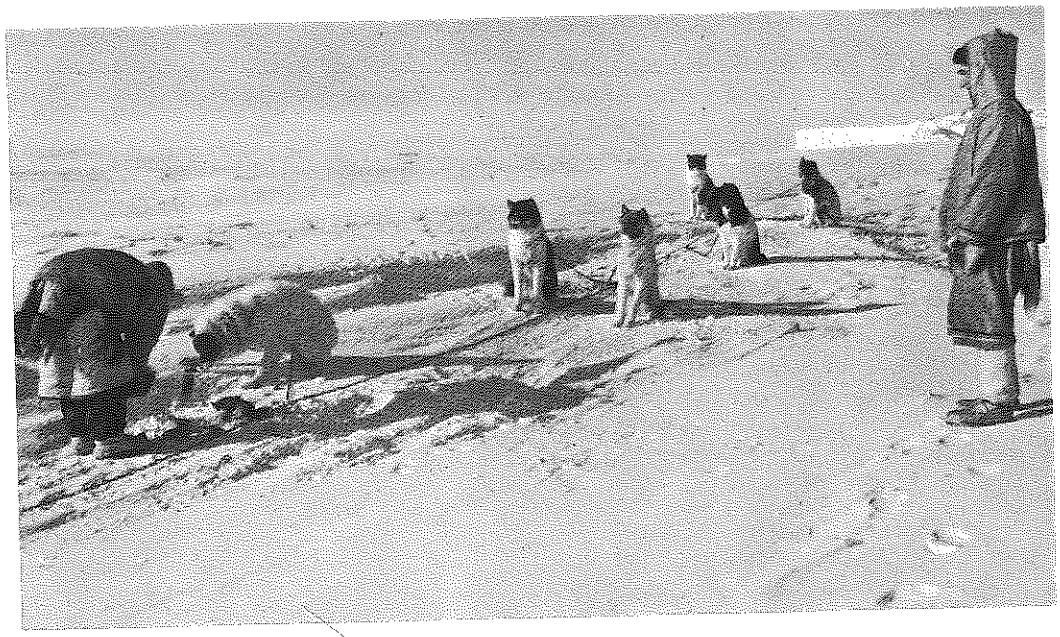
1 Integers

Ste. Agathe Altitude 2350 m
Valleville Altitude 350 m

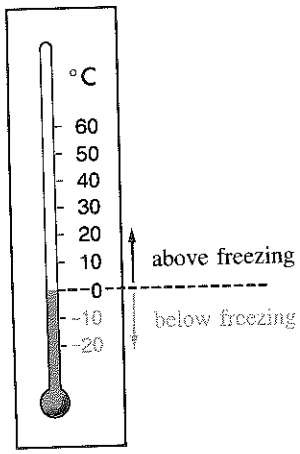
It is 8°C in Valleville.
The temperature decreases 6.5°C for every 1000 m increase in altitude. If there is precipitation at the village of Ste. Agathe, will it be rain or snow? (See Section 1-3, *Example 4*.)



1-1 USING INTEGERS

On a thermometer, 0° represents the freezing point of water on the Celsius scale.

Temperatures below freezing are indicated on this thermometer by *negative integers*; temperatures above freezing by *positive integers*.



Newspapers publish charts of the temperatures at different places in the country at a certain time of the day.

From the chart, it can be seen that only Victoria, Vancouver, and Calgary were above freezing.

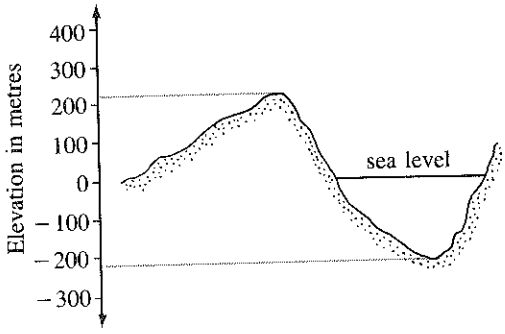
Which was the coldest place?

Temperatures at 2 p.m. EST yesterday:

	°C
Whitehorse	−1
Yellowknife	−7
Victoria	7
Vancouver	6
Calgary	4
Edmonton	−6
Prince Albert	−2
Regina	−5
Winnipeg	−10

In land elevations, positive integers indicate heights *above* sea level and negative integers indicate heights *below* sea level.

The sectional drawing shows a high point of about +220 or 220 m above sea level. The lowest point is about −220 or 220 m below sea level.

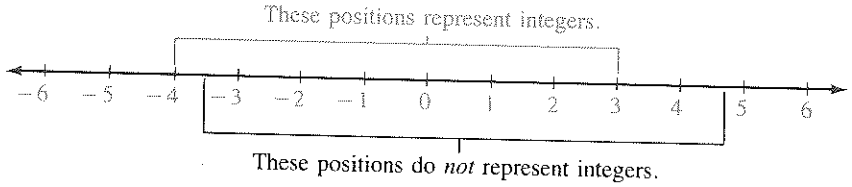


In general, numbers such as 1, 2, 3, . . . (sometimes written $+1$, $+2$, $+3$, . . .) are called positive integers; and numbers such as -1, -2, -3, . . . are called negative integers. The negative integers, zero, and the positive integers make up the set of integers, denoted by I.

$I = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Integers such as 3 and -3 are called *opposite integers*. We say that 237 is the opposite of -237.

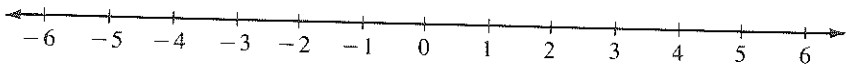
Integers can be represented on a number line.



Any integer on the number line is *greater than* all the integers to its *left* and *less than* all the integers to its *right*.
For example, -2 is greater than -5. This is written $-2 > -5$.
Conversely, -5 is less than -2. This is written $-5 < -2$.

Example 1. Compare.
a) -6 and 2 b) -5 and -1 c) 4 and -3 d) 0 and -5

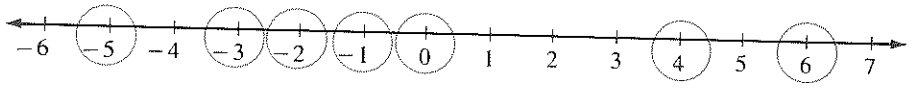
Solution. Draw a number line.



a) -6 is to the left of 2, so -6 is less than 2, or $-6 < 2$.
b) -5 is to the left of -1, so -5 is less than -1, or $-5 < -1$.
c) 4 is to the right of -3, so $4 > -3$.
d) 0 is to the right of -5, so $0 > -5$.

Example 2. Arrange -5, 6, -3, -1, 0, 4, -2 in order from:
a) least to greatest b) greatest to least.

Solution. Draw a number line. Circle the given integers.



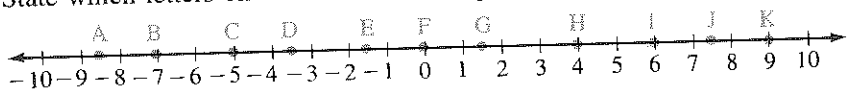
a) The integers are ordered from least to greatest when written in order from left to right. -5, -3, -2, -1, 0, 4, 6

b) The integers are ordered from greatest to least when written in the reverse order. 6, 4, 0, -1, -2, -3, -5

EXERCISES 1-1

(A)

1. State which letters on this number line represent integers.



Write the integer represented by each letter.

2. Write using integers.
   a) a gain of \$9
   b) a loss of \$21
   c) 80°C above freezing
   d) 20°C below freezing
   e) a profit of \$50
   f) a loss of \$75
   g) a debt of \$81
   h) a depth of 12 000 m
   i) an altitude of 3000 m

3. If +100 represents a gain in altitude of 100 m, state what these integers represent.
   a) −300 b) +25 c) −100 d) +2

4. If −5 represents a debt of \$5, state what these integers represent.
   a) −12 b) +7 c) +15 d) −53

5. State the opposite.
   a) a gain of \$10
   b) an altitude loss of 500 m
   c) a 3 kg loss of mass
   d) a temperature of −14°C
   e) 18
   f) −11
   g) the opposite of −7
   h) the opposite of 5

(B)

6. Compare.
   a) −3 and 2 b) 5 and −6 c) −4 and −1 d) −3 and 0
   e) 2 and −5 f) −9 and −1 g) −11 and 10 h) −8 and −9

7. State the least integer.
   a) 1, −2, 0
   b) −6, −3, 1
   c) −1, 4, −8
   d) 0, −2, −4
   e) 2, −9, −3
   f) −5, 1, −1
   g) −4, 7, −10, 1
   h) 3, −2, 15, −18, 7
   i) −5, 3, 0, −16, 17

8. Arrange in order from least to greatest.
   a) 3, −1, 5, −4 b) −2, 8, −10, 5 c) −1, 4, −8, −2, 5, 0

9. Arrange in order from greatest to least.
   a) −2, 7, 1, −4 b) −3, 0, 2, −1 c) 5, −8, −2, 8, 0, 3

(C)

10. State which integer is
    a) 3 less than 1.
    b) 2 more than −1.
    c) 6 more than −4.
    d) 8 less than 5.
    e) 7 more than the opposite of 3.
    f) 5 less than the opposite of −1.

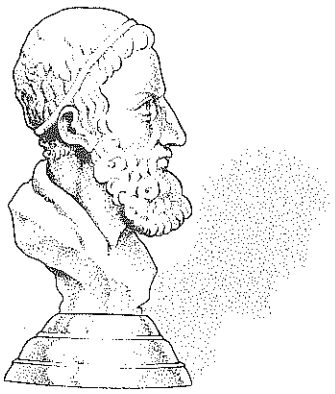
11. State which integer is
    a) 6 less than 2.
    b) 4 more than −9.
    c) 3 more than 0.
    d) 5 less than −2.
    e) 2 more than the opposite of −3.
    f) 1 less than the opposite of −1.

THE MATHEMATICAL MIND

Problems and their Solvers of Times Gone By

Some problems involving mathematics have required far more than correct arithmetic and the application of the right formulas. They have required the discovery of new principles, and the invention of special mathematical techniques. Here are three of the world's greatest mathematicians and the kinds of problems they solved.

Why does a small rock sink and a large block of wood float?



Archimedes
287–212 B.C.

Archimedes is regarded as the greatest problem solver of the ancient world. Apparently his powers of concentration were so deep that, when working on a problem, he became unaware of his surroundings. The story is told that he was in his bathtub when he discovered the principle of buoyancy. So great was his excitement that he leaped from his tub and ran through the streets naked shouting: "Eureka! Eureka!" (I have found it! I have found it!)

What holds up the moon?

Sir Isaac Newton
1642–1727

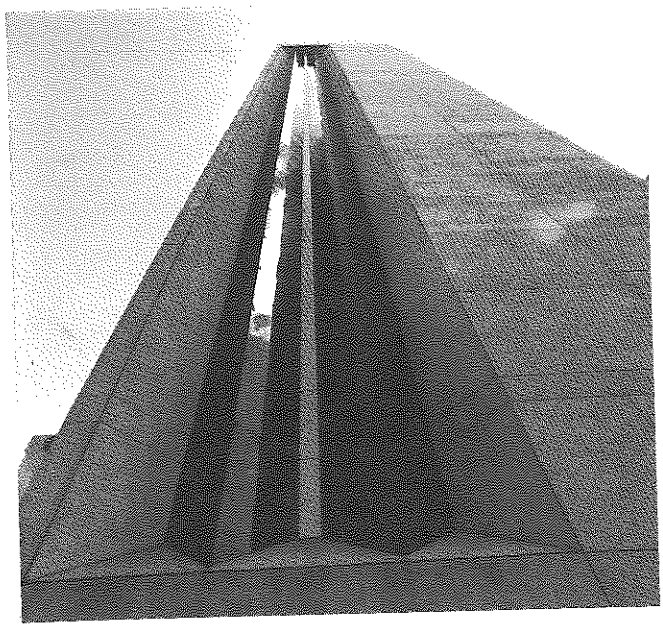
Before Isaac Newton, no one understood the idea of gravity. No one knew why the moon travels in an orbit instead of hurtling off into space or crashing to the Earth.

By the time Newton was 25 years old, he had formulated the law of gravitation and cracked the problem — a problem that had baffled scientists from the beginning of time.

Is there a way to send messages around the world instantly?

Carl Friedrich Gauss
1777–1855

Some consider Carl Friedrich Gauss to be the greatest mathematician of all time. In addition to his computer-like skill in performing mental calculations, he had an almost superhuman ability to solve problems. Though his achievements were mainly in pure mathematics, he is also known for his invention of the telegraph. This invention was a giant step forward in communications, and led the way to the development of the telephone and the radio.

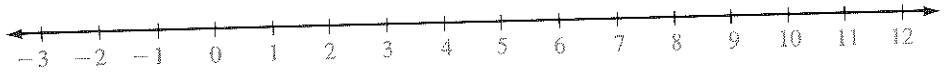


1-2 ADDING INTEGERS

An elevator in an apartment block travels from the top floor (the 12th) to the first floor. Then it continues below ground level to the four parking levels beneath the building.

P3	P2	P1	P	1	2	3	4	5	6	7	8	9	10	11	12

Each floor can be considered as an integer on a number line.



- The elevator is on the 8th floor. It goes up 2 floors. Each upward movement can be represented by a positive integer.

  $$8 + (+2) = 10$$

  The elevator is then at the 10th floor.
- The elevator moves down 4 floors. Each downward movement can be represented by a negative integer.

  $$10 + (-4) = 6$$

  The elevator is then at the 6th floor.

The addition of integers can be shown by moves on a number line. Start at the first integer. Move to the right for positive integers. Move to the left for negative integers.

Example 1. Simplify.

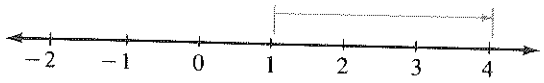
a) $(+1) + (+3)$ b) $(-1) + (-3)$

c) $(+1) + (-3)$ d) $(-1) + (+3)$

Solution. a) $(+1) + (+3)$

Start at 1. Move 3 to the right.

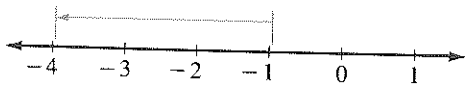
$(+1) + (+3) = 4$



b) $(-1) + (-3)$

Start at -1. Move 3 to the left.

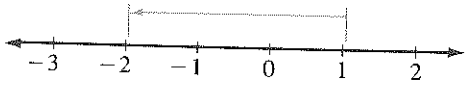
$(-1) + (-3) = -4$



c) $(+1) + (-3)$

Start at 1. Move 3 to the left.

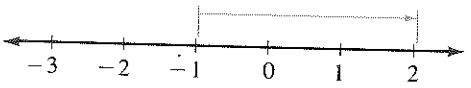
$(+1) + (-3) = -2$



d) $(-1) + (+3)$

Start at -1. Move 3 to the right.

$(-1) + (+3) = 2$



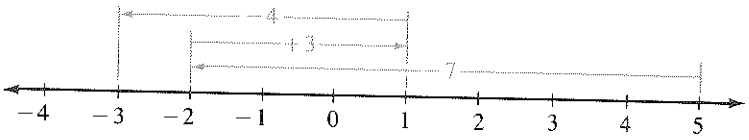
Example 2. Simplify. $(+5) + (-7) + (+3) + (-4)$

Solution. Start at $+5$.

Move 7 to the left to -2.

Move 3 to the right to 1.

Move 4 to the left to -3.



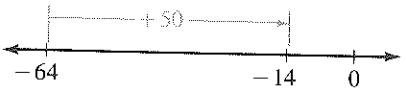
$(+5) + (-7) + (+3) + (-4) = -3$

To add several integers, it is easier to add the integers with the same sign first and then use the number line to obtain the final sum.

Example 3. Simplify. $(-25) + (+16) + (-11) + (-28) + (+34)$

Solution.

$$\begin{aligned} &(-25) + (+16) + (-11) + (-28) + (+34) \\ &= (-25) + (-11) + (-28) + (+16) + (+34) \\ &= (-64) + (+50) \\ &= -14 \end{aligned}$$



Positive integers are usually written without the positive sign. For example, the integer sum in *Example 3* could be written as $(-25) + 16 + (-11) + (-28) + 34$.

The number line is a useful device for showing addition of integers. However, after a little practice you should be able to add integers without its help.

EXERCISES 1-2

(A)

1. Simplify.
   a) $(-6) + (+2)$ b) $(+8) + (-5)$ c) $(+5) + (-8)$ d) $(-3) + (+8)$
   e) $(+3) + (+2)$ f) $(-6) + (-4)$ g) $(+5) + (+3)$ h) $(+7) + (-5)$
   i) $(-4) + (+6)$ j) $(+12) + (-3)$ k) $(-5) + (+11)$ l) $(+9) + (-9)$

2. Simplify.
   a) $(+3) + (-6)$ b) $(-9) + (-6)$ c) $(+4) + (-5)$ d) $(+7) + (-3)$
   e) $(+9) + (+5)$ f) $(-3) + (-6)$ g) $(-4) + (-8)$ h) $(-5) + (-7)$
   i) $(-9) + (-7)$ j) $(+3) + (-12)$ k) $(+14) + (-8)$ l) $(-16) + (+7)$

3. Simplify.
   a) $(-3) + (-4) + (+8)$ b) $(+2) + (-5) + (-7)$
   c) $(-2) + (-5) + (-9)$ d) $(+7) + (-8) + (+2)$
   e) $(-9) + (+3) + (+2)$ f) $(+4) + (-6) + (-2)$
   g) $(-8) + (-5) + (+7)$ h) $(-9) + (+7) + (-10)$

(B)

4. Simplify.
   a) $(-8) + (-7) + (+14) + (+1)$ b) $(-13) + (+2) + (+19) + (-7)$
   c) $(-10) + (+7) + (-12) + (+6)$ d) $(+14) + (-3) + (-9) + (-11)$
   e) $(+16) + (+12) + (-11) + (-15)$ f) $(+32) + (+43) + (-29) + (-11)$

5. Simplify.
   a) $5 + (-3) + 7$ b) $(-5) + (-4) + 6$
   c) $2 + (-9) + 4$ d) $(-8) + 1 + (-2)$
   e) $(-13) + 27 + (-11)$ f) $37 + (-21) + (-52)$
   g) $18 + 39 + (-71)$ h) $(-87) + 78 + (-13)$
   i) $(-21) + (-29) + 50$ j) $91 + (-27) + 19 + (-72)$

6. a) Simplify.
   i) $(+4) + (-4)$ ii) $(-7) + (+7)$ iii) $(-36) + (+36)$
   iv) $(+81) + (-81)$ v) $(-23) + (+23)$ vi) $(-57) + (+57)$
   b) What can you conclude about the sum of an integer and its opposite?

7. Copy each chart. Add horizontally and vertically. Find the sum of the integers in each chart in two different ways.

a)

+4	−7	+6	
−3	+10	−9	
+8	−3	−11	

b)

−7	+3	−8	
−6	+5	+2	
+9	−4	+10	

8. An elevator is at the 14th floor. It goes down 8 floors, then down 5 more floors, then up 4 floors, then down 1 floor. At which floor is the elevator now?

9. The chart shows the normal body temperature and the range of body temperatures that certain animals have survived in.

Animal	Normal Body Temperature	Temperature Range in °C	
		Minimum	Maximum
Catfish	+20°C	+6	+34
Crocodile	+26°C	+23	+29
Garter Snake	+22°C	+4	+39
Horned Lizard	+35°C	+25	+45
Human	+37°C	+16	+44

a) Which animal survived the greatest temperature range?
b) How much did its normal temperature increase before the animal reached the maximum of its range?
c) How much did its normal temperature decrease before the animal reached the minimum of its range?

10. Write an addition statement for each statement.
a) A football team is on the 15-yard line. It gains 15 yards, and on the next play it is penalized 5 yards.
b) Deposit $85 in an account. Write cheques for $29 and $37. Deposit a further $52, and write a cheque for $66.
c) A man's mass was 80 kg. He went on a diet and lost 7 kg. While on vacation he gained 5 kg.

11. Copy and complete the chart. Add +3 when moving to the right. Add +2 when moving up.

↑
Add +2

−11				

Add +3 →

What pattern do you notice in the diagonals on the chart?

12. Write the integer represented by each square.
a) $7 + \square = 3$
b) $2 + \square = -5$
c) $(-6) + \square = -2$
d) $1 + \square = 4$
e) $-7 = (-7) + \square$
f) $-11 = (-5) + \square$

13. Copy and complete the chart. Each square should contain an integer such that the rule beneath the chart is obeyed.

			+7				

Add −3 →

14. Find the sum of the integers in the chart in *Exercise 13*.

15. Copy each chart. Insert integers so that their horizontal and vertical sums have the same total.

a)

+8		+5
	−1	
+2		

b)

	−3	−4
+6		
	−2	

c)

	+5	−7
	−3	−9

16. Copy the chart. Find the integer that should be added to each square. Write the rule beneath the chart, then complete the chart.

		−9			6		

Add ? →

17. Copy the chart. Find the integer that should be added in each direction. Complete the chart.

			−3	
	3			9

↑
Add ?

Add ? →

INVESTIGATE

Some integers can be written as the sum of consecutive integers. For example,

$+7 = (+3) + (+4)$ and
$+7 = (-2) + (-1) + (0) + (+1) + (+2) + (+3) + (+4)$
$-21 = (-6) + (-7) + (-8)$ and
$-21 = (-1) + (-2) + (-3) + (-4) + (-5) + (-6)$

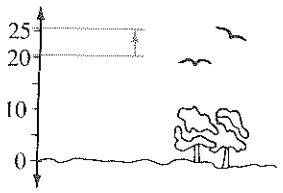
a) Write each integer as the sum of consecutive integers.
i) −11 ii) 18 iii) −14 iv) −17 v) 21 vi) 20

b) List the different ways that these integers can be written as the sum of consecutive integers.
i) 15 ii) −25 iii) 30 iv) −35

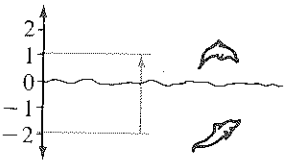
c) Find the five negative integers greater than −40 that *cannot* be written as the sum of consecutive integers.

1-3 SUBTRACTING INTEGERS

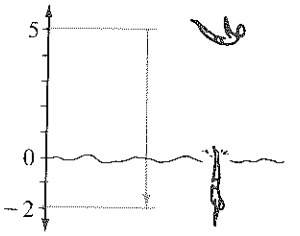
Consider these examples of changes in height.



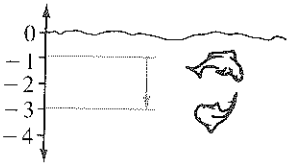
- A hawk at 20 m rises to a height of 25 m.
  Change in height = final height − initial height
  = 25 − 20
  = 5
  The hawk rose 5 m.



- A dolphin at a depth of 2 m leaps to a height of 1 m above the water surface.
  Change in height = final height − initial height
  = 1 − (−2)
  = 3
  The dolphin rose 3 m.



- A diver 5 m above a pool dives to a depth of 2 m.
  Change in height = final height − initial height
  = (−2) − (+5)
  = −7
  The diver dropped 7 m.



- A trout at a depth of 1 m swims to a depth of 3 m.
  Change in height = final height − initial height
  = (−3) − (−1)
  = −2
  The trout dropped 2 m.

Each example on the previous page illustrates the subtraction of one integer from another. The same results could have been obtained if each expression were written as the sum of integers.

We saw that $25 - 20 = 5$; we know that $25 + (-20) = 5$
Therefore, $25 - 20 = 25 + (-20)$

Similarly,
$1 - (-2) = 3$ but $1 + (+2) = 3$ so $1 - (-2) = 1 + (+2)$
$(-2) - (+5) = -7$ but $(-2) + (-5) = -7$ so
$(-2) - (+5) = (-2) + (-5)$
$(-3) - (-1) = -2$ but $(-3) + (+1) = -2$ so
$(-3) - (-1) = (-3) + (+1)$

This pattern suggests that adding the opposite of an integer gives the same result as subtracting the integer.

> To subtract an integer, add its opposite.

Example 1. Simplify.
a) $(+6) - (-2)$ b) $(-7) - (+3)$ c) $7 - 9$

Solution.
a) $(+6) - (-2) = (+6) + (+2)$
$= 8$
b) $(-7) - (+3) = (-7) + (-3)$
$= -10$
c) $7 - 9 = -2$

Frequently, expressions involving integers are written without the brackets.

Example 2. Simplify.
a) $3 - 5 + 6 - 7$ b) $5 - 9 + 2 - 8 - 3 + 6$

Solution.
a) $3 - 5 + 6 - 7$ can be written as $(+3) + (-5) + (+6) + (-7)$.
Rearranging, $(+3) + (+6) + (-5) + (-7) = (+9) + (-12)$
$= -3$

b) $5 - 9 + 2 - 8 - 3 + 6$ can be written as
$(+5) + (-9) + (+2) + (-8) + (-3) + (+6)$
$= (+5) + (+2) + (+6) + (-9) + (-8) + (-3)$
$= (+13) + (-20)$
$= (-7)$

With practice, integer expressions can be simplified directly.

Example 3. Simplify. $-3 + 6 - 5 + 4 - 3 - 7$

Solution. $-3 + 6 - 5 + 4 - 3 - 7 = -3 - 5 - 3 - 7 + 6 + 4$
$= -18 + 10$
$= -8$

The problem posed at the beginning of the chapter can now be solved.

Example 4. It is 8°C in Valleville (altitude 350 m). The temperature decreases 6.5°C for every 1000 m increase in altitude. If there is precipitation in Ste. Agathe (altitude 2350 m), will it be rain or snow?

Solution. The difference in altitude is 2000 m.

Therefore, the temperature in Ste. Agathe is 2(6.5°C) or 13°C lower than the temperature in Valleville.

The temperature in Ste. Agathe is 8°C − 13°C = −5°C.
Any precipitation in Ste. Agathe will probably be snow.

EXERCISES 1-3

(A)

1. The temperature was −6°C. It is now 4°C. How much did the temperature change?
2. A balloon was 600 m above the ground. It is now 250 m above the ground. What is its change in altitude?
3. State the temperature change.
   a) from −12°C to 8°C b) from −17°C to −5°C c) from 27°C to −27°C
   d) from −6°C to 18°C e) from 7°C to −1°C f) from −1°C to −11°C
4. State the altitude change.
   a) from 3170 m to 525 m b) from −265 m to 425 m
   c) from −350 m to −580 m d) from −900 m to −250 m
5. Simplify.
   a) (+4) − (+6) b) (+7) − (+2) c) (−8) − (+4)
   d) (+3) − (+1) e) (+6) − (−1) f) (−4) − (−3)
   g) (−6) − (+3) h) (−2) − (+5) i) (+2) − (+1)
   j) (+1) − (−5) k) (−4) − (−2) l) (+2) − (−4)
   m) (−8) − (−9) n) 0 − (−2) o) 0 − (+3)

(B)

6. Simplify.
   a) (+45) − (−15) b) (−23) − (−13) c) (−14) − (+66)
   d) (−145) − (−35) e) (+68) − (+98) f) (−72) − (−42)
   g) (+75) − (−15) h) (−187) − (−42) i) (−27) − (+43)
   j) (+26) − (+31) k) (−18) − (+42) l) (+37) − (+34)
   m) (−29) − (−18) n) (+54) − (−17) o) (−99) − (+91)
7. Simplify.
   a) (−9) − (+2) + (−3) − (+5) b) (+8) + (+4) − (+6) − (−3)
   c) (+8) − (+3) − (−4) − (−7) d) (−6) − (−3) − (−7) + (−8)
   e) (−10) + (+6) − (+5) − (+7) f) (+1) − (−6) − (+3) − (−4)
   g) (−7) + (−3) − (−5) − (+8) h) (−2) − (+6) − (−4) − (+7)

8. Simplify.
   a) $(-3) + (-8) - (+9) - (-7)$
   b) $(+8) - (+13) + (-6) + (+3)$
   c) $(-7) - (-11) + (+3) - (+6)$
   d) $(+15) - (-3) - (+11) + (-8)$
   e) $(+5) - (+10) - (+2) - (-12)$
   f) $(+4) + (-5) + (-11) - (-1)$
   g) $(-9) + (+11) + (-14) - (+5)$
   h) $(-16) - (+8) + (-4) - (-7)$

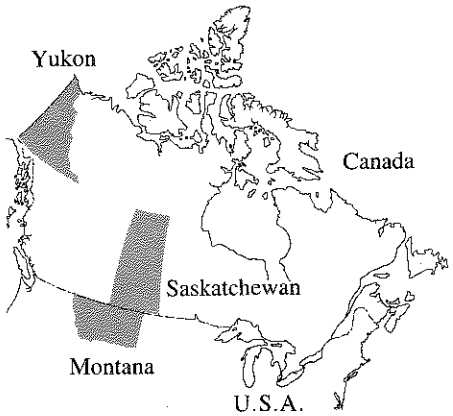
9. Simplify.
   a) $5 - 2 - 8 + 3 - 1$
   b) $-4 + 6 + 2 - 7 - 3$
   c) $-1 - 5 + 9 - 2 + 3$
   d) $7 - 2 - 6 + 4 - 8 + 2$
   e) $-3 - 9 + 1 - 5 + 7 - 4$
   f) $17 - 14 - 2 + 13 - 9 - 10$
   g) $-7 - 2 + 3 - 6 + 8 - 4$
   h) $14 - 11 - 8 + 12 - 2 - 14$
   i) $36 - 27 + 41 - 81 + 16$
   j) $-40 - 22 + 31 + 17 - 54$

10. Simplify.
   a) $(-4 + 6) - (3 - 7)$
   b) $(8 - 5) - (-4 + 6)$
   c) $(-8 + 3) + (-2 - 5)$
   d) $(-9 + 4) + (-3 - 7)$
   e) $(7 - 4) + (8 - 3)$
   f) $(-6 + 2) - (7 - 9)$
   g) $(8 - 6) + (9 - 4)$
   h) $(-10 + 4) - (8 - 12)$
   i) $(10 - 9) + (8 - 10)$
   j) $(-3 - 6) - (-1 - 8)$

11. Simplify.
   a) $(-3 + 7) + (6 - 4 + 10)$
   b) $(9 - 6 + 13) - (-4 + 5 - 11)$
   c) $(13 - 10 + 1) - (-2 - 8 + 1)$
   d) $(-15 + 18 + 4) + (12 - 3 - 13)$
   e) $(-2 - 16 + 10) + (17 + 9 - 27)$
   f) $(14 + 15 - 17) - (21 - 11 + 3)$
   g) $(18 + 21 - 32) + (-12 - 13 - 15)$

12. The lowest temperature ever recorded in Canada was $-63°C$ in the Yukon in 1947. The highest temperature was $45°C$ in Saskatchewan in 1937. What is the difference between these temperatures?

13. The greatest temperature change in North America in a single day was from $+7°C$ to $-49°C$ in Montana. What is the difference between these temperatures?

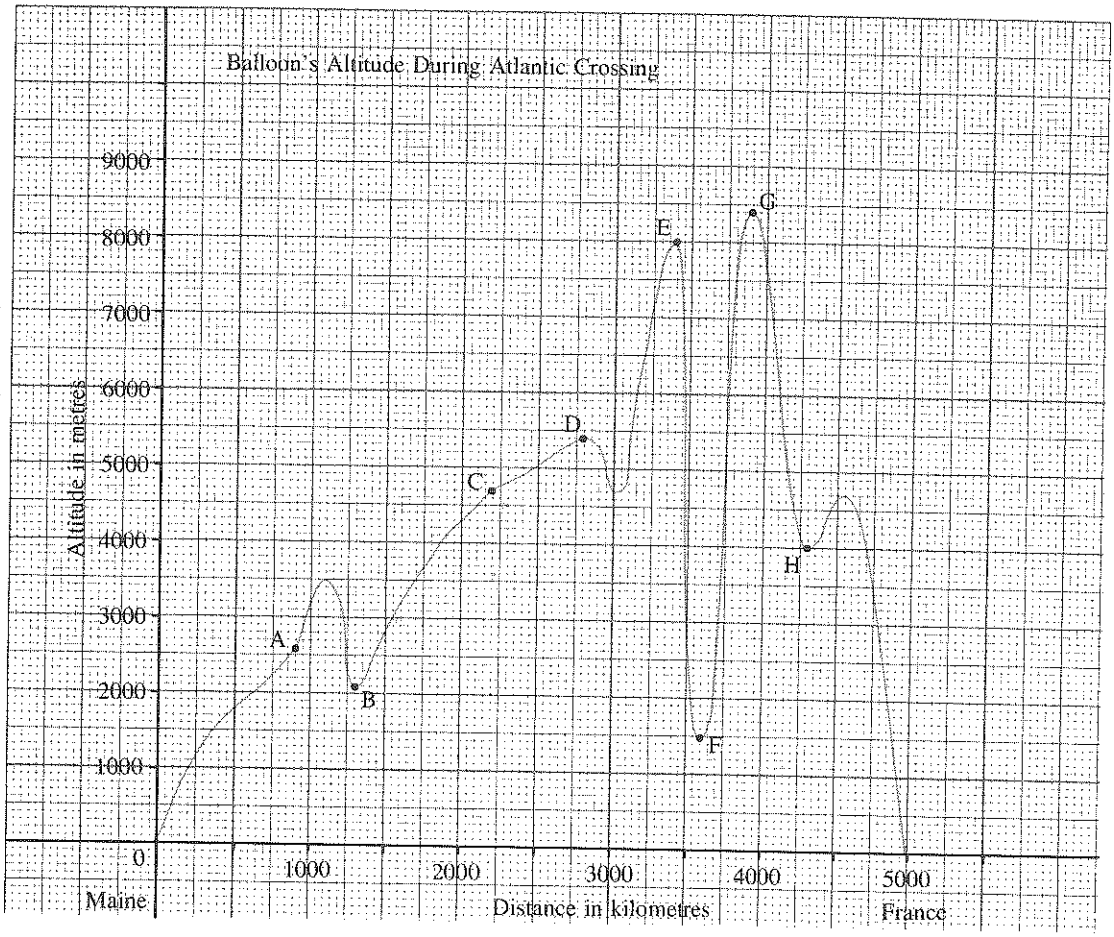


14. Write the integer represented by each square.
   a) $17 - (+19) = \square$
   b) $\square - (+6) = -4$
   c) $7 - (+3) = \square$
   d) $-8 - (-5) = \square$
   e) $11 - \square = 3$
   f) $\square - (+3) = -6$
   g) $-23 - \square = 23$
   h) $\square - (+5) = 3$
   i) $-21 - \square = -12$

15. Write the integer represented by each square.
   a) $\square - (+5) = 13$
   b) $\square - (+6) = 4$
   c) $7 - \square = 45$
   d) $-8 - \square = -3$
   e) $-11 - \square = 3$
   f) $\square - (+3) = 6$

16. The time difference between Toronto and Vancouver is 3 h.
    a) An airplane leaves Vancouver for Toronto at 08:00 and the flying time is 4 h 10 min. What time does it arrive in Toronto?
    b) On the return flight the airplane leaves Toronto at 07:30 and the flying time to Vancouver is 4 h 50 min. What time does it arrive in Vancouver?

17. In August 1978, three Americans made the first crossing of the Atlantic Ocean by balloon. The graph shows the altitude of the balloon along the flight path.



State the approximate change in altitude between each pair of points.

a) from A to B b) from B to C c) from A to C d) from E to F
e) from F to G f) from E to G g) from D to H h) from B to F

MATHEMATICS AROUND US

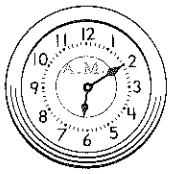
Times of the Day around the World

The time of day changes 1 h for every 15° difference in longitude. This means that when it is midday in London, England (0° longitude), it is midnight at the date line (180° longitude); a new day is just starting there. A place on the date line is 12 h ahead of London.

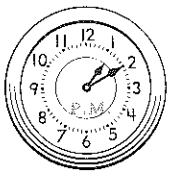
Find the date line and London, England on a globe.

Times are usually compared with the time in London.

London
England



Halifax
Nova Scotia



Moscow
U.S.S.R.

Standard time difference in hours between London (England) and other cities

Athens	+2	Halifax	−4	Peking	+8
Bangkok	+7	Jakarta	+7	Rome	+1
Bogota	−5	Jerusalem	+2	Santiago	−4
Brasilia	−3	Mexico City	−6	Washington	−5
Canberra	+10	Moscow	+3	Wellington	+12
Dublin	0	Ottawa	−5	Vancouver	−8

Halifax −4 means that Halifax is 4 h behind London.
Moscow +3 means that Moscow is 3 h ahead of London.
Therefore, Moscow is 3 h − (−4 h) or 7 h ahead of Halifax.

QUESTIONS

1. It is 08:00 in London. State the time in each city.
   a) Ottawa b) Mexico City c) Jerusalem d) Canberra
2. It is 22:00 in Jakarta. State the time in each city.
   a) Bangkok b) Bogota c) Wellington d) Halifax
3. It is 21:00 in Ottawa. State the time in each city.
   a) Vancouver b) Dublin c) Washington d) Peking
4. Team Canada is playing Moscow Selects and the game is being televised live via satellite.
   a) The game is in Moscow at 8 P.M. State the time that a viewer would be watching the game in each city.
      i) Halifax ii) Vancouver iii) Ottawa
   b) The game is in Toronto (the same time zone as Ottawa) at 8 P.M. State the time that a viewer in Moscow would be watching it.
5. The scenes of an earthquake in Chile are sent via television satellite from Santiago at 16:00. State the time that the transmission is received in these places.
   a) London b) Ottawa c) Bogota d) Mexico City
   e) Moscow f) Vancouver g) Brasilia h) Wellington

CALCULATOR POWER

If your calculator has a [+/−] key, it can be used to simplify expressions involving integers.

Example 1. Simplify. $(-5) - (-20)$

Solution. Key in: [5] [+/−] [−] [2] [0] [+/−] [=] to display 15

$(-5) - (-20) = 15$

If your calculator also has a memory, it can be used to simplify similar expressions.

Example 2. Simplify. $(-58) - (-39)$

Solution. Key in: [CM] [3] [9] [+/−] [M+] [5] [8] [+/−] [−] [RM] [=]

[CM]: ↑ clears memory

[M+]: ↑ adds −39 to memory

[RM]: ↑ recalls −39 from memory

to display −19

$(-58) - (-39) = -19$

1. Use the methods of these examples to check the answers to *Exercises 5* and *6* on page 13.
2. Modify the strategy of each example to check the answers to *Exercises 7* and *8* on pages 13 and 14.

1-4 MULTIPLYING INTEGERS

To multiply integers, we must define what we mean by the following products.

$(+4)(+5)$ $(+4)(-5)$ $(-4)(+5)$ $(-4)(-5)$

To help us define these products, we recall certain facts about multiplication of whole numbers.

Multiplication of whole numbers is defined as repeated addition. For example, 4×5 means $5 + 5 + 5 + 5$.

The multiplication table contains patterns such as this.

The first numbers decrease by 1;

$4 \times 5 = 20$
$3 \times 5 = 15$
$2 \times 5 = 10$
$1 \times 5 = 5$
$0 \times 5 = 0$

the products decrease by 5.

We would like integers to behave as much as possible like whole numbers. Therefore, we will extend these facts to apply to the multiplication of integers. For example, we can use repeated addition to define what we mean by $(+4)(+5)$ and $(+4)(-5)$.

$$(+4)(+5) \text{ means } 4(+5) = (+5) + (+5) + (+5) + (+5)$$
$$= +20$$

$$(+4)(-5) \text{ means } 4(-5) = (-5) + (-5) + (-5) + (-5)$$
$$= -20$$

To define what we mean by $(-4)(+5)$, we write the above number pattern using integers, and extend it to negative integers.

The first numbers decrease by 1;

$$\begin{aligned}
(+4) \times (+5) &= +20 \\
(+3) \times (+5) &= +15 \\
(+2) \times (+5) &= +10 \\
(+1) \times (+5) &= +5 \\
0 \times (+5) &= 0 \\
(-1) \times (+5) &= -5 \\
(-2) \times (+5) &= -10 \\
(-3) \times (+5) &= -15 \\
(-4) \times (+5) &= -20
\end{aligned}$$

the products decrease by 5.

We see that we should define $(-4)(+5)$ to be equal to -20.
To define what we mean by $(-4)(-5)$, we use another number pattern.

The first numbers decrease by 1;

$$\begin{aligned}
(+4) \times (-5) &= -20 \\
(+3) \times (-5) &= -15 \\
(+2) \times (-5) &= -10 \\
(+1) \times (-5) &= -5 \\
0 \times (-5) &= 0 \\
(-1) \times (-5) &= +5 \\
(-2) \times (-5) &= +10 \\
(-3) \times (-5) &= +15 \\
(-4) \times (-5) &= +20
\end{aligned}$$

the products increase by 5.

We see that we should define $(-4)(-5)$ to be equal to $+20$.

Summarizing the above results, we define the multiplication of two integers.

> The product of two integers with the same signs is positive.
> The product of two integers with different signs is negative.

These results can be illustrated in a multiplication table.

x	Positive	Negative
Positive	Positive	Negative
Negative	Negative	Positive

Example 1. Simplify.

a) $(-2)(+6)$ b) $(-7)(-8)$ c) $(+16)(-14)$

Solution. a) $(-2)(+6) = -12$

b) $(-7)(-8) = 56$

c) $(+16)(-14) = -224$

Example 2. Simplify. $(-3)(+2) - (-6)(-2)$

Solution. $(-3)(+2) - (-6)(-2) = (-6) - (+12)$

$= (-6) + (-12)$

$= -18$

EXERCISES 1-4

(A)

1. Simplify.

a) $(-5)(+6)$ b) $(+7)(-8)$ c) $(-7)(-9)$ d) $(+6)(+9)$

e) $(-12)(+5)$ f) $(-3)(-13)$ g) $(+8)(+9)$ h) $(-5)(+5)$

i) $(-5)(-5)$ j) $(+7)(-6)$ k) $(+3)(+4)$ l) $(-4)(-8)$

m) $(+6)(-7)$ n) $(+4)(-7)$ o) $(+9)(+3)$ p) $(0)(-7)$

2. Simplify.

a) $(+16)(-5)$ b) $(-18)(+3)$ c) $(-14)(-4)$ d) $(-17)(+9)$

e) $(-11)(+28)$ f) $(+19)(+11)$ g) $(+36)(+72)$ h) $(+47)(-16)$

i) $(-69)(-89)$ j) $(-74)(-18)$ k) $(+37)(-31)$ l) $(-44)(-22)$

3. Simplify.

a) $(-2)(+5)(-7)$ b) $(-3)(-4)(-2)$ c) $(+6)(-5)(+4)$

d) $(-1)(+3)(-3)$ e) $(+2)(-3)(-3)$ f) $(-2)(-2)(-3)$

g) $(+5)(-1)(-1)(-1)$ h) $(-1)(-2)(-3)(-4)$ i) $(-5)(+4)(+3)(-2)$

(B)

4. Simplify.

a) $(-2)(+3) + (-6)(-2)$ b) $(-4)(-3) + (-1)(-2)$

c) $(-2)(-6) - (+5)(-2)$ d) $(-3)(+7) - (+1)(+5)$

e) $(-2)(+8) - (-3)(-3)$ f) $(+4)(-7) + (-8)(+6)$

g) $(-3)(+9) + (-2)(+7)$ h) $(-7)(-9) - (-6)(-7)$

i) $(-2)(-2)(+1) + (-3)(-3)(-2)$ j) $(-5)(-2)(-2) - (+2)(-1)(-1)$

5. What must be true of two integers if their product is:

a) positive b) negative c) zero?

6. By comparing the answers in *Exercise 3*, what appears to be true for:

a) the product of an even number of negative numbers;

b) the product of an odd number of negative numbers?

(C)

7. Find the integer represented by each square.

a) $(+5) \times \square = -20$ b) $(-2) \times \square = 16$ c) $\square \times (+7) = -56$

d) $\square \times (-6) = 54$ e) $(-8) \times \square = -72$ f) $\square \times (-12) = -96$

g) $-32 = (+8) \times \square$ h) $48 = (-16) \times \square$ i) $-39 = \square \times (-13)$

j) $-15 \times \square = -75$ k) $\square \times 18 = -72$ l) $98 = (-7) \times \square$

8. Find the integer represented by each square.
   a) $(-2)(+3) \times \square = -24$
   b) $(+4) \times \square \times (+2) = -32$
   c) $\square \times (-1)(-6) = 42$
   d) $(-3) \times \square \times (+5) = -45$
   e) $96 = (-2)(+6) \times \square$
   f) $-36 = (-6)(-6) \times \square$
   g) $56 = \square \times (-2)(-2)$
   h) $-81 = (-9) \times \square \times (+3)$

Models for Multiplying Integers

When mathematicians began working with integers, they had difficulties seeing how the product of two negative numbers can be positive. For example, if you have a negative number, such as -3, and then do something "negative" to it, such as multiplying it by -2, where does the "positive" answer $+6$ come from?

We can gain some insight into this situation by considering models for multiplying integers.

Gains and Losses

Suppose Mr. Ziegler spends \$3 each week on lotteries. We represent this using the integer -3. Suppose also that he never wins anything.

2 weeks from now, he will have \$6 less than he has now.

$(+2)(-3) = -6$

2 weeks ago, he had \$6 more than he has now.

$(-2)(-3) = +6$

Travelling on a Number Line

Suppose the integers on a number line are 1 cm apart. A toy car travels to the left along the number line at 3 cm/s. We represent this using the integer -3.

2 seconds from now, the car will be 6 cm to the left of its present position.

$(+2)(-3) = -6$

2 seconds ago, the car was 6 cm to the right of its present position.

$(-2)(-3) = +6$

Good People and Bad People

In a certain town, all people are identified as being good $(+)$ or bad $(-)$.

1. For each of the following moves, determine whether it would be good for the town, or bad for the town.
   a) A good person moves to the town.
   b) A bad person moves to the town.
   c) A good person leaves the town.
   d) A bad person leaves the town.
2. Write an expression involving integers for each move in *Question 1*.
3. Suppose there are two families each containing three bad people. Write an expression involving multiplication of integers to represent the effect on the town if these two families leave the town.

MATHEMATICS AROUND US

Glaciers on the Move

Glaciers are large masses of ice that move very slowly down a mountain or along a valley. They are formed when the winter snowfall exceeds the summer melting.

The Columbia Glacier (below) in Alaska is retreating because the ice is melting at the bottom faster than it comes down from above. In recent years, it has been retreating at about 2 m per day.

But the nearby Hubbard Glacier, whose source is in Canada, is now advancing, because the ice is coming down the valley faster than it melts at the bottom. It is advancing at about 3 m per day.

QUESTIONS

1. Write an integer to represent the daily change in the length of each glacier.
2. Write an expression involving multiplication of integers to represent the difference between the length of each glacier now and:
   a) 5 days from now
   b) 50 days from now
   c) 5 days ago
   d) 50 days ago.
3. Explain how the expressions in *Question 2* illustrate the rules for multiplying two integers.

1-5 DIVIDING INTEGERS

Division is the inverse of multiplication.
Since $7 \times 4 = 28$, then $28 \div 4 = 7$ and $28 \div 7 = 4$.
The same is true for integers.
Since $(+5)(-4) = -20$, then $(-20) \div (+5) = -4$ and
$(-20) \div (-4) = +5$

Example 1. Simplify.
a) $(+6) \div (-3)$ b) $(-24) \div (-6)$

Solution. a) $(+6) \div (-3)$
Since $(-3)(-2) = 6$, then $(+6) \div (-3) = -2$
b) $(-24) \div (-6)$
Since $(-6)(+4) = -24$, then $(-24) \div (-6) = 4$

Example 2. Simplify.
a) $(+15) \div (+3)$ b) $(-20) \div (-2)$
c) $(+24) \div (-8)$ d) $(-33) \div (+3)$

Solution. a) $(+15) \div (+3) = 5$ b) $(-20) \div (-2) = 10$
c) $(+24) \div (-8) = -3$ d) $(-33) \div (+3) = -11$

The above examples suggest the following rules.

> The quotient of two integers with *like* signs is *positive*.
> The quotient of two integers with *unlike* signs is *negative*.

Example 3. Simplify.
a) $\dfrac{63}{-9}$ b) $\dfrac{-42}{-7}$ c) $\dfrac{(-8)(-9)}{3(-4)}$

Solution. a) $\dfrac{63}{-9} = -7$ b) $\dfrac{-42}{-7} = 6$ c) $\dfrac{(-8)(-9)}{3(-4)} = \dfrac{+72}{-12} = -6$

Example 4. Simplify. $\dfrac{(-24)}{4} - \dfrac{10}{(-2)}$

Solution. Perform the divisions before subtracting.

$$\frac{(-24)}{4} - \frac{10}{(-2)} = (-6) - (-5)$$
$$= -6 + 5$$
$$= -1$$

EXERCISES 1-5

A

1. Simplify.
   a) $(-48) \div (+4)$ b) $(-36) \div (-4)$ c) $(+32) \div (-8)$
   d) $(-18) \div (+3)$ e) $(-60) \div (-12)$ f) $(-40) \div (-5)$

2. Simplify.
   a) $\frac{-36}{4}$ b) $\frac{46}{-2}$ c) $\frac{-18}{-9}$ d) $\frac{-85}{5}$ e) $\frac{-49}{-7}$
   f) $\frac{81}{-9}$ g) $\frac{-76}{-19}$ h) $\frac{-121}{11}$ i) $\frac{132}{-12}$ j) $\frac{91}{13}$

3. Simplify.
   a) $\frac{(-4)(10)}{-8}$ b) $\frac{(6)(-15)}{-5}$ c) $\frac{(-10)(12)}{(5)(-3)}$ d) $\frac{(-15)(-20)}{(-10)(3)}$
   e) $\frac{(-50)(9)}{(15)(6)}$ f) $\frac{(14)(-16)}{(-8)(-7)}$ g) $\frac{(-5)(9)(-24)}{(-3)(4)}$ h) $\frac{(-6)(-8)}{(-2)(-1)(-3)}$

B

4. Simplify.
   a) $\frac{(-30)}{5} + \frac{15}{(-3)}$ b) $\frac{(-20)}{10} + \frac{8}{(-2)}$ c) $\frac{(-9)}{(-3)} - \frac{12}{4}$
   d) $\frac{14}{(-2)} - \frac{(-16)}{8}$ e) $\frac{(-36)}{4} + \frac{(-56)}{(-8)}$ f) $\frac{(-42)}{7} - \frac{54}{(-6)}$
   g) $\frac{(-63)}{(-7)} - \frac{(-56)}{(-8)}$ h) $\frac{(-81)}{(-9)} + \frac{(-72)}{(-8)}$ i) $\frac{35}{7} + \frac{48}{(-6)}$

5. Canada sells products and services to other countries and buys products and services from them. The Canadian balance of payments is a measure of all yearly business transactions between Canada and the rest of the world.
   a) What does a negative balance of payments indicate?
   b) What was the average monthly balance of payments for each year?

Year	Balance of Payments $ Millions
1979	4 319
1980	− 174
1981	10 468
1982	1 106
1983	5 366
1984	5 307
1985	1 901

6. The temperature is falling at the rate of 3°C/h. Find how long it takes to fall through each temperature range.
   a) −4°C to −10°C b) −2°C to −11°C c) 4°C to −11°C

7. Find the integer represented by each square.
   a) $40 \div \square = -10$ b) $\square \div (-5) = -7$ c) $\frac{\square}{(-3)} = 6$
   d) $\frac{(-20)}{\square} = -4$ e) $-3 = \frac{\square}{(-2)}$ f) $(-65) \div \square = -13$

8. Find the integer represented by each square.
   a) $3132 \div \square = -87$ b) $1972 \div \square = -29$ c) $\square \div (-47) = 63$
   d) $(-4676) \div \square = -167$ e) $-33 = \square \div (-44)$ f) $48 = (-1008) \div \square$

9. If $x > 0$, $y > 0$, and $z < 0$, decide which expressions are always positive or always negative.
   a) $\frac{x}{y}$ b) $\frac{x + y}{z}$ c) $\frac{xy}{z}$ d) $\frac{y - z}{x}$
   e) $\frac{z - x}{y}$ f) $\frac{z}{x + y}$ g) $\frac{z - x}{z}$ h) $\frac{y - z}{x + y}$

CALCULATOR POWER

Finding Quotient and Remainder

277 players join a football league.
16 players are needed for each team.
How many teams can be formed?
How many players are left over?

Key in: [2] [7] [7] [÷] [1] [6] [=] to display 17.3125
Hence, 17 teams can be formed. The remainder is 0.3125.
To display this remainder, subtract 17 from the number in the display.
Then, to display the remainder as a whole number, multiply by 16.
Key in: [−] [1] [7] [=] [x] [1] [6] [=] to display 5
Hence, 5 players are left over.

COMPUTER POWER

Finding Quotient and Remainder

The program below can be used to find the quotient and the remainder for any given division problem.

```
100 REM *** QUOTIENTS AND REMAINDERS ***
110 INPUT "WHAT IS THE DIVIDEND? ";A
120 INPUT "WHAT IS THE DIVISOR? ";B
130 Q=INT(A/B)
140 PRINT:PRINT "THE QUOTIENT IS ";Q
150 R=INT((A/B-Q)*B+0.1)
160 PRINT "THE REMAINDER IS ";R
170 END
```

To solve the problem on the facing page, enter the program in a computer. Type RUN and press RETURN. The computer will ask for the dividend. Enter the number 277 and press RETURN. Then the computer will ask for the divisor. Enter 16 and press RETURN. Then the computer will display this result.

```
THE QUOTIENT IS 17
THE REMAINDER IS 5
```

This indicates that 17 teams can be formed, and there will be 5 players left over.

QUESTIONS

1. Use a calculator or a computer to find each quotient and remainder.
   a) $\frac{2079}{76}$ b) $\frac{68\ 075}{82}$ c) $\frac{93\ 726}{158}$ d) $\frac{27\ 938}{356}$ e) $\frac{80\ 079}{283}$
2. Marilyn works in a factory where she makes fruit tarts. On an 8 h shift, her team produced 136 240 tarts. These tarts are packaged in boxes of 12 tarts. The boxes are packed in cases of 24 boxes.
   a) How many cases of tarts were produced?
   b) How many boxes were left over?
   c) How many tarts were left over?
3. Use the computer program to investigate the results of using negative integers.
   a) Use a dividend of -31 and a divisor of 5.
   b) Use a dividend of 38 and a divisor of -6.
   c) Use a dividend of -33 and a divisor of -7.
   Explain the results.

PROBLEM SOLVING

Guess and Check

A theatre charges for admission, as illustrated. A family of 10 paid a total of $58.00. How many children and senior citizens are in the family?

AJAX THEATRE	
TICKET PRICES	
CHILDREN	$4.00
ADULTS	$10.00
SENIORS	$6.00

Understand the problem

- What is the cost of a ticket for a child? an adult? a senior citizen?
- How many people are in the family?
- What was the total cost of admission?
- What are we asked to find?

Think of a strategy

- Try a guess and check strategy.

Carry out the strategy

- List a possible family of 10, for example, 4 children, 2 adults, 4 seniors.
- Calculate the admission for that family. The admission is $60, which is too high by $2.
- To decrease the admission price by $2, exchange 1 senior citizen for a child. There are 5 children, 3 senior citizens, and 2 adults in the family.

Children	Adults	Seniors	Total
4	2	4	$60

$$4(4) + 2(10) + 4(6) = 60$$

Children	Adults	Seniors	Total
5	2	3	$58

Look back

- Is there a total number of 10 people in the family?
- Is the total cost of admission $58.00?
- Is this the only possible solution?

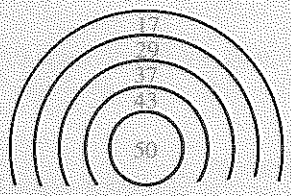
Solve each problem

1. Ms. McCall cashed a cheque for $63.00. She received 6 bills, none of which was a $1 bill. What were the denominations of the bills she received?

2. An edition of the Daily Planet newspaper has its pages numbered starting at 1. The page numbers have a total of 121 digits. How many pages are in the newspaper?

3. Pietro had 20 problems for homework. His mother paid him 25¢ for each one he solved and deducted 35¢ for each one he couldn't solve. Pietro earned 80¢. How many problems was he able to solve?

4. Six darts were thrown into the dart board shown in the diagram. The total score was 211. Inside which rings did the six darts land? Is there more than one correct answer?



5. A gas station sells an average of 4000 L of gas per day when the price is 60¢ per litre. For each 1¢ increase in the price, the number of litres sold per day is reduced by 50 L. What price gives the greatest possible weekly revenue?

6. Erin collected $6.05 in nickels and dimes. She has 8 more dimes than nickels. How many coins does she have?

7. Minibuses seating 10, 12 or 15 passengers are used to transport hotel guests from the airport to the hotel. A hotel has 5 minibuses of each size available when a party of 120 people arrives. In how many different ways can these guests be transported using some of these minibuses if each bus used must be filled?

1-6 ORDER OF OPERATIONS WITH INTEGERS

Now that we have reviewed the four basic operations with integers, let us review the order in which they must be performed.

- Operations within grouping symbols are performed first, starting with the innermost and working outward.
- Multiplication and division are performed in order from left to right.
- Lastly, addition and subtraction are performed in order from left to right.

Example 1. Simplify.
a) $(-6) + (-3)(5)$ b) $(-16) \div (-4) + (-8)$

Solution. a) Perform the multiplication first.

$$\begin{aligned}(-6) + (-3)(5) &= (-6) + (-15)\\ &= -21\end{aligned}$$

b) Perform the division first.

$$\begin{aligned}(-16) \div (-4) + (-8) &= (+4) + (-8)\\ &= -4\end{aligned}$$

Example 2. Simplify.
a) $[(-7) + (-3)] \div (-5) + (-6)$
b) $-3[2 - 5(2 - 8)(-1 + 3)]$

Solution. a) Perform the addition within the brackets first.

$$\begin{aligned}[(-7) + (-3)] \div (-5) + (-6) &= (-10) \div (-5) + (-6)\\ &= (+2) + (-6)\\ &= -4\end{aligned}$$

b) Perform the operations within the innermost brackets first.

$$\begin{aligned}-3[2 - 5(2 - 8)(-1 + 3)] &= -3[2 - 5(-6)(2)]\\ &= -3[2 - 5(-12)]\\ &= -3[2 + 60]\\ &= -3(62)\\ &= -186\end{aligned}$$

A fraction bar is a grouping symbol, like brackets. It indicates that the numerator and the denominator must be simplified before the other operations are performed.

Example 3. Simplify. $\dfrac{4(-3) + 7(-9 + 5)}{(2 - 3)(2 + 3)}$

Solution. Perform the operations in the numerator and in the denominator first.

$$\begin{aligned}\frac{4(-3) + 7(-9 + 5)}{(2 - 3)(2 + 3)} &= \frac{(-12) + 7(-4)}{(-1)(5)}\\ &= \frac{-12 - 28}{-5}\\ &= \frac{-40}{-5}\\ &= 8\end{aligned}$$

EXERCISES 1-6

(A)

1. Simplify.
   a) $(+4) + (-3)(-2)$
   b) $(-5)(-4) \div (-2)$
   c) $(-18) \div (+3) + (-11)$
   d) $(-6) + (-12) \div (+4)$
   e) $(-6)(0) \div (-4)$
   f) $(+11) - (+9) \div (-3)$
   g) $(-21) \div (+7)(-5)$
   h) $(-1)(-8) \div (+2)$
   i) $(-7) - (-5)(-3)$
   j) $(-3) \times (-4) \div (-6)$

(B)

2. Simplify.
   a) $[(-10) + (-2)] \div (+6) - (+4)$
   b) $(-8) + (+10)[(-12) - (-7)]$
   c) $[(-20) + (+4)] \times [(-10) - (-6)]$
   d) $(-16) \div (-2) + (-8)(+3)$

3. Use brackets with the expression $3 + 5 \times 4 - 2$ so that it simplifies to each value.
   a) 16 b) 21 c) 30 d) 13

4. Simplify.
   a) $3(-2 + 6) - 5(4 - 1)$
   b) $(-5)(-4) + (-6)(3)$
   c) $-2(-4 + 3) + 3(-1 - 5)$
   d) $(-3)(-1)(5) - (-2)(-4)(-1)$
   e) $5(2 - 6)(2 - 6)$
   f) $7(7 - 2) - 5(-3 - 8) + 19$

5. Simplify.
   a) $\dfrac{(-15)}{3} - \dfrac{(-10)}{5}$
   b) $\dfrac{(-7) + 3(-1 + 4)}{-2}$
   c) $\dfrac{4(-5 + 3) - 2(-1 + 5)}{-6 + 2}$
   d) $\dfrac{35 - 81}{27 - 4} - \dfrac{(-4)(3 - 10)}{8 - 15}$

6. Use brackets with each expression so that it simplifies to the answer given.
   a) $1 + 3 \times 5 + 7$; answer 27
   b) $4 + 4 + 4 \times 4$; answer 48
   c) $2 \times 4 + 6 + 8$; answer 28
   d) $5 + 5 \times 5 + 5$; answer 100
   e) $48 \div 8 - 2 \times 3$; answer 24

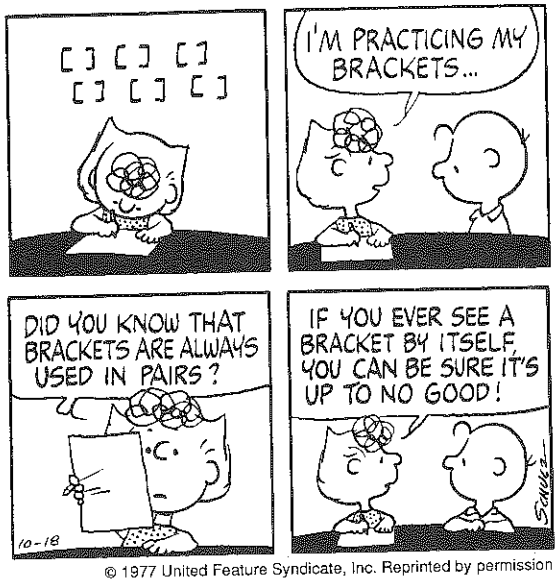
7. Simplify.
   a) $(12 + 8) \div (2 - 6)$
   b) $(-3 + 4)(8 - 10) - (7 - 9)(4 - 1)$
   c) $(6 - 2 + 3)(-7 + 5 - 1)$
   d) $(4 - 9)(2 + 3) + (8 - 2)(-3 + 2)$
   e) $\dfrac{(-5 + 2)(-4 - 6)}{3 - 9}$
   f) $\dfrac{5(-3 - 4) - (-6)(13 - 6)}{(-1)(11 - 4)}$

8. Simplify.
   a) $5(-4) - [3(-6) + (-3) - 4(2(-4) - 7)] + 3(-8)$
   b) $-2[-7 - 3(4) + 5 - 2(-1)] + 3(-6 + 8)$
   c) $4[-6(-2 - 7) - 5(7 + 2)]$
   d) $-7(-4) - 2[-3(-4 + 6) + 6(7 - 3(-4))] - 8(-4)$

9. a) Using all the digits in the year 1986, the operations $+$, $-$, $\times$, $\div$, and brackets, if required, form expressions for all the integers from 1 to 10.
   b) Repeat the process for the current year. Are there any integers which cannot be expressed in this way?

INVESTIGATE

- List 4 different integers, for example, -1, -5, $+8$, -3.
- From each integer in turn, subtract all the integers that precede it. That is,
  $(-5) - (-1) = -4$
  $(+8) - (-1) = +9$; $(+8) - (-5) = +13$
  $(-3) - (-1) = -2$; $(-3) - (-5) = +2$; $(-3) - (+8) = -11$
- Use a calculator to find the product of these differences.
  $(-4)(+9)(+13)(-2)(+2)(-11) = -20\,592$
  a) Is this result divisible by 12?
  b) Rewrite the 4 integers in a different order and repeat the procedure of subtraction and multiplication.
  c) Is the resulting integer divisible by 12?
  d) Repeat the procedures for each list of integers.
  i) -7, $+3$, $+2$, -1 ii) $+1$, -4, -2, -1 iii) $+5$, -3, $+4$, -5
  e) Which of these lists result in numbers that are divisible by 12?
  f) Choose 4 different integers and repeat the above procedures. What do you notice?



1. Arrange in order from least to greatest.
   a) $-5, 3, -4, -7, -9, 6$ b) $-6, 6, -5, -9, 9, -7$

2. Simplify.
   a) $(-5) + (+6)$ b) $(-5) + (-3)$ c) $(-6) + (+9)$ d) $(-1) + (+9)$
   e) $(-7) + (+11)$ f) $(+12) + (-13)$ g) $(-19) + (+11)$ h) $(-17) + (-14)$

3. Simplify.
   a) $(-4) + (+3) + (-5)$ b) $(+2) + (-7) + (-4)$
   c) $(-43) + (-17) + (+5)$ d) $(-39) + (+10) + (+31)$
   e) $(-91) + (+13) + (-26)$ f) $(-7) + (+104) + (-110)$

4. The greatest temperature variation in a single day in Alberta's chinook belt was from 17°C to −28°C. What was the change in temperature?

5. Simplify.
   a) $(+7) - (+4)$ b) $(-8) - (+2)$ c) $(-9) - (-2)$ d) $(+5) - (-2)$
   e) $(-7) - (-2)$ f) $(+8) - (-5)$ g) $(+6) - (-3)$ h) $(-5) - (+5)$

6. Write the integer represented by each square.
   a) $(+5) + \square = 0$ b) $(-13) + \square = 0$ c) $(+7) + \square = -1$
   d) $\square + (-5) = -2$ e) $(+4) + \square = 3$ f) $\square + (-3) = 2$
   g) $\square + (-6) = -4$ h) $\square + (+5) = -4$ i) $\square - (+4) = 2$
   j) $\square - (+3) = -5$ k) $(-5) - \square = 2$ l) $\square - (-9) = 1$

7. Simplify.
   a) $(-8) - (+3) + (-5) - (+7)$
   b) $(-14) - (+12) - (+3)$
   c) $(-103) - (+27) - (-100)$
   d) $(+283) - (-20) + (-60)$
   e) $(+70) - (+90) - (-100)$
   f) $(-100) + (-70) - (+20)$
   g) $(-30) - (-72) + (-43)$
   h) $(+981) - (-19) - (+891)$
   i) $(+45) + (-100) - (-10) - (-40)$
   j) $(-230) - (-300) - (-50) - (-40) + (+230)$

8. Simplify.
   a) $(-8 + 5) - (17 - 9)$
   b) $(-30 - 20) - (-20 - 30)$
   c) $(93 - 84) - (-67 + 89)$
   d) $(-9 - 17) - (11 - 27)$
   e) $(3 - 7) + (5 - 9) - (2 - 7)$
   f) $(284 - 180) - (-3 + 99) + (109 - 47)$
   g) $(33 - 21 + 24) - (47 + 12 - 29)$
   h) $(-74 - 18 - 21) + (-47 + 81 + 10)$

9. Simplify.
   a) $(-7)(+8)$ b) $(-6)(-9)$ c) $(+5)(-7)$ d) $(+3)(-9)$
   e) $(-12)(-6)$ f) $(+10)(+8)$ g) $(-14)(+3)$ h) $(-4)(+15)$
   i) $(-11)(-10)$ j) $(+12)(+12)$ k) $(-16)(-12)$ l) $(-150)(+3)$
   m) $(+13)(+13)$ n) $(-22)(+5)$ o) $(+15)(-15)$ p) $(+16)(-20)$

10. Simplify.
   a) $(-6)(+3)(-4)$ b) $(-8)(-2)(+7)$ c) $(+6)(+5)(+3)$ d) $(-8)(-2)(-3)$
   e) $(+5)(-3)(-4)$ f) $(-7)(-2)(-5)$ g) $(-9)(+9)(-9)$ h) $(+2)(-3)(+4)$

11. Simplify.
   a) $(-2)(-3)(-4)(-5)$ b) $(+8)(-2)(+6)(-3)$ c) $(-5)(+8)(-2)(-10)$
   d) $(+6)(+7)(-2)(0)$ e) $(-1)(+1)(+2)(+9)$ f) $(+5)(+5)(+4)(+4)$

12. Simplify.
   a) $(-16) \div (+4)$ b) $(-18) \div (-9)$ c) $(-48) \div (-16)$
   d) $(+81) \div (-3)$ e) $(-64) \div (-8)$ f) $(-54) \div (+18)$
   g) $(-108) \div (+36)$ h) $(+121) \div (-11)$ i) $(-144) \div (-9)$

13. Simplify.
   a) $\dfrac{(-8)(-12)}{(-24)(-4)}$ b) $\dfrac{(-5)(+39)}{(-13)(-3)}$ c) $\dfrac{(+121)(-7)}{(+77)(-11)}$ d) $\dfrac{(-42)(+6)}{(+14)(-9)}$
   e) $\dfrac{(+65)(-15)}{(+25)(+3)}$ f) $\dfrac{(+85)(+70)}{(-50)(-17)}$ g) $\dfrac{(-51)(-91)}{(-13)(-17)}$ h) $\dfrac{(+92)(+42)}{(+28)(-69)}$

14. Simplify.
   a) $\dfrac{(-40)}{5} + \dfrac{18}{(-6)}$ b) $\dfrac{(-42)}{6} - \dfrac{(-63)}{7}$ c) $\dfrac{42}{(-3)} + \dfrac{(-42)}{7}$
   d) $\dfrac{(-49)}{7} - \dfrac{26}{(-13)}$ e) $\dfrac{(-96)}{(-16)} - \dfrac{132}{12}$ f) $\dfrac{85}{17} + \dfrac{(-95)}{19}$

15. Simplify.
   a) $(-9)(+4) \div (-6)$ b) $(+8) \div (-4) + (-2)(-11)$
   c) $(+25) \div (-5)(-4) - (-3)$ d) $(-3) + (-15) \div (+5)(-6)$

16. Find the integer represented by each square.
   a) $(-4)(+5) \times \square = -220$ b) $(-2) \times \square \times (11) = 154$
   c) $189 = (-3)(7) \times \square$ d) $\square \times (-3)(-13) = -429$

17. A pilot is flying at an altitude of 5000 m where the temperature is $-21°C$. The nearby airport where he intends to land is at an altitude of 1000 m and the control tower reports precipitation. If the temperature increases 6.5°C for every 1000 m decrease in altitude, will the precipitation be rain or snow?

18. Simplify.
   a) $(-36) \div (-4) - (+6)(-5) \div (+3)$
   b) $-2[(-7)(-3) - (+5)(0) + (+2)(-1)]$
   c) $(+8)(-6) \div (+12) - (-3) \div (+1) + (-7)(+1)$
   d) $4[(-11)(-2) + (-6)(+3) - (+4)(+4)]$