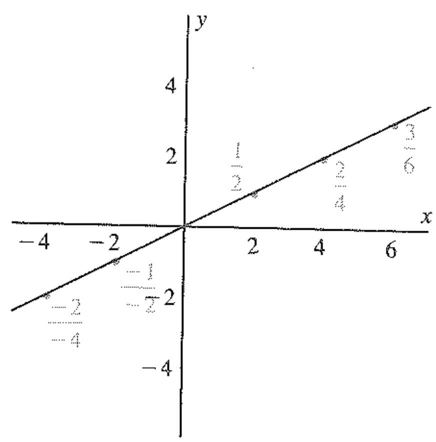
INVESTIGATE

Rational Numbers on a Grid

- Draw a grid, as illustrated, and label each axis from -6 to 6.
- Plot the rational number, $\frac{1}{2}$, on the grid as follows. From the origin, move 2 units in the x-direction and then 1 unit in the y-direction. Draw a dot and label it $\frac{1}{2}$.



- In a similar way, plot other rational numbers equivalent to $\frac{1}{2}$, for example, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{-1}{-2}$, $\frac{-2}{-4}$, and so on.
- Join the dots representing $\frac{1}{2}$ with a straight line. All points on this line can be considered as rational numbers equivalent to $\frac{1}{2}$.



1. Draw a grid and label each axis from -10 to 10. Plot each rational number listed below. Join its dot to the origin and extend the line in each direction. Use this line to list 4 rational numbers equivalent to the number plotted.
   $\frac{-2}{3}$, $\frac{3}{4}$, $\frac{4}{1}$, $\frac{5}{-2}$, -3
   Remember that the denominator of each rational number indicates how far to move in the x-direction. The numerator indicates how far to move in the y-direction.
2. Choose a rational number. Plot in on a grid. List 6 rational numbers equivalent to the number plotted.

THE MATHEMATICAL MIND

The Introduction of the Number Zero

Zero as a placeholder

One of the most important problems encountered by ancient civilizations was that of finding a way to write very large numbers without using new symbols.

In our place-value system, the meaning of each digit in a number, such as 4023, depends on the place where it is written. The 0 is called a placeholder, to indicate in this case that there are no hundreds.

The idea of using a special symbol as a placeholder was introduced many centuries ago by several different civilizations. It is one of the greatest practical ideas of all time. Here are some examples of the placeholder symbols used by these ancient civilizations.

Babylonian (300 B.C.) 	Mayan (A.D. 200–600) 
Hindu (A.D. 600) 	Arab (A.D. 1000) 

Zero as a number

The idea of using zero as a number probably originated about A.D. 800 when the Hindus used positive and negative integers to represent credits and debits. They needed a special number to represent no credit and no debit. This number was gradually picked up by the Arabs, and eventually reached Spain. But mathematicians regarded this number with suspicion, and zero was not completely accepted and used as a number in the western world until after 1500.

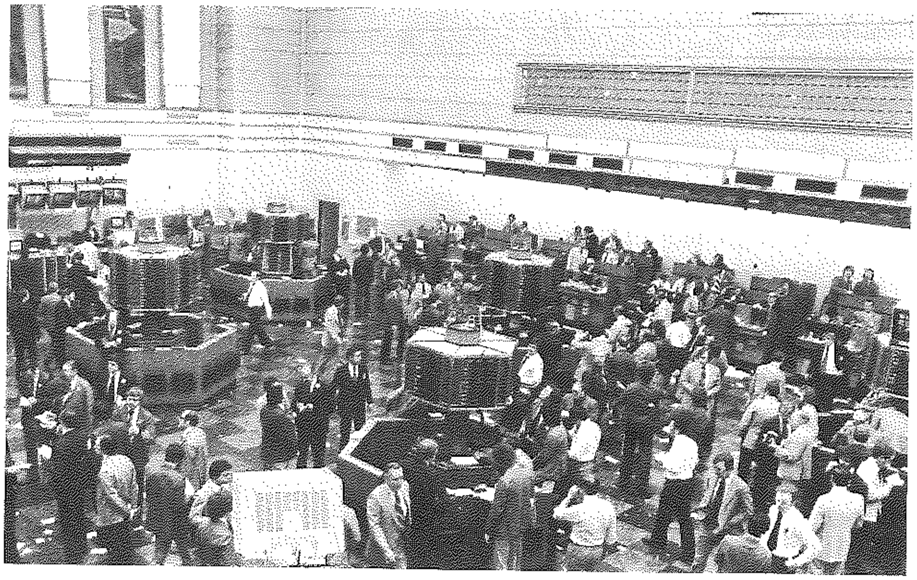
Properties of zero

Because zero is a number, just like any other number, we may operate with it. But it has certain properties that other numbers do not have.

- When 0 is added to or subtracted from any number, that number does not change.
  $5 + 0 = 5$
  $7 - 0 = 7$
- When 0 is multiplied by any number, the product is 0.
  $0 \times 3 = 0$
- When 0 is divided by any number (except 0), the quotient is 0.
  $0 \div 2 = 0$
- Division by 0
  There is a restriction on operating with 0, because we cannot divide by 0. Recall that we write $6 \div 2 = 3$ because $3 \times 2 = 6$. But if we try to find a meaning for $6 \div 0$, we would have to find a number that gives 6 when it is multiplied by 0. Since this is impossible, we say that division by 0 is not defined.

QUESTIONS

1. Simplify, if possible.
   a) $17 + 0$ b) $-4 + 0$
   c) $259 + 0$ d) $23 - 0$
   e) $-9 - 0$ f) $319 - 0$
2. Simplify, if possible.
   a) 0×9 b) -6×0
   c) 107×0 d) 0×0
   e) $0 \div 3$ f) $0 \div (-12)$
   g) $5 \div 0$ h) $-8 \div 0$
   i) $0 \div 0$ j) $0 \div (-3)$
3. Give an example to illustrate why 0 is considered to be a number.



2-2 RATIONAL NUMBERS IN DECIMAL FORM

On the stock market, a change of $+\frac{1}{2}$ means an increase in price of $\frac{1}{2}$ a dollar or \$0.50.

Consider a change of $-\frac{3}{8}$. To express this as a drop in price, $\frac{3}{8}$ of a dollar is expressed as a decimal. Divide 3 by 8.

$$
\begin{array}{r}
0.375 \\
8\overline{)3.000} \\
\underline{2\,4} \\
60 \\
\underline{56} \\
40 \\
\underline{40} \\
0
\end{array}
$$

$\frac{3}{8} = 0.375$

The drop in price is \$0.375.

The value \$0.375 represents $37\frac{1}{2}$ cents. It is not possible to have $37\frac{1}{2}$ cents. However, shares are usually bought and sold in lots of 100, where a change of $37\frac{1}{2}$ cents per share would become \$37.50 per 100 shares.

The decimal, 0.375, is a *terminating* decimal. After the third decimal place, the rest of the digits are zeros.
That is, 0.375 = 0.375 000 000 . . .

This is not always the case. Consider the rational number $\frac{26}{11}$. To express this as a decimal, divide 26 by 11.

$$
\begin{array}{r}
2.3636 \\
11\overline{)26.0000} \\
\underline{22} \\
4\,0 \\
\underline{3\,3} \\
70 \\
\underline{66} \\
40 \\
\underline{33} \\
70
\end{array}
$$

The remainders, after subtracting, alternate between 4 and 7. This produces a sequence of digits which repeats.

$\frac{26}{11} = 2.3636\ldots$

The decimal, 2.3636 . . . , is a *repeating* decimal. It does not terminate. This decimal is written more simply as $2.\overline{36}$ or $2.\dot{3}\dot{6}$ with a line drawn over the repeating digits or periods placed over the first and last digits which repeat.

Example 1. Express $\frac{100}{7}$ as a decimal

a) by dividing. b) with a calculator.

Solution. a)

$$
\begin{array}{r}
14.285\,714 \\
7\overline{)100.000\,000} \\
\underline{7} \\
30 \\
\underline{28} \\
20 \\
\underline{14} \\
60 \\
\underline{56} \\
40 \\
\underline{35} \\
50 \\
\underline{49} \\
10 \\
\underline{7} \\
30
\end{array}
$$

When dividing, bring down zeros until the remainders repeat. This means that the digits in the decimal will repeat. Since there are only 7 possible remainders when dividing by 7 (namely 0, 1, 2, 3, 4, 5, 6) this decimal must repeat on or before the seventh digit.

$\frac{100}{7} = 14.\overline{285\,714}$

b) $\frac{100}{7}$, with a calculator

Key in: $\boxed{1}$ $\boxed{0}$ $\boxed{0}$ $\boxed{\div}$ $\boxed{7}$ $\boxed{=}$ to display 14.285714

Since the calculator displays only 8 digits, it is not always possible to tell what the sequence of repeating digits is. See *CALCULATOR POWER*, page 46.

We can express a terminating decimal as a common fraction.

Example 2. Express these decimals as common fractions in lowest terms.

a) 3.5 b) 0.65 c) -7.4 d) -0.375

Solution. a) 3.5 means 35 tenths.

$$3.5 = \frac{35}{10} = \frac{7}{2}$$

b) 0.65 means 65 hundredths.

$$0.65 = \frac{65}{100} = \frac{13}{20}$$

c) -7.4 means -74 tenths.

$$-7.4 = \frac{-74}{10} = -\frac{37}{5}$$

d) -0.375 means -375 thousandths.

$$-0.375 = \frac{-375}{1000} = -\frac{15}{40} = -\frac{3}{8}$$

Example 3. In football, the pass-completion average of a quarterback is found by dividing the number of passes completed by the number attempted.

a) Calculate the lifetime pass-completion average, to 3 decimal places, for the following quarterbacks.

b) List the averages in order from greatest to least.

	Name	Attempted Passes	Completed Passes
i)	Sam Etcheverry	2829	1630
ii)	Russ Jackson	2530	1356
iii)	Joe Paopao	2136	1230
iv)	Jackie Parker	2061	1089

Solution. a) i) Etcheverry's average $= \frac{1630}{2829}$

$\doteq 0.576\ 175\ 327$

$= 0.576$ to 3 decimal places

ii) Jackson's average is $\frac{1356}{2530}$ or 0.536 to 3 decimal places.

iii) Paopao's average is $\frac{1230}{2136}$ or 0.576 to 3 decimal places.

iv) Parker's average is $\frac{1089}{2061}$ or 0.528 to 3 decimal places.

b) From greatest to least: 0.576, 0.576, 0.536, 0.528

EXERCISES 2-2

Ⓐ

1. Write these numbers to 8 decimal places, rounding where necessary.
   a) $3.\overline{23}$ b) $42.\overline{307}$ c) $-81.4\dot{6}$ d) $690.0\overline{45}$
   e) $-2.6\dot{5}1\dot{3}$ f) $2.\dot{6}51\dot{3}$ g) $0.\overline{069}$ h) $-0.00\dot{7}\dot{4}$

2. Write these repeating decimals, using a dot or a bar over the repeating digits.
   a) 6.3333 . . . b) 0.17171717 . . . c) 42.135135 . . .
   d) 0.0363636 . . . e) −38.348348 . . . f) −46.23333 . . .
   g) −0.717171 . . . h) 813.813813 . . . i) −0.0213232 . . .

3. State which of these numbers are: a) integers b) rational numbers.
   $\frac{1}{12}$, −1.8, −0.611611611 . . ., 0, $2\frac{3}{4}$, $0.\overline{3}$, 7, $-13.85\overline{762}$,
   −17, 6.432432 . . ., 0.625

4. Write in decimal form.
   a) $\frac{3}{5}$ b) $\frac{2}{-3}$ c) $\frac{4}{9}$ d) $-\frac{3}{8}$ e) $\frac{7}{21}$ f) $\frac{-3}{22}$
   g) $\frac{15}{7}$ h) $-\frac{1}{6}$ i) $\frac{5}{16}$ j) $\frac{-17}{27}$ k) $\frac{11}{12}$ l) $\frac{13}{11}$

5. Express in fractional form.
   a) 0.75 b) 3.25 c) −0.625 d) 0.0625
   e) −2.75 f) −5.875 g) 16.4 h) −40.0625

Ⓑ

6. Compare each pair of rational numbers. Replace the comma with > or <.
   a) $6.4, -\frac{25}{4}$ b) $-\frac{23}{7}, -3.5$ c) $\frac{3}{8}, -\frac{5}{11}$ d) $\frac{-57}{100}, -0.5$
   e) $-8.6, -\frac{75}{9}$ f) $-15.8, -\frac{76}{5}$ g) $\frac{51}{16}, 3.175$ h) $\frac{7}{11}, \frac{16}{25}$

7. In the hockey play-offs, a goalkeeper allowed 11 goals in 7 games. The goalkeeper for the opposing team allowed only 8 goals in the same number of games. Calculate "the goals-against" average, to 2 decimal places, for each goalkeeper.

8. A baseball player's batting average is found by dividing the number of hits by the number of times at bat, and rounding to 3 decimal places.
   a) Calculate the batting averages of these players.

	Batter	Year	Times at Bat	Number of Hits
i)	Hugh Duffy	1984	539	236
ii)	Ty Cobb	1911	591	248
iii)	Babe Ruth	1924	529	200
iv)	Lou Gehrig	1927	584	218
v)	Ted Williams	1941	456	185

   b) List the players in the order of their batting averages from greatest to least.

9. Arrange these fractions from greatest to least.

$\frac{6}{7}, \frac{5}{8}, \frac{9}{11}, \frac{10}{13}, \frac{13}{15}$

Ⓒ

10. a) Simplify $\frac{2}{3} + \frac{5}{6}$, and write the result in decimal form.

b) Write $\frac{2}{3}$ and $\frac{5}{6}$ in decimal form and find their sum. How does the result compare with that for part a)?

c) Repeat the procedure of parts a) and b) for these expressions.

i) $\frac{3}{4} + \frac{2}{5}$ ii) $\frac{5}{8} - \frac{1}{4}$ iii) $\frac{1}{6} - \frac{5}{9}$

iv) $\frac{2}{9} - \frac{5}{11}$ v) $\frac{7}{16} + \frac{5}{12}$ vi) $\frac{29}{37} - \frac{11}{37}$

11. Use a calculator to express these fractions in decimal form. What do you notice?

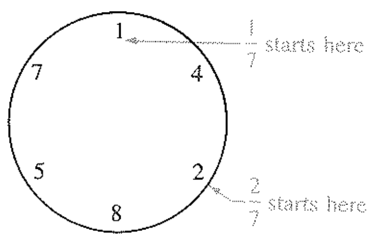
a) $\frac{5}{173}$ b) $\frac{50}{173}$ c) $\frac{500}{173}$ d) $\frac{5000}{173}$

12. Use the result of *Exercise 11* to express each fraction to as many decimal places as possible, with your calculator.

a) $\frac{1}{810}$ b) $\frac{6}{5293}$ c) $\frac{6.9}{9572.6}$ d) $\frac{2.3 \times 6.4}{168.7 \times 24.9}$

INVESTIGATE

a) Use a calculator to express $\frac{1}{7}$ and $\frac{2}{7}$ as repeating decimals. The repeating decimals are the same for both fractions. They can be arranged in a circle.



b) Find the decimal representations of $\frac{3}{7}, \frac{4}{7}, \frac{5}{7}$, and $\frac{6}{7}$. If they also fit the circle of digits, then the digits of the decimal representation of this set of fractions are said to form a *cyclic pattern*.

c) Investigate the cyclic pattern for each set of fractions.

i) $\frac{1}{13}, \frac{2}{13}, \frac{3}{13}, \ldots$ ii) $\frac{1}{14}, \frac{2}{14}, \frac{3}{14}, \ldots$ iii) $\frac{1}{21}, \frac{2}{21}, \frac{3}{21}, \ldots$

CALCULATOR POWER

Investigating Repeating Decimals

When a calculator is used to express a rational number in fractional form, as a decimal, all the repeating decimals may not appear in the display. This happens if the number of repeating digits exceeds the capacity of the display. For example, on a calculator with an 8-digit display:

$$\frac{4}{17} \doteq 0.2352941$$

If we use the calculator skilfully we can obtain more digits in the decimal expansion of $\frac{4}{17}$, and eventually express $\frac{4}{17}$ as a repeating decimal.

Consider the corresponding long division up to the same number of digits. The remainder at the fifth decimal place is 7, and the calculations from this point on are shaded. In fact, at this point, the division is similar to $7 \div 17$. Except for the position of the decimal point, the figures are the same.

$$\begin{array}{r}
0.2352941\\
17\overline{)4.0000000}\\
\underline{3\,4}\\
60\\
\underline{51}\\
90\\
\underline{85}\\
50\\
\underline{34}\\
160\\
\underline{153}\\
70\\
\underline{68}\\
20\\
\underline{17}\\
3
\end{array}$$

This shows that if we use a calculator to express $\frac{7}{17}$ as a decimal, the first two decimals will be 0.41, and the rest will be the additional decimals for $\frac{4}{17}$.

$$\begin{array}{r}
0.41\\
17\overline{)7.00}\\
\underline{6\,8}\\
20\\
\underline{17}\\
3
\end{array}$$

$$\frac{7}{17} \doteq 0.4117647$$

Therefore, $\frac{4}{17} \doteq 0.235294117647$

We can obtain still more decimals if we can find a fraction having a denominator of 17 and a decimal expansion that starts with 0.47. Using a calculator, this can be found by systematic trial.

Key in: $\boxed{1}$ $\boxed{7}$ $\boxed{\times}$ $\boxed{\cdot}$ $\boxed{4}$ $\boxed{7}$ $\boxed{=}$ to display 7.99

This suggests that the fraction required is $\frac{8}{17}$.

Key in: $\boxed{8}$ $\boxed{\div}$ $\boxed{1}$ $\boxed{7}$ $\boxed{=}$ to display 0.4705882

Therefore, $\frac{4}{17} \doteq 0.235\ 294\ 117\ 647\ 058\ 82$

Now we want a fraction with a denominator of 17 and a decimal expansion that starts with 0.82.

Key in: $\boxed{1}$ $\boxed{7}$ $\boxed{\times}$ $\boxed{\cdot}$ $\boxed{8}$ $\boxed{2}$ $\boxed{=}$ to display 13.94

This suggests that the fraction required is $\frac{14}{17}$.

Key in: $\boxed{1}$ $\boxed{4}$ $\boxed{\div}$ $\boxed{1}$ $\boxed{7}$ $\boxed{=}$ to display 0.82 352 94

Therefore, $\frac{4}{17} \doteq 0.235\ 294\ 117\ 647\ 058\ 823\ 529\ 4$

The final six digits are a repeat of the first six digits.
The repeating decimal is now evident.

$$\frac{4}{17} = 0.\overline{235\ 294\ 117\ 647\ 058\ 8}$$

1. Express each fraction as a repeating decimal.

   a) $\frac{4}{21}$ b) $\frac{87}{137}$ c) $\frac{23}{79}$

   d) $\frac{217}{82}$ e) $\frac{19}{84}$ f) $\frac{15}{23}$

2. Express each fraction as a repeating decimal.

   a) $\frac{100}{239}$ b) $\frac{328}{271}$ c) $\frac{55}{202}$

   d) $\frac{4762}{859}$ e) $\frac{424}{757}$ f) $\frac{155}{353}$

3. Investigate the patterns in the repeating decimals for each set of fractions.

   a) $\frac{1}{17}, \frac{2}{17}, \frac{3}{17}, \ldots$ b) $\frac{1}{41}, \frac{2}{41}, \frac{3}{41}, \ldots$

4. Investigate other repeating decimals using this method. Write a report of your findings.

COMPUTER POWER

Investigating Repeating Decimals

Since a computer is capable of repeating a sequence of steps very rapidly and accurately, it can be programmed to perform a division to any desired number of decimal places. Therefore, a computer is an ideal tool for investigating the patterns which occur when rational numbers are expressed as repeating decimals. The following program will cause the computer to print as many decimal digits as desired.

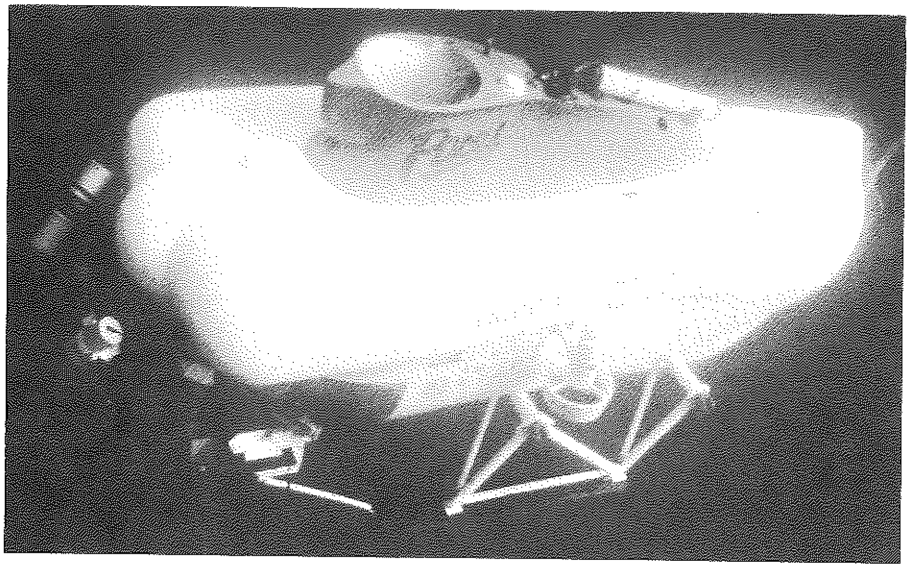
```
100 REM *** REPEATING DECIMALS ***
110 INPUT "WHAT IS THE NUMERATOR? ";N
120 INPUT "WHAT IS THE DENOMINATOR? ";D
130 INPUT "HOW MANY DECIMAL DIGITS? ";T
140 I=INT(N/D):PRINT
150 PRINT "THE DECIMAL EXPANSION TO ";T;" PLACES IS: "
160 PRINT:PRINT I;".";
170 R=N-I*D
180 FOR J=1 TO T
190     A=INT(R*10/D)
200     PRINT A;
210     R=R*10-D*A
220 NEXT J
230 END
```

To express $\frac{39}{17}$ as a repeating decimal, input the program. Type RUN and press $\boxed{\text{RETURN}}$. Answer each question that the computer asks and then press $\boxed{\text{RETURN}}$. Here is a sample of the output.

```
WHAT IS THE NUMERATOR? 39
WHAT IS THE DENOMINATOR? 17
HOW MANY DECIMAL DIGITS? 50
THE DECIMAL EXPANSION TO 50 PLACES IS:
2.29411764705882352941176470588235294117
647058823529
```

The result shows that $\frac{39}{17} = 2.\overline{2941176470588235}$

1. Express each fraction as a repeating decimal.
   a) $\frac{38}{23}$ b) $\frac{27}{31}$ c) $\frac{187}{84}$ d) $\frac{355}{113}$
2. Investigate the patterns in the repeating decimals for these fractions.
   a) $\frac{1}{19}, \frac{2}{19}, \frac{3}{19}, \ldots$ b) $\frac{1}{43}, \frac{2}{43}, \frac{3}{43}, \ldots$



2-3 MULTIPLYING AND DIVIDING RATIONAL NUMBERS

Submersibles like the Pisces III and the Aluminaut can perform numerous deep-water duties, such as search and rescue, repairing oil rigs, undersea exploration, and scientific research. The maximum operational depth of Pisces III is -1.10 km while the Aluminaut can operate at a depth 4.5 times as great.

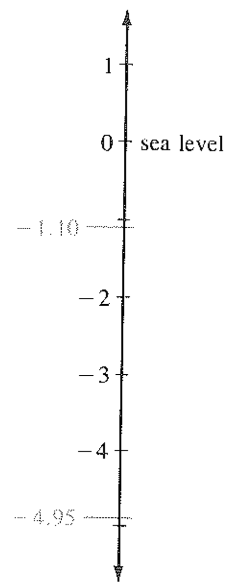
What is the maximum depth the Aluminaut can operate in?

Multiply to find the depth. $4.5 \times (-1.10)$

The rules for multiplying decimals and integers also apply when multiplying rational numbers.

$4.5 \times (-1.10) = -4.95$

The Aluminaut can operate to a maximum depth of -4.95 km.



In 1 min, a diver with scuba gear can descend to a depth of -15.2 m. To secure the legs of an oil-drilling rig the diver must descend to a depth of -76 m. How long will it take to reach this depth?

Divide to find the length of time. $(-76) \div (-15.2)$

The rules for dividing decimals and integers apply when dividing rational numbers.

$(-76) \div (-15.2) = +5$

The diver will take 5 min to descend.

The rules for multiplying rational numbers in fractional form are the same as those for multiplying common fractions. The signs obey the same rules as those for multiplying integers.

Example 1. Simplify. $\left(-\frac{2}{3}\right)\left(-\frac{9}{11}\right)$

Solution. The product of an even number of negative numbers is a positive number.

$$\left(-\frac{2}{3}\right)\left(-\frac{9}{11}\right) = \frac{2}{3} \times \frac{9}{11}$$

$$= \frac{2}{\cancel{3}_{1}} \times \frac{\cancel{9}^{3}}{11}$$

$$= \frac{6}{11}$$

Two rational numbers with a product of 1 are called reciprocals. The reciprocal of a rational number is sometimes called its multiplicative inverse.

> The *reciprocal* of any rational number $\frac{m}{n}$ (where $m, n \neq 0$) is defined to be the rational number $\frac{n}{m}$.

Example 2. a) What is the reciprocal of $-\frac{2}{5}$?

b) What is the multiplicative inverse of 4?

Solution. a) Since $\left(-\frac{2}{5}\right)\left(-\frac{5}{2}\right) = +1$, the reciprocal of $-\frac{2}{5}$ is $-\frac{5}{2}$.

b) The multiplicative inverse of 4 is $\frac{1}{4}$.

The rule for dividing rational numbers in fractional form is the same as the rule for dividing common fractions — multiply by the reciprocal.

Example 3. Simplify. $\frac{25}{4} \div \left(-\frac{5}{8}\right)$

Solution. The reciprocal of $-\frac{5}{8}$ is $-\frac{8}{5}$.

$$\frac{25}{4} \div \left(-\frac{5}{8}\right) = \frac{25}{4} \times \left(-\frac{8}{5}\right)$$

$$= -10$$

Example 4. Simplify. a) $2.54 \times (-3.86)$ b) $(-9.0272) \div 0.52$

Solution. a) $2.54 \times (-3.86)$

The product is negative. Use a calculator to multiply 2.54 by 3.86.

Key in: $\boxed{2}$ $\boxed{\cdot}$ $\boxed{5}$ $\boxed{4}$ $\boxed{\times}$ $\boxed{3}$ $\boxed{\cdot}$ $\boxed{8}$ $\boxed{6}$ $\boxed{=}$ to display 9.8044

Therefore, $2.54 \times (-3.86) = -9.8044$

b) $(-9.0272) \div 0.52$

The quotient is negative. Use a calculator to divide 9.0272 by 0.52.

Key in: $\boxed{9}$ $\boxed{\cdot}$ $\boxed{0}$ $\boxed{2}$ $\boxed{7}$ $\boxed{2}$ $\boxed{\div}$ $\boxed{\cdot}$ $\boxed{5}$ $\boxed{2}$ $\boxed{=}$ to display 17.36

Therefore, $(-9.0272) \div 0.52 = -17.36$

EXERCISES 2-3

(A)

1. Simplify.

a) $\frac{1}{2} \times \frac{8}{5}$ b) $\left(\frac{-2}{3}\right)\left(\frac{6}{-7}\right)$ c) $\left(\frac{-1}{4}\right)\left(\frac{-2}{-3}\right)$ d) $\left(\frac{-3}{-8}\right)\left(\frac{1}{-21}\right)$

e) $\left(\frac{15}{-2}\right)\left(\frac{-2}{45}\right)$ f) $-\left(\frac{-5}{12}\right)\left(\frac{36}{-5}\right)$ g) $\left(-\frac{7}{3}\right)\left(\frac{-6}{5}\right)$ h) $\frac{8}{3}\left(-\frac{9}{4}\right)$

2. Simplify.

a) $(7.2) \times 5$ b) $(-3) \times 6.4$ c) $(-4) \times (-0.8)$ d) $(-0.2) \times 0.6$

e) $1.3 \times (-0.5)$ f) 2.8×0.2 g) $(-1.5) \times 1.1$ h) $(-0.9) \times (-1.4)$

3. Write the multiplicative inverse for each rational number.

a) 9 b) -23 c) $\frac{16}{19}$ d) $-\frac{7}{13}$ e) -1.5 f) 0.8

g) 0.75 h) -2.5 i) $-\frac{1}{16}$ j) -0.6 k) -10 l) 0.01

4. Write the rational number represented by each square.

a) $\frac{3}{8} \times ▒ = 1$ b) $\frac{5}{9} \times ▒ = 1$ c) $\left(-\frac{4}{7}\right) \times ▒ = 1$

d) $2.5 \times ▒ = 1$ e) $\left(-\frac{2}{3}\right) \times ▒ = 1$ f) $\left(-\frac{7}{15}\right) \times ▒ = 1$

g) $\frac{3}{5} \times ▒ = -1$ h) $\left(-\frac{7}{8}\right) \times ▒ = -1$ i) $0.6 \times ▒ = -1$

5. Simplify.

a) $\frac{1}{8} \div \frac{1}{2}$ b) $\left(-\frac{7}{10}\right) \div \left(\frac{4}{-9}\right)$ c) $\left(\frac{5}{-8}\right) \div \left(\frac{-3}{-4}\right)$

d) $\left(\frac{-1}{5}\right) \div \left(\frac{8}{-15}\right)$ e) $\left(\frac{-8}{2}\right) \div \left(\frac{-4}{3}\right)$ f) $\left(-\frac{10}{3}\right) \div \frac{5}{4}$

g) $\frac{5}{4} \div \left(-\frac{5}{2}\right)$ h) $\left(-\frac{2}{3}\right) \div \frac{5}{7}$ i) $\frac{11}{6} \div \left(\frac{-7}{-12}\right)$

6. Simplify.
   a) $(-8.4) \div 2$ b) $(-3.6) \div (-4)$ c) $9.9 \div 0.3$
   d) $(-1.21) \div 1.1$ e) $16.8 \div (-0.8)$ f) $1.69 \div 0.13$
   g) $(-10.8) \div (-0.9)$ h) $0.288 \div (-0.12)$ i) $(-2.4) \div (-0.16)$

B

7. In 1932 the record diving depth in a submersible was about 9 times as deep as the record depth of 1865. The record depth of 1865 was -74.7 m. What was the depth achieved in 1932?

8. The sperm whale is normally found at a depth of -252 m. The greatest depth that a sperm whale has reached is about 4.5 times as great as its normal depth. What depth did the whale reach?

9. Simplify.
   a) $\left(\frac{-18}{7}\right) \times \left(\frac{-21}{9}\right)$ b) $\left(\frac{-3}{28}\right) \div \frac{9}{7}$ c) $\left(\frac{36}{-5}\right) \times \left(\frac{-18}{-35}\right)$
   d) $\frac{4}{39} \div \left(\frac{-64}{13}\right)$ e) $\frac{9}{48} \times \left(\frac{-6}{16}\right)$ f) $\left(\frac{-15}{55}\right) \times \left(\frac{-2}{-11}\right)$
   g) $\left(-\frac{72}{7}\right) \div \left(-\frac{12}{49}\right)$ h) $\left(\frac{-75}{3}\right) \div \left(\frac{-15}{4}\right)$ i) $\left(-\frac{33}{4}\right) \times \left(\frac{7}{22}\right)$

10. Simplify.
   a) $(-2.38) \times 4.47$ b) $(-3.4336) \div (-9.28)$ c) $0.046 \times (-10.08)$
   d) $0.164\ 15 \div (-24.5)$ e) $(-313.7) \times (-0.18)$ f) $(-106.2) \div 236$
   g) $0.000\ 161\ 2 \div 0.031$ h) $57.28 \times (-6.04)$ i) $6.4061 \div (-0.047)$

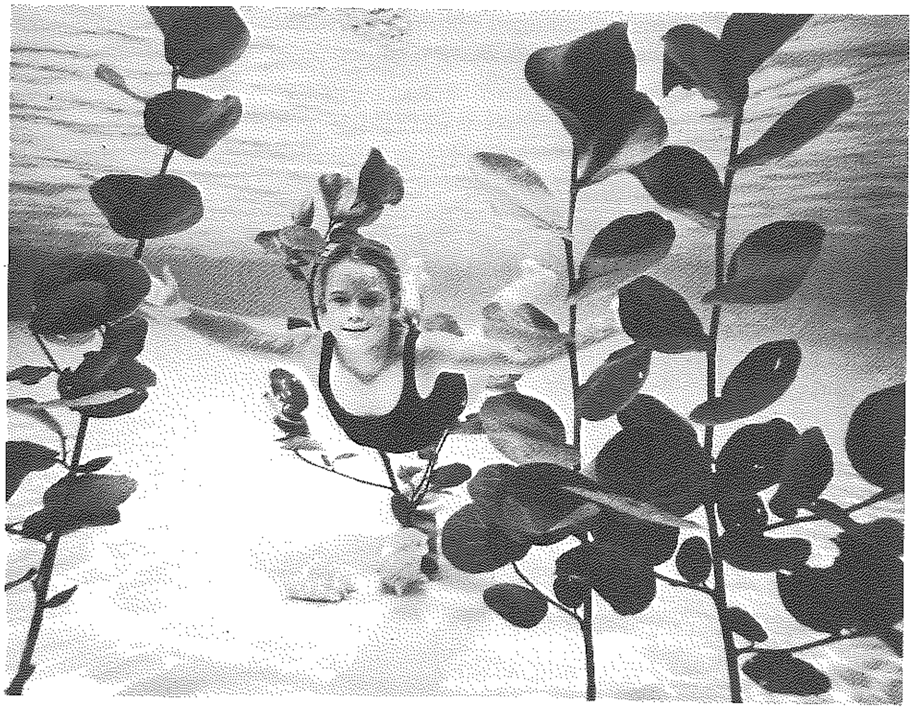
C

11. Without using a calculator or simplifying the expression, replace each comma with $>$ or $<$.
   a) $\frac{13}{-14}, \frac{-14}{13}$ b) $\frac{-6}{7}, \frac{7}{-8}$ c) $\frac{-2387}{3592}, \frac{-2388}{3593}$

12. Give an example of a rational number that can be expressed as a terminating decimal, and whose reciprocal can be expressed as a repeating decimal.

13. Write all the single digit numbers whose reciprocal is:
   a) a terminating decimal
   b) a repeating decimal
   c) neither a terminating decimal nor a repeating decimal.

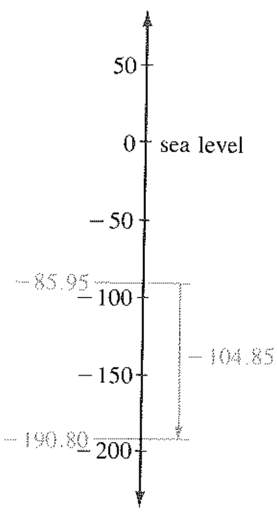
14. Write these expressions in order from least to greatest.
   $\left[(-25) \div \frac{2}{3}\right] \div \left(-\frac{1}{6}\right)$; $-\left[(-5)^2 \div \frac{1}{9} \div \left(\frac{4}{-5}\right)\right]$; $(-3)^2 \div \left[\frac{16}{5} \div \left(\frac{5}{-4}\right)^2\right]$



2-4 ADDING AND SUBTRACTING RATIONAL NUMBERS

To swim underwater from the surface of a pool 6 m deep to the bottom and back can be quite a challenge.

To an expert diver like Jacques Mayol, the challenge is to dive deeper than anyone else. In 1973 he held the world's record for breath-held diving at -85.95 m. In 1986 he still held the record but he had reached a depth of -104.85 m.



What is the total depth descended on both dives?
Add to find the total depth. $(-85.95) + (-104.85)$
The rules for adding decimals and integers also apply when adding rational numbers.
$(-85.95) + (-104.85) = -190.80$
Mayol descended 190.8 m on both dives.

How much deeper is the 1986 record than the 1973 record?
Subtract to find the difference in depth. $(-104.85) - (-85.95)$
The rules for subtracting decimals and integers also apply when subtracting rational numbers.
The opposite of -85.95 is $+85.95$.
Therefore, $(-104.85) - (-85.95)$ becomes
$$(-104.85) + (+85.95) = -18.90$$
The 1986 record is 18.9 m deeper than the 1973 record.

The rules for adding and subtracting rational numbers in fractional form are the same as those for adding and subtracting common fractions.

The operations are easier if rational numbers with negative denominators are first changed to their equivalents with positive denominators.

The signs obey the same rules as those for operating with integers.

Example 1. Simplify.

a) $\frac{3}{4} + \left(\frac{2}{-3}\right)$ b) $\left(\frac{-5}{6}\right) - \left(\frac{12}{-7}\right)$

Solution. a)
$$\begin{aligned}\frac{3}{4} + \left(\frac{2}{-3}\right) &= \frac{3}{4} + \left(-\frac{2}{3}\right)\\ &= \frac{3}{4} - \frac{2}{3}\\ &= \frac{9}{12} - \frac{8}{12}\\ &= \frac{1}{12}\end{aligned}$$

b)
$$\begin{aligned}\left(\frac{-5}{6}\right) - \left(\frac{12}{-7}\right) &= -\frac{5}{6} - \left(-\frac{12}{7}\right)\\ &= -\frac{5}{6} + \frac{12}{7}\\ &= -\frac{35}{42} + \frac{72}{42}\\ &= \frac{37}{42}\end{aligned}$$

Example 2. Simplify.

a) $(-0.928) + 37.089$ b) $1.37 - (-18.40)$

Solution. a) $(-0.928) + 37.089$

Rewrite the expression as $37.089 - 0.928$.

Use a calculator to subtract 0.928 from 37.089.

Key in:

$\boxed{3}\ \boxed{7}\ \boxed{\cdot}\ \boxed{0}\ \boxed{8}\ \boxed{9}\ \boxed{-}\ \boxed{\cdot}\ \boxed{9}\ \boxed{2}\ \boxed{8}\ \boxed{=}$ to display 36.161

Therefore, $(-0.928) + 37.089 = 36.161$

b) $1.37 - (-18.40)$

Rewrite the expression as $1.37 + 18.40$.

Use a calculator to add 18.40 to 1.37.

Key in: $\boxed{1}\ \boxed{\cdot}\ \boxed{3}\ \boxed{7}\ \boxed{+}\ \boxed{1}\ \boxed{8}\ \boxed{\cdot}\ \boxed{4}\ \boxed{=}$ to display 19.77

Therefore, $1.37 - (-18.40) = 19.77$

EXERCISES 2-4

Ⓐ

1. Simplify.
   a) $\frac{3}{4}+\frac{2}{3}$ b) $\frac{5}{7}-\frac{2}{5}$ c) $\frac{3}{8}-\frac{5}{6}$ d) $\frac{-5}{12}+\left(\frac{-3}{8}\right)$
   e) $\frac{2}{-9}+\frac{5}{6}$ f) $-\frac{4}{5}-\frac{2}{3}$ g) $\frac{3}{-4}-\left(\frac{-2}{5}\right)$ h) $-\left(\frac{-3}{8}\right)-\left(\frac{5}{-4}\right)$
2. Simplify.
   a) $(-1.7)+(-3.1)$ b) $2.8-5.9$ c) $(-3.6)-(-2.1)$
   d) $1.7+(-8.9)$ e) $7.6+9.3$ f) $(-6.4)+11.8$
   g) $(-8.7)-(-9.8)$ h) $(-15.6)+23.9$ i) $36.3-(+41.7)$

Ⓑ

3. Simplify.
   a) $-\frac{2}{3}+\left(\frac{1}{-4}\right)$ b) $-\left(\frac{-5}{6}\right)+\frac{3}{2}$ c) $\left(\frac{3}{-8}\right)-\frac{3}{4}$ d) $\frac{-5}{8}+\left(\frac{-1}{-6}\right)$
   e) $-\left(\frac{2}{-3}\right)-\frac{3}{10}$ f) $\frac{3}{4}-\left(\frac{-5}{8}\right)$ g) $\frac{9}{4}+\left(\frac{-7}{3}\right)$ h) $\frac{-20}{6}-\left(-\frac{13}{3}\right)$
4. Simplify.
   a) $\frac{7}{3}+\frac{21}{4}$ b) $\frac{47}{8}-\frac{8}{3}$ c) $\frac{13}{2}-\frac{49}{5}$ d) $\frac{17}{5}-\frac{35}{4}$
   e) $-\frac{14}{3}+\frac{12}{5}$ f) $\frac{9}{7}-\frac{9}{5}$ g) $-\frac{13}{5}+\frac{11}{6}$ h) $\frac{43}{3}-\frac{47}{7}$
5. Simplify.
   a) $-2.387+4.923$ b) $33.78-(-64.35)$ c) $204.9-256.1$
   d) $-0.405-18.924$ e) $-12.37+8.88$ f) $-45.8-(-327.6)$
   g) $4.29+563.08$ h) $84.91-37.08$ i) $-0.046+(-0.104)$
6. Simplify.
   a) $\frac{-7}{10}-\left(\frac{-7}{3}\right)$ b) $\frac{-15}{4}-\frac{13}{6}$ c) $\frac{13}{8}+\left(\frac{-3}{7}\right)$
   d) $\frac{25}{2}-\left(-\frac{13}{4}\right)$ e) $\frac{-11}{4}+\left(\frac{-4}{3}\right)$ f) $\frac{20}{9}-\left(\frac{-22}{3}\right)$
   g) $-\left(-\frac{11}{6}\right)-\left(\frac{11}{-18}\right)$ h) $\frac{14}{-5}+\left(\frac{-3}{7}\right)$ i) $\frac{-3}{11}+\frac{16}{3}$
7. In 1985 the United States had a federal budget deficit of −\$179.0 billion. In 1986 the deficit decreased by +\$7.1 billion over 1985.
   a) What was the total deficit at the end of 1986?
   b) In 1987 the deficit is projected to increase by a further −\$18.2 billion. What is the total deficit expected at the end of 1987?

8. The table shows the record depths achieved by a specially constructed submersible called a bathyscaphe.
   a) Calculate the difference in depth between consecutive records.
   b) Between which two dives was the difference in depth the largest?

Year	Record Depth in metres
1953	−3150.1
1954	−4049.9
1959	−5666.3
1960	−7315.2

9. Find the integer represented by each square.

a) $\frac{2}{5} + \frac{\square}{5} = \frac{6}{5}$ b) $\frac{2}{5} + \frac{\square}{5} = \frac{-6}{5}$ c) $\frac{3}{-7} - \frac{\square}{7} = -\frac{5}{7}$

d) $\frac{3}{-7} - \frac{\square}{7} = \frac{-5}{7}$ e) $\frac{7}{8} - \frac{4}{\square} = \frac{11}{8}$ f) $\frac{5}{9} - \frac{8}{\square} = \frac{1}{9}$

10. An editor assigns a value for each letter and space when laying out a book.

Symbols	Value
I, i, letter l, digit 1, punctuation	$\frac{1}{2}$
Other digits and lower case letters	1
m, w, and mathematical signs	$1\frac{1}{2}$
Spaces and capitals (except M, W)	2
M, W	$2\frac{1}{2}$

Find the total value of each line.
a) The answer given was $109\frac{1}{2}$.
b) Mass is measured in kilograms. Weight is in newtons.
c) Simplify 3.14 ÷ 4 × 7.2. Give the answer to 2 decimal places.

11. The shares of publicly owned companies are bought and sold on the stock exchange. Newspapers list each day's transactions.

COMPANY	Sales	High	Low	Close	Change
Scot Paper	413	$\$9\frac{3}{4}$	$9\frac{3}{4}$	$9\frac{3}{4}$	$-\frac{3}{4}$
Scot York	14 850	$\$7\frac{1}{2}$	7	$7\frac{1}{2}$	$+\frac{1}{8}$
Scotts A	1 000	$\$8\frac{1}{2}$	$8\frac{1}{2}$	$8\frac{1}{2}$	
Seagram	3 815	$\$35\frac{1}{2}$	$35\frac{1}{4}$	$35\frac{1}{4}$	$-\frac{3}{4}$
Seco Cem	100	$\$9\frac{1}{2}$	$9\frac{1}{2}$	$9\frac{1}{2}$	$-\frac{1}{4}$
Selkirk A	1 200	\$18	17	18	$+2\frac{1}{4}$
Shaw Pipe	10 900	$\$12\frac{1}{2}$	$12\frac{1}{4}$	$12\frac{3}{8}$	$-\frac{1}{8}$

The clipping shows that on one day 10 900 shares of Shaw Pipe were traded. The highest price paid for a share was $\$12\frac{1}{2}$ and the lowest price was $\$12\frac{1}{4}$. This was down $\frac{1}{8}$ on the previous day's closing price.

a) What happened to Scot York shares that day?
b) How much would 500 shares of Seagram's cost at the low price for the day?
c) How many shares of Seco Cem could have been bought for $1900?
d) Calculate the previous day's closing price for each share.

Ⓒ

12. If $x > 0$, $y < 0$, and $z < 0$, which expressions are always positive?

a) $\frac{x}{y}$ b) $\frac{xy}{z}$ c) $\frac{x}{yz}$ d) $\frac{y}{xz}$ e) $\frac{x}{y + z}$

f) $\frac{x - y}{z}$ g) $\frac{x}{x - y}$ h) $\frac{x - y}{x - z}$ i) $\frac{x}{y} + \frac{x}{z}$ j) $\frac{y}{z} - \frac{x}{y}$

CALCULATOR POWER

Adding and Subtracting Rational Numbers in Fractional Form

Many scientific calculators will add and subtract fractions if the numbers are keyed in, in the order in which they appear.

For example, to simplify $\frac{2}{5} + \frac{3}{4}$,

key in: $\boxed{2}$ $\boxed{\div}$ $\boxed{5}$ $\boxed{+}$ $\boxed{3}$ $\boxed{\div}$ $\boxed{4}$ $\boxed{=}$ to display 1.15

However, this is usually not the case when using a 4-function calculator. When the above sequence is keyed in, the result, 0.85, is incorrect. This is because the calculator adds 3 to the result of $2 \div 5$ before dividing by 4. However, by altering the sequence of operations, the correct result can be obtained.

Consider the sum $\frac{a}{b} + \frac{c}{d}$.

This can be written as $\frac{ad + bc}{bd} = \left(\frac{ad + bc}{b}\right)\frac{1}{d}$

$$= \left(\frac{ad}{b} + c\right)\frac{1}{d}$$

To simplify $\frac{2}{5} + \frac{3}{4}$,

key in: $\boxed{2}$ $\boxed{\times}$ $\boxed{4}$ $\boxed{\div}$ $\boxed{5}$ $\boxed{+}$ $\boxed{3}$ $\boxed{\div}$ $\boxed{4}$ $\boxed{=}$

to display the correct result of 1.15

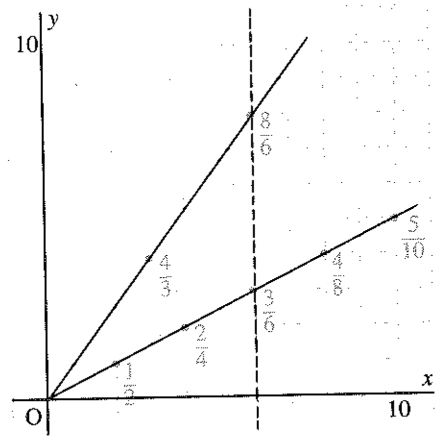
1. Use your calculator to simplify *Exercises 1* and *3* on page 55.

INVESTIGATE

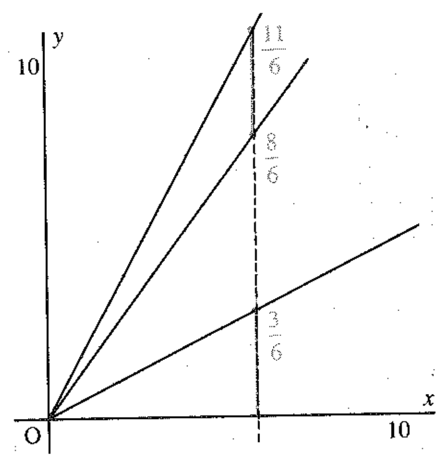
Adding Rational Numbers on a Grid

- Recall how to plot a rational number, in fractional form, on a grid. For example, to plot $\frac{4}{3}$, from the origin move 3 units in the x-direction and then 4 units in the y-direction. Draw a dot and label it $\frac{4}{3}$.

To simplify the expression $\frac{4}{3} + \frac{1}{2}$



- Plot each number on a grid. Join each point to the origin and extend the line.
- Move along each line from the origin. Label those equivalent rational numbers that can be written in the form $\frac{m}{n}$, where m and n are integers.
- On the two lines drawn, look for two points in the same vertical line, that is, $\frac{8}{6}$ and $\frac{3}{6}$.



- Add the vertical segments. Slide the line segment, with its upper end on $\frac{3}{6}$, vertically so that its lower end coincides with $\frac{8}{6}$. Then the upper end coincides with the point $\frac{11}{6}$.
- This point, $\frac{11}{6}$, is the sum of $\frac{4}{3}$ and $\frac{1}{2}$.

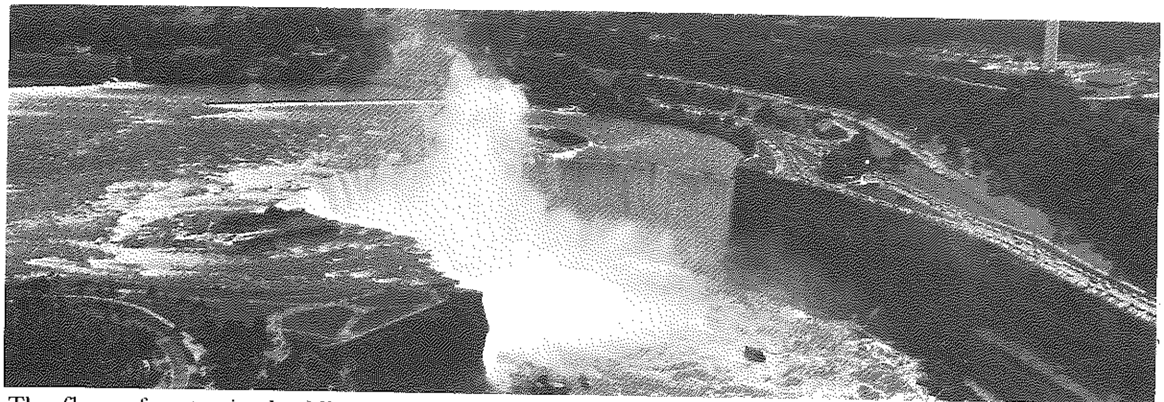
1. Use a grid to find each sum.

   a) $\frac{3}{4} + \frac{5}{3}$ b) $\frac{2}{5} + \frac{3}{2}$ c) $\frac{7}{9} + \frac{5}{6}$ d) $\frac{3}{5} + \frac{5}{3}$

2. Can you think of a method of subtracting rational numbers using a grid?

MATHEMATICS AROUND US

Niagara Falls is Moving!

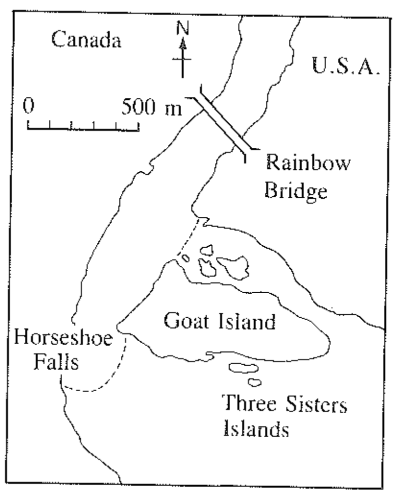


The flow of water in the Niagara River is about 5700 m^3/s. This great volume of water causes erosion at Niagara Falls and they moved upstream about 264 m, between the years 1700 and 1900.

Hydro-electric power plants, requiring the diversion of some of the water around the Falls, were opened in 1905, 1922, 1954, and 1960. Nowadays, as much as 75% of the water may go around, instead of over, the Falls. This has halved the rate of erosion.

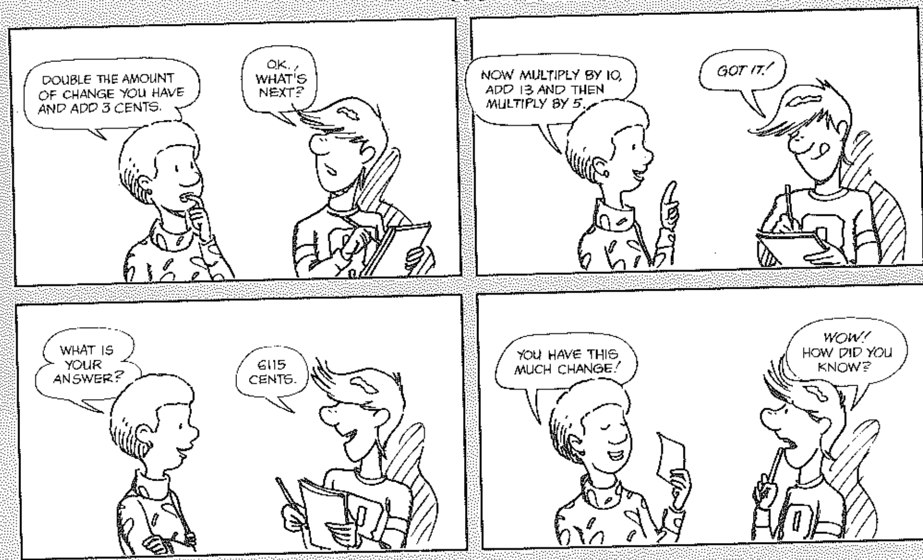
QUESTIONS

1. About how far did Niagara Falls move upstream in 1800?
2. About how far does Niagara Falls move upstream each year now?
3. a) How far has Niagara Falls moved since you were born?
   b) How far will it move in your lifetime?
4. How long would it take Niagara Falls to move the length of your classroom?
5. Use the map to answer these questions.
   a) When will Niagara Falls reach the Three Sisters Islands?
   b) When was Niagara Falls at the location of the Rainbow Bridge?
6. What assumptions did you make when answering *Questions 1* to *5*?



PROBLEM SOLVING

Work Backwards



How much change did Karl have?

Understand the problem

- What operations did Karl perform on the number he started with?
- What are we asked to find?

Think of a strategy

- Try working backwards from the final number to the starting number by reversing the operations.

Carry out the strategy

- Complete a flow chart for the operations that Karl performed.

Starting Number → $\times 2$ → $+3$ → $\times 10$ → $+13$ → $\times 5$ → Final Number

- Reverse the flow chart by changing multiplication into division and addition into subtraction.
- Start with the final number 6115 and follow the flow chart in reverse to determine the starting number.

Starting Number ← $\div 2$ ← -3 ← $\div 10$ ← -13 ← $\div 5$ ← Final Number

59 ← 118 ← 121 ← 1210 ← 1223 ← 6115

From the reverse flow chart, Karl had 59¢ change.

Look back

- Start with 59 and follow the instructions given to Karl. Do you obtain the final number 6115?

Solve each problem

1. Laura cashed a cheque for \$28.65 and put the cash in her purse. Then she purchased 2 magazines for \$1.95 each, a book for \$6.95, and a record for \$5.89. She had \$21.60 left in her purse. How much money did she have before cashing her cheque?
2. Todd had more milk than his younger brother Alan so he poured into Alan's glass as much milk as Alan's glass already contained. Then he poured from Alan's glass into his own, as much as his own glass contained. Finally he poured back into Alan's glass as much milk as Alan presently had. Then, both glasses contained 256 mL of milk. How much did each boy start with?
3. Stacey has a 3 L bucket and an 8 L bucket. How can she use these two unmarked buckets to obtain exactly 4 L of water?
4. Mr. Barter didn't have enough money to purchase a book he wanted. He said to the clerk, "If you will give me as much money as I have in my hand, I will spend \$6.00." The clerk agreed and after Mr. Barter spent the \$6.00 he repeated the offer. The clerk matched the amount Mr. Barter had left, and he spent another \$6.00. After Mr. Barter repeated the offer a third time, spending another \$6.00, he had no more money. How much money did he start with?
5. Ahmed's plane is to arrive in St. John's at 11:10 Newfoundland time. The flight time is 2 h and 55 min. What time should Ahmed leave his house if it takes about 1 h to drive to Lester B. Pearson airport in Toronto and he wants to arrive 45 min before departure? (Newfoundland is 1.5 h ahead of Toronto.)
6. Each year a car depreciates to $\frac{4}{5}$ of its value one year before. What was the original value of a car that is worth \$8000 after 4 years?
7. Lesley boarded a school bus at Oak Street along with 5 other students. The bus drove to Elm Street where it picked up 7 more students. The next stop was Beech Crescent where the bus picked up 6 students. There were now 1.5 times as many students on the bus as there were when it arrived at Oak Street. How many students were on the bus when it left Beech Crescent?
8. The total value of any sum of money that earns interest at 9% per annum doubles every 8 years. What amount of money invested now at 9% per annum will accumulate to \$30 000 in 32 years?

2-5 ORDER OF OPERATIONS WITH RATIONAL NUMBERS

CONTEST WINNERS
will be asked to answer correctly the following skill-testing question. What is the value of
$\frac{3}{4} - \left(-\frac{1}{2}\right)\left(\frac{5}{8}\right) \div \left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)$?

Merlin wrote:

$$\frac{3}{4} - \left(-\frac{1}{2}\right)\left(\frac{5}{8}\right) \div \left[\left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)\right]$$
$$= \frac{3}{4} - \left(\frac{-5}{16}\right) \div \frac{1}{16}$$
$$= \frac{3}{4} + \frac{5}{16} \div \frac{1}{16}$$
$$= \frac{17}{16} \div \frac{1}{16}$$
$$= 17$$

Jasmine wrote:

$$\frac{3}{4} - \left(-\frac{1}{2}\right)\left(\frac{5}{8}\right) \div \left[\left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)\right]$$
$$= \frac{3}{4} - \left(\frac{-5}{16}\right) \div \frac{1}{16}$$
$$= \frac{3}{4} - \left(\frac{-5}{16}\right) \times \frac{16}{1}$$
$$= \frac{3}{4} - (-5)$$
$$= \frac{3}{4} + 5$$
$$= \frac{23}{4}$$

Who is correct?
The order of operations with rational numbers is the same as it is for integers.

- Operations within brackets are performed first.
- Multiplication and division are performed in order from left to right.
- Lastly, addition and subtraction are performed in order from left to right.

In the example above, Jasmine's solution is correct. Merlin made an error when he subtracted $\left(\frac{-5}{16}\right)$ from $\frac{3}{4}$ before he divided by $\frac{1}{16}$.

Example 1. Simplify. $\left(\frac{-9}{4}\right) \times \left(\frac{-10}{21}\right) \div \left(\frac{-45}{7}\right)$

Solution. Multiply and divide in order from left to right.

$$\left(\frac{-9}{4}\right) \times \left(\frac{-10}{21}\right) \div \left(\frac{-45}{7}\right) = \left(\frac{\cancel{-9}}{\cancel{4}}\right) \times \left(\frac{\cancel{-10}}{\cancel{21}}\right) \times \left(\frac{\cancel{-7}}{\cancel{45}}\right)$$
$$= -\frac{1}{6}$$

Example 2. Simplify. $3\left(\frac{-5}{6}\right) + 5\left(\frac{-9}{8}\right) - 2\left(\frac{-3}{4}\right)$

Solution. Brackets imply multiplication, so perform these operations first.

$$3\left(\frac{-5}{6}\right) + 5\left(\frac{-9}{8}\right) - 2\left(\frac{-3}{4}\right) = 3\left(\frac{-5}{6}\right) + 5\left(\frac{-9}{8}\right) - 2\left(\frac{-3}{4}\right)$$
$$= \frac{-5}{2} - \frac{45}{8} + \frac{3}{2}$$
$$= -\frac{20}{8} - \frac{45}{8} + \frac{12}{8}$$
$$= -\frac{53}{8}$$

Example 3. Simplify. $3.78 - \frac{14.91}{4.26}(3.8 - 5.9)$

Solution. Use a calculator to do the arithmetic.
Evaluate the expression in the brackets first. Multiply and divide next.

$$3.78 - \frac{14.91}{4.26}(3.8 - 5.9) = 3.78 - \frac{14.91}{4.26}(-2.1)$$
$$= 3.78 - (-7.35)$$
$$= 3.78 + 7.35$$
$$= 11.13$$

Example 4. Simplify. $\left(-\frac{3}{5} + \frac{1}{2}\right) \times \left(-\frac{2}{3}\right)$

Solution. Simplify the expression in brackets first.

$$\left(-\frac{3}{5} + \frac{1}{2}\right) \times \left(-\frac{2}{3}\right) = \left(-\frac{6}{10} + \frac{5}{10}\right) \times \left(-\frac{2}{3}\right)$$
$$= \left(-\frac{1}{10}\right) \times \left(-\frac{2}{3}\right)$$
$$= \frac{1}{15}$$

EXERCISES 2-5

1. Simplify.
   a) $\frac{-2}{3} + \left(\frac{1}{-4}\right) - \left(\frac{-5}{6}\right)$
   b) $\frac{3}{2} - \left(\frac{3}{-8}\right) - \frac{3}{4}$
   c) $\frac{5}{-8} + \left(\frac{-1}{-6}\right) - \left(\frac{2}{-3}\right)$
   d) $\frac{3}{-10} - \frac{3}{4} - \left(\frac{-5}{8}\right)$
   e) $\frac{9}{4} + \frac{17}{3} - \frac{29}{6}$
   f) $\frac{-3}{5} + \left(\frac{-7}{10}\right) - \frac{1}{2}$
   g) $-\frac{7}{2} + \frac{4}{3} - \left(-\frac{5}{6}\right)$
   h) $-\frac{5}{9} - \left(-\frac{2}{3}\right) + \left(-\frac{7}{6}\right)$
   i) $\frac{13}{2} + \left(\frac{-2}{3}\right) - \frac{7}{4} + \left(\frac{4}{-3}\right)$
   j) $\frac{4}{7} - \left(\frac{3}{-5}\right) + \left(\frac{-1}{2}\right) - \frac{3}{35}$

2. Simplify.
   a) $\left(\frac{4}{-9}\right) \times \left(\frac{-21}{-32}\right) \times \left(\frac{-3}{14}\right)$
   b) $\left(\frac{-10}{27}\right) \times \left(\frac{-8}{20}\right) \times \left(\frac{-45}{-28}\right)$
   c) $\left(\frac{-6}{-25}\right) \div \left(\frac{-2}{-21}\right) \div \left(\frac{14}{-25}\right)$
   d) $\left(\frac{12}{-39}\right) \div \left(\frac{-10}{-9}\right) \div \left(\frac{18}{-5}\right)$
   e) $\left(\frac{15}{-32}\right) \times \left(\frac{-4}{5}\right) \div \left(-\frac{9}{16}\right)$
   f) $\left(\frac{-12}{28}\right) \div \left(\frac{-8}{-15}\right) \times \left(\frac{-14}{-25}\right)$
   g) $\frac{5}{2} \div \left(-\frac{10}{3}\right) \times \frac{8}{3}$
   h) $\left(-\frac{15}{4}\right) \times \frac{8}{5} \div \left(-\frac{6}{5}\right)$
   i) $\left(\frac{20}{-3}\right) \div \left(\frac{-35}{9}\right) \times \left(\frac{-14}{-6}\right) \div \frac{4}{3}$
   j) $\frac{22}{3} \times \left(\frac{-6}{77}\right) \times \left(\frac{-3}{-2}\right) \div \left(\frac{2}{-7}\right)$

3. Simplify.
   a) $3.7 + 0.4 - 17.6$
   b) $-0.38 + 2.09 - 8.11$
   c) $54.68 + (-18.07) - (+38.46)$
   d) $-25.3 - (-27.9) + 60.0$

4. Simplify.
   a) $(-14.6) \times (-23.7) \times 10.4$
   b) $(-12.958) \div (-2.2) \div 1.9$
   c) $(145.0) \times (-14.6) \div (-12.5)$
   d) $(966.52) \div (-29.2) \times 0.9$
   e) $(0.017\,67) \div (-0.95) \div (-0.31)$
   f) $0.08 \times (-1.03) \times 0.5$

5. Simplify.
   a) $\frac{4}{5} \times \left[\frac{3}{8} + \left(\frac{-7}{4}\right)\right]$
   b) $\left[\frac{-3}{7} - \left(\frac{-7}{2}\right)\right] \div \left(\frac{-7}{3}\right)$
   c) $\left(\frac{-6}{7}\right) + \left[\frac{3}{4} \times \left(\frac{-16}{7}\right)\right]$
   d) $\left[\left(\frac{-18}{5}\right) \div \frac{27}{5}\right] - \left(\frac{-6}{11}\right)$
   e) $\left[\left(\frac{-5}{9}\right) - \frac{7}{6}\right] \times \frac{9}{5}$
   f) $\left(\frac{-4}{9}\right) \div \left[\left(\frac{-3}{8}\right) + \left(\frac{-4}{3}\right)\right]$

6. Simplify.
   a) $\left(-\frac{5}{6}+\frac{2}{3}\right)\times\left(-\frac{3}{4}\right)\div\frac{5}{6}$
   b) $\left(-\frac{3}{5}\times\frac{2}{3}\right)+\frac{5}{6}\div\left(-\frac{5}{3}\right)$
   c) $\left(-\frac{3}{4}\right)\div\frac{1}{5}+\left[-\frac{1}{3}\times\left(-\frac{5}{2}\right)\right]$
   d) $\left[\frac{3}{16}+\left(-\frac{3}{4}\right)\right]\times\left(-\frac{3}{8}\right)\div\frac{1}{4}$
   e) $\frac{3}{5}+\left(-\frac{2}{3}\right)\times\left[-\frac{3}{4}\div\left(-\frac{1}{2}\right)\right]$
   f) $\left[\frac{7}{12}\div(-14)\right]-\frac{3}{8}\times\frac{5}{3}$
7. Simplify.
   a) $[(-3.8)+(-0.9)]\times[7.2-4.7]$
   b) $\frac{79.12}{9.2}(-2.18+5.27)$
   c) $(-4.91)\times(-3.78)+\left(\frac{50.827}{-6.85}\right)$
   d) $(-74.52)\div(9.2)+(-23.9)\times 16.7$
   e) $(-0.65)-(-11.82)\times(21.65)\div(-17.32)$
   f) $[88.48\div(-15.8)]-[(-34.9)+47.0]$

2-6 RATIONAL NUMBERS AND FORMULAS

A survey was conducted on the heights and the masses of grade 9 students. It was discovered that there is a relationship between the height and the average mass of the students. This relationship can be expressed in a *formula*.

$M = \frac{3}{4}H - 72$

M represents the average mass of a grade 9 student who is H centimetres tall.

The formula can be used to find the average mass of students of any given height.

Suppose a student, who is 1.5 m tall, wants to know if her mass is above or below the average. She *substitutes* for H in the formula. First she converts her height to centimetres.

H = 1.5 m or 150 cm

Then, her mass is given by $M = \frac{3}{4}(150) - 72$

$= 40.5$

The student knows that students of her age and height have an average mass of 40.5 kg.

Many formulas used in science, business, and industry involve rational numbers. Consider the problem that was posed at the beginning of this chapter.

Example 1. The rate of fuel consumption of a certain model of car is given by this formula.

$$R = -\frac{36}{5}F + \frac{29}{2}$$

F is the fraction of driving on the highway. R is the rate of fuel consumption in litres per 100 km. What will be the rate of fuel consumption, to 1 decimal place, when:

a) three-quarters of the driving is on the highway

b) two-thirds of the driving is in the city?

Solution. a) To find the value of the rate, R, substitute $F = \frac{3}{4}$ into the formula.

$$\begin{aligned} R &= -\frac{36}{5}F + \frac{29}{2} \\ &= -\frac{36}{5}\left(\frac{3}{4}\right) + \frac{29}{2} \\ &= -\frac{27}{5} + \frac{29}{2} \\ &= -5.4 + 14.5 \\ &= 9.1 \end{aligned}$$

When three-quarters of the driving is on the highway, the rate of fuel consumption is 9.1 L/100 km.

b) If two-thirds of the driving is in the city, then one-third is on the highway. Substitute $F = \frac{1}{3}$ in the formula to find the rate, R.

$$\begin{aligned} R &= -\frac{36}{5}F + \frac{29}{2} \\ &= -\frac{36}{5}\left(\frac{1}{3}\right) + \frac{29}{2} \\ &= -\frac{12}{5} + \frac{29}{2} \\ &= -2.4 + 14.5 \\ &= 12.1 \end{aligned}$$

When two-thirds of the driving is in the city, the rate of fuel consumption is 12.1 L/100 km.

Why is there such a difference between the fuel consumption rates for city and highway driving?

Example 2. An old reference book gives temperatures in Fahrenheit degrees. The formula relating Celsius degrees, C, and Fahrenheit degrees, F, is

$$C = \frac{5}{9}(F - 32).$$

Find the Celsius temperature which corresponds to $-30°F$.

Solution. Substitute $F = -30$ into the formula.

$$\begin{aligned} C &= \frac{5}{9}(-30 - 32) \\ &= \frac{5}{9}(-62) \\ &\doteq -34 \end{aligned}$$

A temperature of $-30°F$ is equivalent to approximately $-34°C$.

EXERCISES 2-6

Ⓐ

1. Use the formula in *Example 2* to find the Celsius temperature equivalent of each temperature.
   a) 59°F b) $-4°F$ c) 86°F

2. Find the rate of fuel consumption, R litres per 100 km, when the fraction of the distance the car in *Example 1* is driven:
   a) on the highway is two-thirds of the total
   b) on the highway is five-sixths of the total
   c) in the city is seven-twelfths of the total.

Ⓑ

3. The cost of operating a certain type of aircraft is given by this formula.
   $$C = 900 + \frac{m}{200} + \frac{20\ 000\ 000}{m}$$
   m is the cruising altitude in metres. C is the cost in dollars per hour. Find the hourly cost of operating the aircraft at each altitude.
   a) 8000 m b) 10 000 m
   Why is it cheaper to operate the aircraft at higher altitudes?

4. The power delivered by a high-voltage power line is given by this formula.
   $$P = I(132 - \frac{1}{10}I)$$
   I is the current in amperes and P is the power in kilowatts. Find the power, to the nearest kilowatt, that is available when the current is 6.6 A.

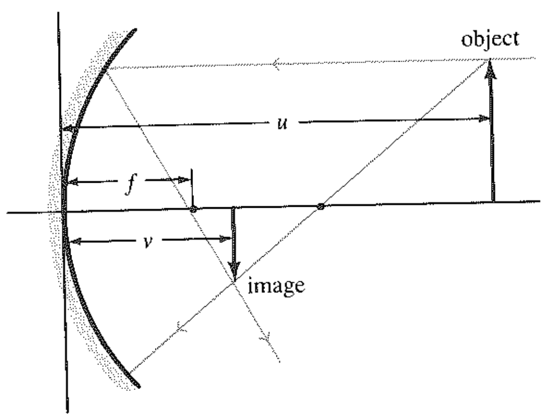
5. The efficiency of a jack is given by this formula. E is the efficiency and h is determined by the pitch of the thread. Find the efficiency of a jack with each value of h.
   $$E = \frac{h(1 - \frac{1}{2}h)}{h + \frac{1}{2}}$$
   a) $\frac{3}{5}$ b) $\frac{2}{3}$

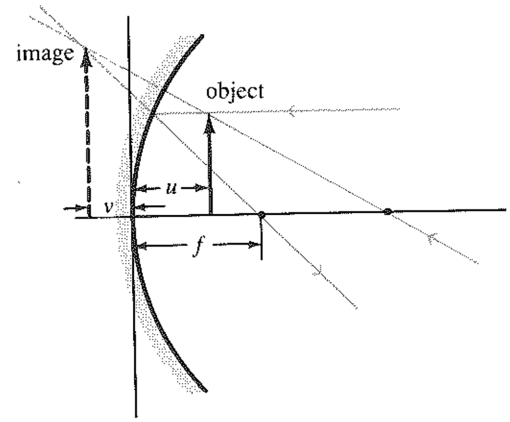
6. The focal length of a concave spherical mirror is related to the distances of the object and the image from the mirror by this formula.

$$f = \frac{uv}{u + v}$$

f is the focal length of the mirror. u is the distance of the object from the mirror. v is the distance of the image from the mirror. All lengths are measured in the same units. Find the focal length of each spherical mirror.

a) When the object is 3 cm from the mirror, the image is 2 cm from the mirror.





b) When the object is 1.6 m from the mirror, the image is -2.4 m from the mirror.

7. A car-rental firm uses this formula, $C = 28.5D + 0.095(d - 75)$, to calculate the cost, C dollars, of using one of their cars for D days to travel d kilometres ($d > 75$). Find how much the firm will charge a customer who uses the car for 5 days and travels 955 km.

8. A daily-interest savings account pays 9% per annum. The interest is calculated daily and added to the account at the end of each month. The formula, $A = P\left(1 + \frac{0.09d}{365}\right)$, gives the value, A dollars, of the money in the account d days after depositing P dollars. Calculate the value of the account for each deposit.

a) a deposit of \$90 after 30 days

b) a deposit of \$10 000 after 5 days

9. A car brakes and decelerates uniformly. The distance, d metres, that it travels in t seconds is given by this formula. $d = ut - 3.5t^2$

u is its speed, in metres per second, before the brakes are applied. Find how far the car travels under these conditions.

a) It is travelling 25 m/s before the brakes are applied. It travels for 5 s after the brakes are applied.

b) It is travelling 35 m/s before the brakes are applied. It travels for 5.5 s after the brakes are applied.

C

10. A car-rental firm uses the formula $C = 28.5D + 0.095(d - 75)$ to compute the cost, C dollars, to customers who use one of their cars for D days to travel d kilometres ($d > 75$). If a customer's bill was \$125.88 for three days of use, how far did the customer drive?

11. Use the formula in *Exercise 8* to find how long a deposit of \$1000 remains in such an account before it has a value of:
    a) \$1001 b) \$1003 c) \$1005.

12. Use the formula in *Example 2* to find the Fahrenheit equivalent of each temperature.
    a) 100°C b) 215°C c) 5°C d) −20°C

13. At what temperature will a Fahrenheit thermometer and a Celsius thermometer show the same reading?

INVESTIGATE

1. Start with any rational number in fractional form, for example, $\frac{3}{5}$. Let n represent its numerator and d its denominator. Form a new fraction by substituting for n and d into this formula.
   $$\frac{2n + d}{n + 2d}$$
   The result is $\frac{2(3) + 5}{3 + 2(5)} = \frac{11}{13}$ which is 0.8461 . . . as a decimal.
   Repeat the process with the new fraction. That is, substitute $n = 11$ and $d = 13$.
   The result is $\frac{2(11) + 13}{11 + 2(13)} = \frac{35}{37}$ or 0.9459 . . .
   Continue this process. What does the result appear to be?

2. Repeat the process several times, each time beginning with a different fraction. What do you notice?

3. Repeat the process with a different formula. For example:
   $$\frac{3n + d}{n + 3d}; \quad \frac{n + d}{4n + d}; \quad \frac{n + 2d}{n + d}; \quad \frac{2n + d}{n + d}.$$

4. Write a report of your findings.

COMPUTER POWER

Investigating Rational Numbers in Formulas

The formula, $\frac{2n + d}{n + 2d}$, which was introduced in the previous *INVESTIGATE* section, can be evaluated using this computer program.

```
100 REM *** FRACTION INVESTIGATION ***
110 INPUT "ENTER COEFFICIENTS: ";A,B,C,D
120 INPUT "WHAT IS THE NUMERATOR? ";NUM
130 INPUT "WHAT IS THE DENOMINATOR? ";DEN
140 FOR K = 1 TO 10
150 X = A * NUM + B * DEN:Y = C * NUM + D * DEN
160 F = INT (10000 * X/Y) / 10000
170 PRINT X,Y,F:NUM = X: DEN = Y
180 NEXT K
190 PRINT : INPUT "PRESS S TO STOP, RETURN TO REPEAT ";Y$
200 PRINT : IF Y$ <> "S" THEN 120: END
```

This program is written for a general formula, $\frac{An + Bd}{Cn + Dd}$.

Hence, it is necessary to input values for A, B, C and D, the coefficients of n and d. Then this formula can be used to evaluate *Question 3* of the previous *INVESTIGATE* section.

The printout for *Question 1* of the *INVESTIGATE* section is shown below.

```
ENTER COEFFICIENTS: 2,1,1,2
WHAT IS THE NUMERATOR? 3
WHAT IS THE DENOMINATOR? 5
11          13          .8461
35          37          .9459
107         109         .9816
323         325         .9938
971         973         .9979
2915        2917        .9993
8747        8749        .9997
26243       26245       .9999
78731       78733       .9999
236195      236197      .9999

PRESS S TO STOP, RETURN TO REPEAT S
```

1. Input the program. Run the program. Choose values for A, B, C, and D for the formula. Select a fraction to be substituted in the formula.
2. Write a report of your findings.

1. Express as fractions in lowest terms.
   a) -2.5 b) 0.125 c) -1.8 d) -1.11 e) 8.475
2. State which of these rational numbers are equivalent.
   $\frac{-6}{9}, \frac{16}{-25}, \frac{21}{28}, -\frac{14}{21}, \frac{-30}{-45}, \frac{10}{-15}, \frac{-9}{12}$
3. Arrange these rational numbers from least to greatest.
   $-\frac{15}{16}, \frac{-43}{48}, \frac{11}{12}, \frac{5}{6}, -\frac{-31}{-32}, \frac{2}{3}$
4. a) List four rational numbers between 0 and 1.
   b) List four rational numbers between -1 and -2.
5. Express each rational number as a decimal.
   a) $\frac{3}{8}$ b) $\frac{-4}{7}$ c) $\frac{7}{-12}$ d) $\frac{20}{9}$ e) $-\frac{35}{16}$ f) $\frac{2}{3}$
6. Express each decimal as a fraction in lowest terms.
   a) -1.5 b) 20.25 c) 2.007 d) -41.6 e) 10.75 f) -3.875
7. Simplify.
   a) $-\frac{2}{3} \times \frac{7}{8}$ b) $\left(\frac{5}{-8}\right)\left(\frac{-9}{12}\right)$ c) $\left(\frac{3}{-5}\right)\left(\frac{-7}{-8}\right)$
   d) $\left(\frac{13}{15}\right)\left(\frac{30}{-39}\right)$ e) $\left(-\frac{6}{5}\right)\left(\frac{12}{-15}\right)\left(\frac{-25}{36}\right)$ f) $\left(\frac{8}{13}\right)\left(-\frac{6}{5}\right)\left(\frac{-4}{7}\right)$
8. Simplify.
   a) $\frac{3}{7} \div \left(\frac{-9}{14}\right)$ b) $-\frac{13}{4} \div \left(\frac{2}{-3}\right)$
   c) $\frac{7}{-8} \div \left(-\frac{9}{4}\right)$ d) $\frac{-24}{-35} \div \left(\frac{16}{-21}\right) \div \frac{9}{10}$
   e) $\frac{-3}{5} \div \left(\frac{-5}{-12}\right) \div \left(\frac{-9}{10}\right)$ f) $\frac{18}{5} \div (-3) \div \frac{3}{2}$
9. Simplify.
   a) $(-56.28) \times (0.09)$ b) $(14.46) \div (-24.1)$
   c) $(143.7) \times (-206.8)$ d) $(-1433.36) \div (43.7)$
   e) $(-7.9808) \div (-92.8)$ f) $(-0.029) \times (-33.370)$
10. Simplify.
    a) $\frac{3}{5} + \frac{4}{7}$ b) $\frac{5}{12} + \frac{3}{8}$ c) $\frac{2}{9} + \frac{7}{12}$ d) $\frac{3}{11} + \left(\frac{-5}{11}\right)$
    e) $\frac{13}{-24} + \left(\frac{-7}{24}\right)$ f) $\frac{-2}{3} + \left(\frac{-4}{9}\right)$ g) $\frac{4}{-5} + \frac{14}{15}$ h) $\frac{-3}{-7} + \left(\frac{-2}{5}\right)$
    i) $\frac{-4}{9} + \left(\frac{17}{-21}\right)$ j) $-\frac{5}{12} + \left(\frac{7}{-9}\right)$ k) $-\frac{32}{15} + \frac{19}{6}$ l) $\frac{14}{3} + \left(-\frac{31}{4}\right)$

11. Simplify.
a) $\frac{7}{9} - \frac{1}{6}$ b) $\frac{5}{6} - \frac{3}{10}$ c) $\frac{7}{8} - \frac{5}{12}$ d) $\frac{-5}{8} - \frac{3}{8}$
e) $\frac{17}{-20} - \left(\frac{-12}{20}\right)$ f) $\frac{-7}{8} - \left(\frac{-1}{4}\right)$ g) $\frac{9}{11} - \left(\frac{-3}{5}\right)$ h) $-\frac{16}{5} - \left(-\frac{7}{4}\right)$

12. Simplify.
a) $98.37 - (+102.89)$ b) $(-39.10) + (-9.22)$
c) $(-254.6) - (-748.9)$ d) $58.73 - (-102.99)$
e) $301.7 + (-76.8)$ f) $(-401.01) + (-0.96)$

13. Simplify.
a) $\frac{2}{3} \div \frac{5}{6} + \left(-\frac{1}{4}\right)$ b) $-\frac{5}{4} \times \frac{-2}{5} + \frac{2}{3}$
c) $-\frac{7}{6} - \frac{3}{5} \div \left(-\frac{6}{7}\right)$ d) $\frac{3}{8} \times \left(-\frac{4}{3}\right) + \frac{5}{8} \div \left(-\frac{3}{2}\right)$

14. Simplify.
a) $\frac{5}{2} - \frac{11}{3} + \frac{5}{4}$ b) $\frac{5}{2} - \frac{5}{4} \div \frac{5}{4}$ c) $\frac{-6}{5} + \left(\frac{10}{-2}\right)\left(\frac{-3}{5}\right)$
d) $\left[\frac{3}{-4} - \left(\frac{-3}{4}\right)\right] \div 2$ e) $-6\left(\frac{4}{5} - \frac{1}{2}\right)$ f) $\left(\frac{3}{5}\right)\left(-\frac{1}{2}\right)\left(\frac{-6}{3}\right) + \frac{1}{5}$

15. a) $-9.6 + (-3.2) \times 6.4 \div 1.6$
b) $\frac{-27.36}{-5.7} + (37.42)(-0.81)$
c) $(-88.7 + 43.9) \times (-65.96) \div 9.7$
d) $[(-15.5) + (6.2)(-3.4)] - (-7.7) \times (-8.2)$

16. Simplify.
a) $\left(\frac{3}{4}\right)\left(\frac{1}{-2}\right) + \left(\frac{5}{6}\right)\left(\frac{-1}{3}\right)$
b) $\frac{3}{8} \times \frac{2}{3} - \left(\frac{1}{2}\right)\left(\frac{-5}{6}\right) + \left(\frac{3}{5}\right)\left(\frac{3}{-4}\right)$
c) $\left[\frac{5}{2} \div \left(\frac{-4}{5}\right)\right] - \left(\frac{3}{-4}\right)\left(\frac{-8}{9}\right)$

17. For the years 1975 to 2000, the approximate population, P in thousands, of a city is given by this formula.
$P = (y - 1981)(1995 - y) + 500$
y is the year. Find the population in each year.
a) 1977 b) 1984 c) 1990 d) 1999

18. A car-rental firm uses the formula $C = 20.5D + 0.125(d - 80)$ to compute the cost, C dollars, to a customer who uses one of their cars for D days to travel d kilometres ($d > 80$). How much will a customer pay for using a car for 4 days and driving 800 km?